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# The Development of Mathematics-Related Beliefs in Danish Upper Secondary School Students

Sif Ingibjörg Magnúsdóttir Skjoldager
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Roskilde University,

Department of Science, Systems and Models, IMFUFA

P.O. Box 260, DK - 4000 Roskilde

Tel: 4674 2263  Fax: 4674 3020

**The Development of Mathematics-Related Beliefs**

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Denne undersøgelse har til formål at afdække udviklingen i gymnasielevers forestillinger om matematik (beliefs) ved besvarelsen af følgende forskningsspørgsmål: #1: "Hvordan udvikler gymnasieelevers forestillinger om matematik sig gennem de tre år i gymnasiet?" og #2: "Hvilken indflydelse har gymnasieelevers forestillinger om matematik på matematikindholdet i deres foreløbige planer om videregående uddannelse?" De fire beliefs-aspekter: Matematik i Skolen, Matematik som Fagområde, Matematik i Samfundet samt Mig & Matematik samt deres indflydelse på valg eller fravalg af matematik-relaterede studier i elevernes foreløbige planer for videregående uddannelse blev undersøgt med udgangspunkt i et longitudinalt undersøgelsesdesign bestående af et spørgeskema (N=147) samt opfølgende interviews (1.år: N=24, 3. år: N=21) i henholdsvis 1.g. og 3.g. har det været muligt at følge udviklingen hos individuelle elever over tid. Undersøgelsen viser et fald i elevernes valg af matematikrelaterede uddannelser i deres foreløbige planer for videregående uddannelse og udpeger en sammenhæng mellem fravalg af matematikindhold og elevers oplevelser med manglende forståelse indenfor matematiske emner på A-niveau.

# The Development of Mathematics-Related Beliefs in Danish Upper Secondary School Students

*Ph.D. Dissertation*

*Author:* *Sif Ingibjörg Magnúsdóttir Skjoldager*

*Supervisor:* *Prof. Dr. Mogens Niss*

September 2014

IMFUFA ~ NSM ~ Roskilde University

## ABSTRACT

This study aims at characterising students' interpretations of their experiences in A-level mathematics, the highest level in Danish upper secondary school, by addressing these research questions: #1: "*How do the mathematics-related beliefs of students' develop during their three years in upper secondary school?*" and #2: "*How are the students' ideas for their choice of mathematics-related future study programmes influenced by their mathematics-related beliefs?*". The development of the four aspects of beliefs; *Mathematics at School*, *Mathematics as a Discipline*, *Mathematics in Society* and *Mathematics & Me,* and the influence from these aspects on the selection or deselection of mathematics related study programmes in students' preliminary plans for tertiary education for answering the research questions, were investigated by means of a longitudinal research design consisting of a questionnaire (N=147) complemented by follow-up interviews (1st year: N=24, 3rd year: N=21) in the 1st and the 3rd year of upper secondary school, respectively, it has been possible to trace the development in individual students over time. Among other things, the study shows a decline in the willingness to choose mathematics-related study programmes for tertiary education, especially as a result of experienced lack of understanding of the mathematics taught.

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# PROLOGUE

*In the last weekend of the summer of 2013, my friends Luna, Arno, Trond and Heine had planned a biking trip. They wanted me to join them, and I really wanted to, but I thought it would become too much for me if I spent the whole weekend biking, leaving no time for writing my thesis. However, it turned out they had reserved a shelter in a forest near me, and we agreed that I should join them for dinner at the bonfire in the evening after a whole day of writing.*

*When I arrived to the forest, the sky was still somewhat bright, but it was clear to me that it would soon change. I had not brought a map with me, but I had memorised how to get from the eastern gate to the shelter in the middle of the forest. Nevertheless, I ended up arriving from the western gate. Before entering the forest, I checked a map on a stand, not far from the gate. But when I entered the forest I was in doubt of whether I should turn left or right, since the angles of the path did not match what I had memorised from the map. I decided to try the right path first and see if it would lead me to the branching of the path which I expected to meet not far from the forest gate. I ended up walking quite far along that path before I was convinced that it could not be right. Now the sky was not so bright any more, but darkening quite fast. I hurried back to the forest gate and tried the left path. This was much better. Now I soon reached the branches I expected, but anyway, the next time I had to make a decision between two paths, I was in doubt whether I should choose the one with a name that seemed right, or the path with a symbol that seemed right. I chose the left path with the symbol that I thought would lead me to my friends. However, just to check, I called my friends, and even though my phone was to run out of battery soon, I got Heine, and he could tell me that it was the path with the right name that lead to the cottage. I went back along the wrong path and turned down the right one. Now Heine was biking ahead to find me and the forest had turned completely dark. The light from the bike was a very welcome sight when it appeared ahead of me behind the trees from the turning path. We could now*

*walk the rest of the route together and without that guidance I do not know how I should have found the path leading to the bonfire and the shelter. But there they were, my friends, the bonfire and the cottage. They had been singing and shouting for me to hear them and find the way. With all of us insisting and doing our best, we managed.*

*The story is true, and it ended happily. I decided to write it into my thesis, so as to remind me that even though it may feel like you turn down dead ends and the path you follow ends up becoming much longer and darker than you intended, with the help of people around you, and your own dedication, the end goal is still there and you will arrive at it. And this is the important part.*

*At the moment when I write this, I do not know when I will be done walking down dead ends, taking wrong turns or even how dark it will become before I arrive. But I will set my mind on the bonfire and the friends and the cottage and keep moving in the pace I am able to keep, continuing until I make it.*

11 September 2013/ Sif Skjoldager

This tale concerns one specific event that happened with a specific group of people at a specific point of time, but it may represent the help I have received and the trust I have met all along the path of carrying out and reporting my Ph.D. project.

## PREFACE

First of all, I wish to thank my supervisor, Prof. Dr. Mogens Niss. I am most grateful that you accepted me as your Ph.D. student, but even more grateful for the way you have believed in me all along the ups and downs of the path and through all the challenges I have met. Quite a few, actually, but you have helped me create and maintain a room for continuously progressing on the project no matter the circumstances around me. I have learned tremendously much from you already, but at the same time I also hope to continue learning from you further ahead from here.

Very warm thanks go to the upper secondary school teachers involved in my research project, for your help and advice concerning my empirical work. Your efficiency and support has enabled me to collect material of high quality for the project. Also very warm thanks to the students in the project classes who have volunteered for my empirical investigations and deep thanks to every student who filled out the questionnaires and thus enabled me to perform a longitudinal study. In order to maintain your anonymity, I cannot thank you by name in this context, but wish you to know that I am most grateful for your help.

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Colleagues from Aalborg, Bergen and other places whom I have learned to know through the NOMUS workshops; thank you so much for reminding me of the

joyful spirit of sharing ideas in the early phases of the research process, and for teaching me facets of the Norwegian and Danish languages I never imagined I would come to know.

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*Espergærde 25 April 2014, Sif Skjoldager*

# CHAPTER I: INTRODUCTION

During the years in compulsory school and upper secondary school, students encounter the school subject of mathematics with these settings. The experiences gained in school together with experiences from students' lives in other social contexts, provide sources for their interpretations of both the discipline of mathematics and themselves. This research project aims at characterising these interpretations, and the conclusions students draw from them, as well as the development of the interpretations over the three years of upper secondary school studies in mathematics.

As a former student I have sometimes wondered why I did not think of studying mathematics directly after upper secondary school, but only later in my career, since it was a subject I have enjoyed learning and which has provided me with meaningful perspectives for my personal and professional life. As a doctoral student in mathematics education research I have been given the opportunity to follow four upper secondary school classes in science study programmes during their three years in Danish upper secondary school, which has enabled me to investigate how students' interpretations of mathematics and their ideas about whether or not it could or should be part of their future life in society developed over time.

I have chosen to conduct the study with a conceptual framework from research on students' beliefs about mathematics. I will now provide a preliminary brief

overview of research within this field. A more thorough analytic review will be given in Chapter III.

## SIGNIFICANT RESEARCH

The concept of beliefs about mathematics, has received increasing attention in mathematics education research since Alan Schoenfeld in his book Mathematical Problem Solving (1985) concluded that cognition alone (resources, heuristics and control) did not suffice to explain students success or failure in problem solving. Also the student's beliefs about mathematics had an influence which had not been considered previously.

These findings worked as an inspiration for Douglas McLeod (1989, 1992, 2002) who took up researching the influence from the affective domain on students' problem solving activities. McLeod introduced the idea of linking research into the affective domain with cognitive psychology, more precisely what is known as "Mandler's Theory" (McLeod & Adams, 1989; McLeod, 1992): If a student experiences that her or his plan for a problem solving activity is failing this can be interpreted as a mental schema which is being disrupted, leading to an affective response in terms of emotions, attitudes and beliefs, emotions being less stable and less cognitive and beliefs being more stable and more cognitive, while attitudes are in between (McLeod, 1992). Hence, it is suggested that beliefs are part of a wider affectively oriented domain, but nevertheless, the more cognitive category of beliefs are in focus in this study.

Beside distinguishing between the holders of beliefs, another way of organising research on beliefs is by distinguishing between research dealing with either a) "*analysis and classification of beliefs*" (Goldin, Rösken & Törner, 2009) or b) "*monitoring changes in beliefs*" (ibid.). Also Goldin and his colleagues accentuate that

except for a few studies (Liljedahl, P., Rolka, K., & Rösken, B. 2007a), (Liljedahl, P., Rolka, K., & Rösken, B., 2007b) research dealing with beliefs mainly belongs to the former category.

## RESEARCH QUESTIONS

***RESEARCH QUESTION #1***

> *How do the mathematics-related beliefs of students' develop during their three years in upper secondary school?*

***RESEARCH QUESTION #2***

> *How are the students' ideas for their choice of mathematics-related future study programmes influenced by their mathematics-related beliefs?*

## SIGNIFICANCE OF THE RESEARCH QUESTIONS

This research project contributes to the research field by incorporating both social and societal aspects to belief systems of upper secondary school students. Further, researchers, e.g. (Goldin, Rösken & Törner, 2009), have advocated that more research on the change of beliefs is needed. This may be achieved by studying students' beliefs in longitudinal settings, thus complementing research characterising the nature of belief systems by important insights concerning the development of beliefs over time.

Mathematics teachers in upper secondary school could benefit from this research in the sense that it provides insight into students' interpretations of experiences in upper secondary school mathematics. To members of the *millieu* in which these types of experiences are gained, this study provides a ground for reflection on one's own practice.

For me, researching these questions enables me to go back and re-interpret my own beliefs and experiences and thus to deepen my own understanding of some influences on important decisions in my life.

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# CHAPTER II: METHODOLOGY

In this chapter I shall describe the methods chosen for answering the research questions of the study and discuss the consequences of these decisions in terms of the appropriateness of the design for answering the research questions. More specifically, I shall discuss the coverage of the methods, the possible redundancy stemming from the methods and explicate what is not covered by choosing this design.

## RESEARCH DESIGN

In order to provide grounds for answering the research questions, a research design has been developed. The decisions involved in the design of the study concerns the following areas:

I. Choosing conceptual framework
II. Choosing empirical methods, population, recruitment of informants, strategy as well as timing of the empirical work
III. Choosing and implementing a strategy for the analysis of the empirical material

The discussion and justification of the appropriateness of this design for enabling the answering of the research questions, is the aim of the following sections of this chapter. In some sense, it may appear as if everything is possible and that many possible strategies can be applied to arrive at different but appropriate types of answers. However, as the process of making decisions on these issues, some strategies seemed more promising than others for the quality of the intended

findings. Other decisions have been made from a normative stance, or even for reasons of convenience. I shall try to be explicit about these matters, when I guide the reader through the decisions concerning the design of the study.

I will first recall the two research questions to support the reader in assessing the details of the research design towards the goal of providing research based answers to them:

***RESEARCH QUESTION #1***

> *How do the mathematics-related beliefs of students' develop during their three years in upper secondary school?*

***RESEARCH QUESTION #2***

> *How are the students' ideas for their choice of mathematics-related future study programmes influenced by their mathematics-related beliefs?*

With these questions in mind, we can now move on to the attempt to setting up a research design to provide answers to them.

## I. CHOICE OF CONCEPTUAL FRAMEWORK:

As a first delineation of what is studied and in what way, I have decided to adopt a conceptual framework deriving from research on students' beliefs about mathematics. I choose to focus on four groups of belief objects; *Mathematics at School*, *Mathematics as a Discipline*, *Mathematics in Society* and *Mathematics and Me (*see Chapter III for further description). This is a conceptual specification from the initial 'views' or the more general ' interpretations'.

### *CONSEQUENCES OF THE CONCEPTUAL FRAMEWORK*

The conceptual framework is a set of lenses accentuating what is in focus in this study. A consequence of this framework is the potential body of research findings from this research area. Particularly, the metaphor of a belief system, and

the nature of this system, has resulted in a refinement of the methods for studying the development of it.

The focus is thus of a somewhat cognitive nature. Nevertheless, even with this focus, social aspects may be included in terms of their role as a basis for experiences of the individual. Experiences may be interpreted and contribute to the process of formation of beliefs.

Other sets of lenses would have drawn the attention to other aspects of the phenomena studied; Conceptualisation in terms of motivation would invite incorporation of behaviour, as suggested in Middleton & Spanias' first approximation to a definition of the concept: "*Simply stated, motivations are reasons individuals have for behaving in a given manner in a given situation*" (Middleton & Spanias, 1999). But since motivation for the sake of the research aim should have shed light on students ideas for further study, it would have been necessary to include an account of their actual choices of further education after upper secondary school graduation, which is not within the scope of the project.

A conceptual framework focusing on social aspects of the decision process have been opted out, due to my interest in the individuals' interpretations. Moreover, students' families and friends might come in as major influence on the actual decisions, but the aim of this research project is to point to the influence from mathematics education. This is not to indicate that social aspects are not of interest in mathematics education research, but this project has a different focus.

As I have now introduced the reader to my decision to apply a conceptual framework in terms of beliefs, I can now refine the statement of the research aim and specify that this specifically involves studying the development of beliefs about mathematics in the students over time, in particular with regard to the four aspects

of beliefs.

## II. EMPIRICAL METHODS

How can I, in an insightful way, provide empirical material for describing and characterising the development of students' beliefs about of mathematics? In order to provide answers to Research Question #1 (RQ1), it appears to be necessary to include a longitudinal element in the design.

This criterion will evidently be met by the empirical approach adopted for the study, namely a longitudinal study.

### *LONGITUDINAL STUDY*

A longitudinal study which enables the research to follow the development of something over time can be set up by means of different empirical methods. I have chosen to capture the longitudinal development in students beliefs by means of questionnaires combined with semi-structured qualitative research interviews (Kvale & Brinkmann, 2009).

The way the questionnaires are implemented in the study is somewhat different from what is usually connoted with this form of inquiry. Firstly, the questionnaires combine closed and open-ended items, secondly, the questionnaires have been used as a means for initiating a dialogue in the interviews, and, which will be elaborated further when discussing area III of decisions in the research design, Strategy for Analysis, they will be analysed both in terms of general trends in the population and for tracking the development in individuals.

The interviews are semi-structured because this enables the researcher to make permutations in the order of the research topics and thus be sensitive to the interviewee and what could be a natural flow of conversation. At the same time, the

interview guide worked as a tool for not unintendedly omitting something important. What has been discarded is then to do ethnographic fieldwork in terms of long-term stays in classrooms. This can be justified referring to the desired object for analysis, which is students' beliefs. In general, beliefs are not easily observed, but are better studied by different genres of interviews or questionnaires.

Also randomly chosen quantitative questionnaire surveys have been discarded. Random sampling large scale survey methods have the potential of providing statistical significance by means of statistical tests and generalisation from a representative sample to the entire population. These methods have been implemented in research on students' attitudes towards or beliefs about mathematics (see e.g. Fennema & Sherman, 1976; Kislenko, 2011). In a longitudinal design these methods might lend themselves to causal inferences due to the time order of the variables (Bryman, 2008). However, this kind of empirical method presupposes an operationalisation of pre-determined concepts, which has not been possible in this study, since the conceptual framework was not decided on beforehand but rather was developed as a refinement of some more vaguely defined terms from the outset of the research process, terms that nevertheless provided an initial orientation for the research.

### *CONSEQUENCES OF THE EMPIRICAL APPROACH*

Even though the implementation of the questionnaires does not provide a means for determining statistical significance, it does contribute to the study by giving indications of quantitative variation within a wider group of students sharing some of the same characteristics as the population from the interviews possesses. The interviews enable me to hear the students' perspectives on mathematics expressed in their own words whilst the feature of dialogue allow for continuous validation of the researcher's interpretation of the student's interpretations (Kvale,

2007). Applying these two empirical methods, questionnaires and interviews, in the longitudinal study has the potential to provide answers, not only to the first research question, but also to the second research question: Relating students' experiences, or at least their interpretations of their experiences, in mathematics teaching to their ideas for further study after upper secondary school is possible by means of questionnaires and interviews.

It has now been argued that a longitudinal study, drawing on the empirical methods of questionnaires and interviews for data generation, has the potential to provide answers to the first two research questions. Both methods have their strength in somehow covering the individuals' interpretation of experiences and thus also beliefs about mathematics derived from the mathematical activities encountered in mathematics teaching.

### *POPULATION AND RECRUITMENT*

The students participating in interviews and teaching experiments had all chosen mathematics at the highest possible level in upper secondary school (A-level) and were enrolled in science study programmes. The four classes, one from each of four different high schools (ALFA, BETA, GAMMA & DELTA), were selected to participate in the research project due to the support from the principals of those high schools. In each class six students were recruited for interviews.

It was emphasised that it was voluntary to participate. Each student received a letter from me explaining the purpose of the interview, how data would be treated and who would be responsible for the data. I repeated the content of the letter when I arrived for the interviews and the students could then tick off whether they agreed in participating or not. From those who agreed I chose a selection covering as wide a variety of answers as possible in a certain item in the questionnaire[1]. The proportion

1 Item #1, see chapter IV for details.

of each gender in the interviews represented the proportion in the class. In three of the classes it meant 3 of each gender but in one school (GAMMA), 2 female and four male students reflected the general proportion in the class better.

The schools represent both general (ALFA & BETA) and technical upper secondary schools (GAMMA & DELTA), situated geographically in both suburban and provincial areas. Further five classes from other schools were invited to answer the questionnaires. These were selected from a principle of ensuring adding more variation to the answers, and thus schools from rural areas were included. Even though the influence from socio-economic backgrounds of the parents has not been selected as a focus in this study, the regions in which the schools are situated geographically have been chosen to represent a great variety in socio-economic background in the parents according to data from 'Statistics Denmark'.

*TIMING*

The longitudinal study was carried out in stages spread over the three years of upper secondary school. Having a delay between the students answering the questionnaires and the follow-up interviews allowed for the interviewee to think back and interpret differences from answers given in the questionnaires and the circumstances leading to these answers and the process that had taken place since then.

| Longitudinal study | Data generation 1A | Data generation 1B | Data generation 3A | Data generation 3B |
|---|---|---|---|---|
| Method | 1st Year Questionnaire | 1st Year Interviews | 3rd Year Questionnaire | 3rd Year Interviews |
| Timing[2] | 1st year Ultimo November to primo January | 1st year March and April | 3rd year Ultimo November to primo February | 3rd year March |

Table 3 Sequence for longitudinal study

Furthermore, each stage of the data collection informed the design of the following stages. Further information about the data collection is given in Chapter IV.

## III. STRATEGY FOR ANALYSIS

The third element in the ways in which I go about to become able to answer the research questions, is the way I have chosen to analyse my material. The analysis consist of four parts:

1. Quantitative Analysis
2. Data Exploration
3. Case Analysis
4. Ideal Typical Beliefs

The first part, the Quantitative Analysis, provides information about the general trends with in the full population. The second part, Data Exploration, investigates types of trends in individuals from the fist to the third year. The third part, the Case Analysis, investigates the full range of empirical material on a selected group of case informants on whom both interview material and questionnaire answers are available for both the 1st and in the 3rd year. The case analysis provides

2 The school term lasts from August to May with examinations in June

an authentic background for investigating types of trends spotted in the quantitative analysis and in the data exploration. The case analysis provides an account of the four aspects of beliefs in the 1st year and in the 3rd year as well as influences on the students' plans in the future. Based on these, a characterisation of the transposition of beliefs from the first to the third year was possible. The fourth part, the Ideal Typical Beliefs, is an aggregate description, based on all previous analysis, of the typical beliefs of a typical A-level mathematics student in a science oriented study programme in Danish upper secondary school. This part is meant as a distillation of a summary of the previous beliefs analysis from the quantitative analysis, the data exploration and the case analysis.

### IDEAL TYPICAL BELIEFS

An *ideal type* is an analytical construct (Weber, 1949/1904) which allows the researcher to accentuate certain features of a phenomenon that one wishes to describe. It is a synthesis of a multitude of "c*oncrete individual phenomena*"(Weber, 1949/1904, p. 90), but it may "*not be found empirically anywhere*" (ibid.). In Weber's own words it is described like this:

> *"[E]r ist keine »Hypothese«, aber er will der Hypothesenbildung die Richtung weisen. Er ist nicht eine D a r s t e l l u n g des Wirklichen, aber er will der Darstellung eindeutige Ausdrucksmittel verleihen"* [3]*(Weber, 1988/1904, p. 190)*

A main point is that it is not to be counted as a description of reality, but rather a help for directing the formation of hypotheses. Weber continues;

> *„Es wird gewonnen durch einseitige S t e i g e r u n g e i n e s oder e i n i g e r Gesichtspunkte und durch Zusammenschluss einer fülle von diffus und diskret, hier mehr, dort weniger, stellenweise gar nicht, vorhandenen E i n z e l e rscheinungen, die sich jenen einseitlichen G e d a n k e n b ilde"* [4]*(Weber, 1988/1904, p. 191).*

3 "[I]t is no »hypothesis« but it offers guidance to the construction of hypotheses. It is not a *description* of reality, but it aims to give unambiguous means of expression to such a description" (Weber, 1949/1904, p.90)

4 "An ideal type is formed by the one-sided *accentuation* of one or more points of view by the

which explains that the material for the construction of an Ideal Type may be scattered, and the Ideal Type in itself may not exist in reality, but by accentuating a certain set of features of the phenomenon, a "unified analytical construct" may be found.

### *CONSEQUENCES OF THE STRATEGY FOR ANALYSIS*

Students' individual development as a guide for the analysis is essential for providing answers to the research questions. This approach is at the same time tightly connected to the conceptual framework, since this is setting the agenda for the analysis. Organising the notion of beliefs into four aspects suggest an analysis of each of these aspects as well as of the possible relationships between some of them (see chapter III).

### *THE PROCESS OF ANALYSIS & DATA GENERATION*

During the period of the research project, an ongoing process of interpretation, analysis and data generation has taken place, before entering the final stage of analysing the material. For the longitudinal study, each part of the data generation process has been informed by en interim analysis of previously generated data.

| Longitudinal study | Data generation 1A | Data generation 1B | Data generation 3A | Data generation 3B |
|---|---|---|---|---|
| Method | 1st year Questionnaire | 1st year Interviews | 3rd year Questionnaire | 3rd year Interviews |

Especially, responses to the questionnaires formed a basis for determining the questions to be included in the interview guide for the subsequent interviews.

For each of the four parts of the analysis, a more detailed account of the

synthesis of a great many diffuse, discrete, more or less present and occasionally absent *concrete individual* phenomena, which are arranged according to those one-sided emphasized viewpoints into a unified *analytical* construct (Gedankenbild)." (Weber, 1949/1904, p. 91).

strategy and the tactics adopted, is given in Chapter IV: ANALYSIS, after the conceptual framework of *Beliefs about Mathematics* has been introduced, and is discussed in the next Chapter: III. BELIEFS.

## SUMMARY

The elements of the research design of the study have been discussed in order to assess its appropriateness for providing answers to the research questions. The areas for decision were:

I. Choosing a conceptual framework

II. Choosing empirical methods, population, recruitment strategy as well as timing of the empirical work

III. Choosing and implementing a strategy for the analysis of the empirical material

Each decision gives rise to the need for a justification, which should cover these questions:

1. What is necessary for providing satisfactory answers to the research questions, covering an appropriate breadth of the research aim?

2. What is sufficient? Is it necessary to prioritise and make a selection?

I. For the first area, it has been argued that beliefs provide a sufficient coverage for answering both RQ1 and RQ2.

II. An empirical approach has been selected for the research design. The empirical approach is a longitudinal study, which can justifiably be expected to provide empirical data leading to findings concerning the development of beliefs over a longer period of time. An abundance of methods exist, and ethnographic fieldwork was discarded in favour of interviews and questionnaires.

Necessity and sufficiency have not been explicitly discussed for the choice of population, recruitment strategy or timing of the research.

III. The choice of strategy for analysis has been discussed in terms of the quality of potential answers to the research questions, and the longitudinal research approach seems to give rise to a necessary and sufficient contribution to answering both Research Question #1 and #2.

# CHAPTER III: BELIEFS

In this chapter I describe distinct features of the concept of *beliefs* and define what is meant in this study by the concept of students' mathematics-related beliefs.

## STUDENTS' MATHEMATICS-RELATED BELIEFS

An overview of research concerning mathematics-beliefs may be organised with respect to the holders of the beliefs; teachers, students and specific sub groups of these groups (Goldin, Rösken & Törner, 2009). Research on teachers' beliefs and students' beliefs has been fuelled by different research agendas, but these have not developed independently of each other. Research on teachers' beliefs has its roots in the attempt to understand teaching from a teacher's perspective (Thompson, 1992), whereas research on students' beliefs derived from research on problem-solving activities in mathematics education. However, both Thompson (1992) and Philipp (2007) in their chapters in NCTM's Handbooks of Research on Mathematics Teaching and Learning (first and second, respectively) supplement their reviews of research on teachers' beliefs with research on students' beliefs. This goes both ways; also McLeod (1992) includes references to research on teachers' belief in his chapter on affect in mathematics education research, which otherwise primarily focuses on students. It seems that these areas of research have the potential for enriching each other with perspectives of common usefulness. Nevertheless, the distinction may in some aspects be of importance and thus should not be ignored.

# CONCEPTUAL CLARIFICATION

Before discussing the definition(s) of beliefs, I will outline distinctions between the concept of beliefs and some of its neighbouring concepts; *knowledge*, *values* and *conceptions*.

## *BELIEFS AND KNOWLEDGE*

Both knowledge and beliefs are seen as mental constructs of the individual, based on experiences, and both can be seen as organised in clusters around situations or concepts. Some researchers view beliefs as subjective knowledge (Furinghetti & Pehkonen, 2002). Not all researchers find it necessary to distinguish between knowledge and beliefs, but two important differences are often emphasised: One distinction is that beliefs tend to be held "*with varying degrees of conviction*"(Philipp, 2007, p. 259); one can believe something strongly, but it is not commonly said about knowledge that you know something strongly. But, maybe more precisely, knowledge requires a truth condition whereas beliefs do not (Scheffler, 1965). In line with the latter, I will adapt a distinction proposed by Richardson, according to which knowledge is characterised as "*a set of warranted propositions held by a community of experts*" (2003, p. 3f). Accordingly, beliefs are "*propositions that are accepted as true by the individual holding the belief, but they do not require epistemic warrant*" (ibid).

## BELIEF SYSTEMS

Much like knowledge can be thought of as a cognitive structure in a particular conceptual domain, beliefs can be conceived as organised in a *belief system*:

> *"The* belief system *is conceived to represent all the beliefs, sets, expectancies, or hypotheses, conscious and unconscious, that a person at a given time accepts as true of the world" (Rokeach, 1960, p. 33)*

This metaphor permits the researcher to study the organisations of the

individual's beliefs.

The way beliefs are related to each other in a belief system is characterised by Green (1971) as three dimensions of the system. These dimensions concern only how the beliefs are related to each other and not to the content of the beliefs.

> *"We may, therefore, identify three dimensions of belief systems. First there is the quasi-logical relations between the beliefs. They are primary or derivative. Secondly, there are relations between beliefs having to do with their spatial order or their psychological strength. They are central or peripheral. But there is a third dimension. Beliefs are held in clusters, as it were, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs. Each of these characteristics of belief systems has to do not with the content but with the way we hold them. "(Green, 1971, p. 47)*

The first dimension he describes is the dependency of a belief on other beliefs. Beliefs are not isolated from other beliefs, and some beliefs can be *derivative* of other, *primary*, beliefs.

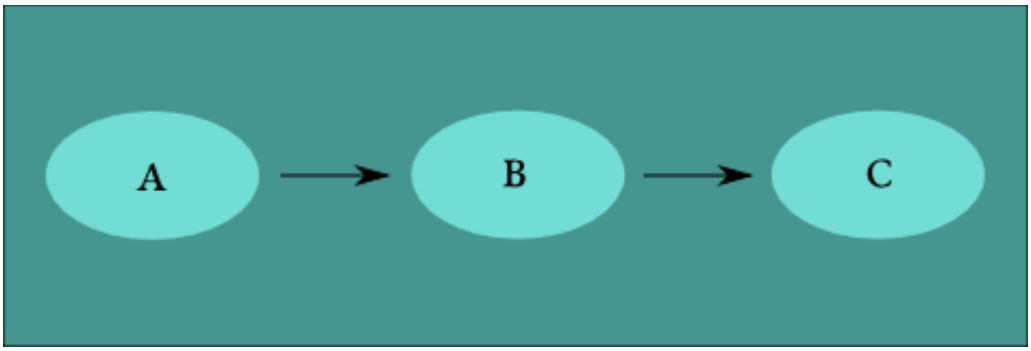



*Illustration 1: Derivative Beliefs: A belief A may be seen as primary to another belief B, if the holder of the beliefs sees A as a reason for B. B is then a derivative belief of A. But B may be seen as a reason for a third belief C, which is then a derivative of B.*

As an example, a primary belief could be that '*school mathematics is not applicable outside school*', and a derivative believe could be that '*studying mathematics in school is something you do to be granted admission to further education*'. Green suggests this dependancy between beliefs to have a structure that resembles a logical structure:

> *"...I suggest simply that belief systems have a kind of logical structure. Some belief are related to others in the way that reasons are related to conclusions." (Green, 1971, p. 44ff)*

But, as opposed to a knowledge system, the organisation of beliefs does not necessarily follow a profound logical structure, but can instead be referred to as a quasi-logical structure:

> *"The quasi-logical relation between beliefs that makes some of them derivative and some primary is not the fixed or stable sort of order that logicians establish between propositions. The actual logical order of beliefs is based upon their content and structure. It has to do with* **what** *is believed. But the order I am concerned to describe has to do with the order beliefs receive in somebody's belief system. It has to do with* **how** *they are believed. Thus, the structure of a belief system will not be defined by the actual logical relations between propositions, but by the quasi-logical order they receive in a belief system."(Green, 1971, p. 45)*

Secondly, Green describes the degrees of conviction, or psychological strength, with which beliefs can be held: *Central* beliefs are held more strongly than *peripheral* beliefs, and the latter are thus more susceptible to change.

> *"It seems to be a fact that some beliefs are more important to people than others. And the same belief may be more important to some believers than to others."(Green, 1971, p. 46)*

If the belief '*studying mathematics in school is something you do to be granted admission to further education'* is more central to the student than '*school mathematics is not applicable outside school*' the latter may be challenged or even changed without changing the former. This implies that these two first dimensions are independent of each other: A peripheral belief (held less strongly) can be primary and a central belief (held strongly) can be a derivative of an other, primary belief.

With the third dimension, Green notes that beliefs can be held in clusters, allowing sets of beliefs to be protected from comparison or cross-fertilisation with sets of conflicting beliefs (Green, 1971).

> *"It is possible to hold conflicting sets of beliefs as psychologically central because we tend to order our beliefs in little clusters encrusted about, as it were, with a protective shield that prevents any cross-fertilization among them or any confrontation between them"(Green, 1971, p. 47)*

My summary of Green's (1971) three dimensions for the organisation of beliefs in a belief system:

1. Dependency (Primary or derivative beliefs)
2. Degree of conviction (Central or peripheral beliefs)
3. Clustering (Sets of beliefs can be more or less protected from conflicting sets of beliefs)

These distinctions are also recognised by Op t'Eynde, de Corte and Verschaffel describing them in the following way (2002, p. 26): "[A] *belief system has a quasi-logical structure, where a knowledge system has a logical structure*". Moreover, they elaborate on this, trying to capture the underlying differences: "*[T]he equilibrium a belief system is trying to achieve is psychological in nature. The underlying rationale are the needs, desires and goals of the self*". These differences in rationales may be useful for interpreting the differences between knowledge systems and belief systems.

### *BELIEFS AND CONCEPTIONS*

Another term associated with 'beliefs' is the notion 'conception'. Anna Sfard defines the *conception* as the subjective counterpart of an 'official' mathematical *concept* (Sfard, 1991):

> *"[T]he word 'concept' (...) will be mentioned whenever a mathematical idea is concerned in its "official" form - as a theoretical construct within "the formal universe of ideal knowledge"; the whole cluster of internal representations and associations evoked by the concept – the concept's counterpart in the internal, subjective "universe of human knowing" - will be referred to as a "conception".*

The definition implies that 'conception' is closely connected to a 'concept' and that it represents the individual's subjective knowing about it. Also Alba Thompson

describes the notion 'conception' in connection to concepts (among other things). Teachers conceptions are seen as (Thompson, 1992, p. 130): *"[A] more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like"*. It seems that conceptions are mentioned as associated with concepts in some sense, which is in accordance with Sfard (1991), but it is also stated that the term 'conception' goes beyond 'concepts' and is even viewed as a more general category than beliefs; beliefs are seen as a subset of conceptions. It should be mentioned that Thompson refers to teachers' conceptions, but at this point of the analysis of concepts, this distinction is not of critical importance. Opposite Thompson and Sfard, Erkki Pehkonen uses the term 'conception' to refer to a subset of beliefs, namely the cognitive part, rather than its affective sides (Pehkonen, Ahtee, Tikkanen & Laine (2011)):

> *"... conceptions are explained as conscious beliefs. In the case of conceptions, we understand that the cognitive component of beliefs is stressed, whereas in basic (primitive) beliefs the affective component is emphasized."*

What seems to be explained here is a distinction between the *cognitive* component of beliefs, and the *affective* component of beliefs. The cognitive component is by Pehkonen termed *conception*, whereas the affective component is termed *basic* or *primitive* beliefs. Hence, we observe that there is no general agreement in the literature about the issue of whether beliefs should be perceived as a subset of conceptions or conceptions should be interpreted as a subset of beliefs. Nevertheless, for this study, the definition of conceptions provided by Sfard is adopted.

### *BELIEFS AND VALUES*

Values differ from beliefs in at least two different ways. Values are described as context-free and thus not tied to any object. If one values beauty, it will be desirable in any context, whereas if one believes that mathematics is beautiful it

would not necessarily apply to other objects as well. A second difference between beliefs and values is that beliefs can be correct or incorrect, whereas values refer to the dichotomy "desirable or undesirable" (Bishop, Seah & Chin, 2003; Rokeach, 1968). In terms of beauty as a value then beauty is seen as desirable and worth striving towards achieving. The belief that mathematics is being beautiful is something which would reflect a true/false judgement.

> *I consider value to be a type of belief, centrally located within one's total belief system, about how one ought or ought not to behave, or about some end-state of existence worth or not worth attaining. Values are thus abstract ideals, positive or negative, not tied to any specific attitude object or situation, representing a persons beliefs about ideal modes of conduct and ideal terminal goals" (Rokeach, 1968, p. 124).*

In Rokeach's terminology, value is a type of belief with some special features; they are not tied to a specific object, representing ideals in terms of modes of behaviour or end goals worth striving for. Distinguishing between values and beliefs then relates to the two criteria that *values are transcendental*, or context-free and that *values represent ideals of what is desirable or undesirable.* These two criteria will be applied to guide the discussion in this study.

## BELIEFS DEFINITIONS IN LITERATURE

Even though beliefs seem to have great importance in students' learning of mathematics, defining the concept of beliefs is not a trivial task. This phenomenon has been the subject of debate for decades, and still is (Pajares, 1992; Törner, 2000; Op't Eynde, de Corte & Verschaffel, 2002; Leder & Forgasz, 2006; Goldin, Rösken & Törner, 2009; Rösken, Pepin & Toerner, 2011; Felbrich, Kaiser & Schmotz, 2012). As a point of departure, I will now turn to some of the main references for beliefs definitions and characterisations, starting with Alan Schoenfeld. In 1985 Schoenfeld explains the meaning of the concept of belief in the following way (Schoenfeld, 1985, p.45):

> *Belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. Beliefs establish the context within which resources, heuristics, and control operate."*

This definition is especially developed for the context of mathematical problem solving in school. However, since the aim of my study is to study students beliefs around some wider domains rather than problem solving situations particularly, the definition may not apply completely to my approach. Nevertheless, I will still study the process of students' development of beliefs about mathematics, namely in terms of connecting experiences with mathematics in school contexts, with the students' ideas of the role of mathematics in their future life in society. In his chapter "Mathematics Teachers' Beliefs and Affect" (in Lester, 2007), Randolph A. Philipp attempts to "*distil meanings that capture distinctions that emerge in usage by researchers..."(*Philipp, 2007, p.258). To this end, Philipp provides the reader with a "*Definition/description*" of beliefs:

> [Belief are]*"(...) Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt lass intensely and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitude (…)"(ibid, p. 259)*

As formulated above, this definition/description express beliefs as a general construct, not necessarily related to mathematics education. It reflects some of the same aspects as McLeod introduced in his 1992 chapter in (Grouws, 1992) by connecting beliefs to attitudes and emotions and by distinguishing between different levels of intensity and degrees of being more or less cognitive. Still, it is addressed that the definition/description is not intended to stand alone as a definition:

> *"...These definitions/descriptions are not intended to stand alone as definitions, but instead I provide them to support the reader in drawing distinctions among commonly*

*used meanings of terms..."(Philipp, 2007, p.258)*

Nevertheless, Philipp also quotes Richardson who gives a definition of beliefs, which is presented by Richardson as a generally accepted definition in the field:

> *"There is considerable agreement on the definition of beliefs as psychologically held understandings, premises or propositions about the world that are felt to be true"*
> (Richardson, 1996, p. 2)

The definition by Richardson is the point of departure for my definition of beliefs. However, in the context of the present study the very broad category *the world* will be replaced by *mathematics*, but in the broadest possible sense, which will be specified when presenting the definition.

### *SOCIAL & SOCIETAL CONTEXTS OF BELIEFS*

There is no rule limiting research on students' beliefs about mathematics from taking into account a wide range of *social* and *societal* contexts. To clarify the discussion, I will here distinguish between *social* context, referring to the specific contexts the student participate in, and *societal* context referring to society at large as a context. Nevertheless, traditionally the context around students' beliefs about mathematics which has been considered, has involved the discipline of mathematics, its teaching and learning (Thompson, 1992) and sometimes the social context in class, possibly around problem solving activities, rather than any societal context. The social context in class plays a central role in the definition of students' "mathematics-related" beliefs suggested by Peter Op t'Eynde, Erik de Corte and Lieven Verschaffel (2002, p. 27):

> *Students' mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context. These beliefs determine in close interaction with each other and with students' prior knowledge their mathematical learning and problem solving in class.*

This definition is developed from an analysis of the nature and structure of beliefs and belief systems indicating that (Op t'Eynde et al., 2002, p.27): "*...the social context, the self, and the object in the world that the beliefs relate to, are constitutive for the development and the functioning of these systems*". But even though it is the social context in class which is emphasised in this definition, their discussion of the role of social context opens for a wider interpretation of the term (ibid., p. 22): "*Beliefs are the product of social life (de Abreu, Bishop, & Pompeu, 1997). They are determined by the socio-cultural environment one lives and works in.*" This of course includes the classroom, since it is the main scene in which the students experience mathematics, but Op t'Eynde et al. continue by referring to (Pehkonen & Törner, 1996) and (Underhill, 1988) (Op t'Eynde et al., 2002, p. 22):

> *"As a member of different social contexts (i.e., family, peers) students are subjected to a very complex and diverse network of influences, that determines the "unique" way in which they find themselves and look at the classroom context."*

So even though they emphasise the social context of the class as one of the their three constituting elements of students' beliefs, they seem to acknowledge the influence from a wider range of social and societal contexts in which the student participates or has participated, to some extent explaining differences in students' beliefs even though they take part in the same social context in the mathematics class. Op t'Eynde et al. are not alone in finding it important to consider the influence from a wider social context on students' beliefs. Also McLeod & McLeod (2002, p. 116) plead for taking into account a wider view of the social context. They turn to Evans (2000) for inspiration, and they write:

> *"We should broaden the notion of "context" to include the two fundamental meanings of the term suggested by Wedege*[5] *(1999):* task-context *(the linguistic features or wording of the task and the assumption a student must make in order to solve the problem mathematically) and* situation-context *(the social, historical, psychological and other circumstances in which problem-solving and learning takes place." (McLeod &*

5 My emphasis. Last name of author corrected to 'Wedege'

*McLeod, 2002, p. 115)*

By introducing Tine Wedege's distinction between *task-context* and *situation-context*, (Wedege, 1999), an invitation to take into account what I refer to as the societal context is formed. However, what Wedege refers to as situation-context includes both what I call *social* context, which is the actual contexts the student participate in, e.g. during a problem-solving activity, and the *societal* context referring to society at large as a context (Wedege, 1999).

Outside beliefs research, societal influence on students' learning has long been of interest. Stieg Mellin-Olsen uses the notions of *social* and *instrumental* rationales for learning to capture some of these influences (Mellin-Olsen, 1981; 1987, p. 157). On the instrumental rationale for learning, Mellin-Olsen writes (1987, p. 157):

> *"It is the rationale which is related to school's influence on the future of the pupil, by the formal qualifications it can contribute. This role as an instrument for the pupil will provide the pupil with an instrumental rationale (I-rationale). In its purest form the I-rationale will tell the pupil that he has to learn, because it will pay out in terms of marks, exams, certificates and so forth."*

Thus, under the *instrumental rationale* (the I-rationale), learning is motivated through its usefulness for the student; as a *means* for passing exams and getting admission to what he or she finds important for his or her future life in society. As such, it is not the value of the knowledge or the skills in themselves, but purely its utility for gaining formal qualifications that counts. The social rationale for learning, however, relates to the value of the knowledge learned (Mellin-Olsen,1987, p. 157-158):

> *"On the other hand there is a rationale which relates to knowledge as such, saying that "this knowledge has a value besides its importance for the external examination." It is a rationale which says that knowledge has an importance beyond its status as school knowledge. I call this rationale the S-rationale. The S stands for "social", indicating that such an evaluation of knowledge is made by the individual as a social subject"*

Hence, under the *social rationale for learning* (the S-rationale), the knowledge itself is valued by the student, and the learning is not merely motivated by its instrumental qualities. Also Ole Skovsmose has contributed to conceptualising societal influence on students' learning. Skovsmose developed the notion of *students' foregrounds* to address this issue (2005; 2011) and together with his colleagues Helle Alrø and Paola Valero he has explored the influence of *foreground* on students' interpretation of the relevance of learning mathematics (Alrø, Skovsmose & Valero, 2009).

## BELIEF OBJECTS

In this section, I will first introduce a discussion of belief objects versus constituting elements of beliefs. Then I will proceed to presenting a range of possible belief objects related to mathematics.

### *CATEGORIES OF BELIEF OBJECTS*

In the review of belief definitions, it became obvious that not all belief definitions referred to objects related to mathematics. In the following, however, the focus will be on belief objects that are related to mathematics in some sense. As mentioned in the previous section, Op t'Eynde and his colleagues organise mathematics-related belief-objects in three main categories: Beliefs about the *classroom context*, beliefs about the *self* and belief about *mathematics education* (Op t'Eynde, 2002, p. 27), each with a set of sub-categories (ibid, p. 28). Conjointly the categories of belief objects developed by Op t'Eynde and his colleagues' (Op t'Eynde et al, 2002, p. 28) cover a field of belief objects comparable, but not necessarily equal to the categories of belief objects developed by other researchers in the field; Underhill (1988), McLeod (1992), Kloostermann (1996) and Pehkonen (1995) each have their way of organising belief objects into categories (ibid, p. 19). For a further discussion of differences and similarities between these categorisations, see (Op

t'Eynde, 2002, p. 20). Uffe T. Jankvist added a fourth dimension to the three main categories from Op t'Eynde et al., namely *mathematics as a discipline*, since this category is not fully covered as a subset of mathematics education (Jankvist, 2009). This idea has also been adopted in my definition.

## CHARACTERISATION OF DEFINITIONS

The differences in the definitions of beliefs might have their root in the different purposes for the definition and its use, or as a consequence of addressing different audiences. D. B. McLeod & S. H. McLeod suggest distinguishing between *informal, formal* and *extended definitions* of beliefs: *the informal definitions* of beliefs are those that are intended for a general audience, and that can be characterised as mere "rules of thumb". The *formal definition* should encompass (McLeod & McLeod, 2002, p. 118): "*the term to be defined, the class of objects or concepts to which it belongs and the distinguishing characteristics that separate it from all other objects or concepts*".

This kind of definition is said to be intended for a broad, but more sophisticated audience, and finally for a specialist audience the third category of definitions is the *extended definition*:

> *"[T]hese start with a formal definition, but continue in more technical language to include more complete characteristics and instantiations of the term." (ibid., p. 118)*

An extended definition has been proposed by Günter Törner (2002). This definition almost has the character of being a meta-definition providing criteria for any definitions of beliefs. I will give a description of Törner's definition, since some of the distinctions he offers are applied in the discussion of my own definition of students' beliefs about mathematics.

### *TÖRNER'S FOUR-COMPONENT-DEFINITION:*

Drawing on features of fuzzy sets, Törner suggests a "*four-component-*

*definition*" of beliefs (Törner, 2002, p. 82):

> *"A belief **B** constitutes itself by a quadruple* $\mathbf{B} = \{\mathbf{O}, \mathbf{C}_O, \mu_i, \varepsilon_j\}$ *where* **O** *is the debatable belief object,* $\mathbf{C}_o$ *is the content set of mental associations,* $\mu_i$ *the membership degree function, and* $\varepsilon_j$ *the evaluation map."*

Moreover, Törner emphasises two variables that he thinks should be taken into account: the person **P** "who has *professed* the beliefs or to whom the belief is *attributed*" and also the time **t** of constitution (Törner, 2002, p. 82). I will now explain the meaning of the elements more thoroughly.

### BELIEF OBJECTS O

As mentioned earlier, beliefs are thought of as concerning something, which is called belief objects. When discussing beliefs concerning mathematics Törner suggests that (Törner, 2002, p. 78): "*anything that shares a direct or indirect connection to mathematics can function as a belief object*". He continues by explaining (ibid.): "*Some belief objects are abstract, for example the nature of mathematics (…) others are more concrete (e.g. the theorem of Pythagoras)*". And thus also everything (related to mathematics) in between. Törner gives a list of examples (Törner, 2002, p. 78):

> *"Subject-specific mathematical facts (mathematical objects): Division, definition of a square, the number Pi, angles, continuity, tangent, function; Domains within mathematics: Geometry, algebra, calculus; Mathematics as a whole; Mathematics as a discipline (School mathematics, university mathematics, industrial mathematics, mathematics within society, symbolism and mathematics); Relations where mathematics or a sub-unit of mathematics (...) is a substantial part: Mathematics and application, mathematics and history, usefulness of mathematics, the role of definitions, the role of proof; Relations where mathematics as well as the individual is a substantial part: Self-concept as a learner of mathematics (Pajares & Miller, 1994) or personal anxiety and mathematics; The learning of mathematics itself"*

Since the belief objects have different sizes, Törner suggests addressing this as "*the breadth*" of the belief object.

### THE CONTENT SET $C_O$

Beliefs are in Törner's terms the "*mental constructs of the individual*" (p.78.). This way of approaching beliefs derives from Schoenfeld, or as Törner (2002, p. 78) puts it:

> *"Accepting Schoenfeld's definition one needs to accept that "mental constructs" may include individual statements, suppositions, commitment and ideologies, and also attitudes, stances, comprehensive episodical knowledge, rumors, perceptions and finally even pictures"*

More precisely he refers to the "*content set* $\mathbf{C}_O$ of a belief related to the object **O**" as the "*range*" of "*mental associations*". Moreover, he describes the content set as "*highly open*" (ibid.).

### THE MEMBERSHIP DEGREE FUNCTION $\mu_i$

The third feature Törner ascribes the content set $\mathbf{C}_O$ stems from the theory of fuzzy sets. In a fuzzy set, each element does not necessarily have the same "*value*"; rather different membership degrees are allowed (Zimmermann, 1990). This idea is adapted by assigning a *membership degree function* $\boldsymbol{\mu}_i$ to each element in the set of beliefs $\mathbf{C}_O$ for which the value vary from zero to one. The interval can be reduced to some discrete values, often in terms of linguistic items (Törner, 2002, p. 80). As an example, Törner mentions Green's categories: Beliefs can be characterised as either *central* or *peripheral*, depending on the psychological strength with which they are held, and also that they can be seen as *primary* or *derivative*, from a quasi-logical perspective (Green, 1971; see also the section on belief systems, p. 28, in this chapter). Since these features are independent of each other, they can each be modelled by a membership degree function, and the number of membership degree functions related to a belief is not restricted (Törner, 2002, p. 81). Each membership degree function will then be associated with a number *i*. Determining specific values for these functions, however, may not be possible.

### EVALUATION MAPS $\varepsilon_j$

The fourth part of the quadruple in Törner's extended definition of beliefs contains the *evaluations maps*. This part of the belief definition concerns the evaluative part of a belief. It can be *responses* in terms of *"favour or disfavour, liking or disliking, approach or avoidance, attraction or aversion(...)"* (Törner, 2002, p. 81). These reactions are modelled through evaluation maps $\varepsilon_j$ which are defined for "*the range of a belief* $\boldsymbol{C_O}$". As suggested above, the evaluation maps have a linguistic value scale, which can be bipolar, appearing as evaluative reactions.

### TÖRNER'S DEFINITION

These four components, the *belief object* $\boldsymbol{O}$, the *content set of the beliefs* $\boldsymbol{C_O}$ , the *membership degree functions* $\mu_i$ and the *evaluative maps* $\varepsilon_j$ form, in addition to the *person* **P** to which they are attributed and the *time* **t** of constitution, Törner's suggestion for a definition of beliefs. Moreover, he adds some characteristics that the belief should fulfil "*in a probabilistic sense*". This part of his definition will not be referred to here. Finally, Törner introduces a discussion of a subject-specific clustering of beliefs in a beliefs-system.

### LEVELS OF BELIEFS

With the term "*subjects-matter beliefs*", Törner refers to beliefs of a mathematical object or a mathematical procedure (Törner, 2002, p. 86). *Domain-specific beliefs* are those that have a mathematical domain as an object. It could be algebra, calculus or geometry (Törner, 2002, p. 87). Very general beliefs are called g*lobal beliefs*. They have belief objects of considerable breadth. In this category Törner mentions belief about the nature of mathematics, beliefs about mathematics teaching or beliefs about mathematics learning. It is then questioned by Törner if anything could be said of whether global beliefs are primary to subject-matter beliefs or vice versa.

## MY DEFINITION OF BELIEFS

In this section I will define and justify what is to be understood by the concept of S*tudents' Mathematics-Related Beliefs* in this study. The definition is accompanied by an argument for organising the concept into four aspects of students' beliefs about mathematics. The definition and the four aspects will constitute the main instrument for the analysis of the empirical material in this study.

### *WHY DEVELOP MY OWN DEFINITION?*

As discussed earlier in this chapter (see pages 33ff.) defining beliefs about mathematics is not a trivial task and in general researchers tend to develop their own definition rather than using existing definitions developed by others. I will therefore justify why I need my own definition for this study. The two main reasons why I develop my own definition relate to

a) What should be *covered* by the definition, and
b) What should be *emphasised* in the definition.

In my study I intend to *emphasise* the *discipline of Mathematics* beyond the part of mathematics that the student encounter in a school context. Also, I aim at *covering* societal influences, in addition to contextual influences, which include the students' foregrounds (Skovsmose; 2005, 2011) thus having a wider contextual focus. The idea of this emphasis is to allow for a very broad interpretation of mathematics, which is conducive to the manner in which the beliefs are organised into the four aspects representing this idea. Beside emphasising these elements related to my focus, my definition will be built on material already developed and present in beliefs research, but appropriated for my specific research aim.

### *WHAT ARE STUDENTS' MATHEMATICS-RELATED BELIEFS?*

I state my definition of *students' mathematics-related beliefs* in the following way:

> *Students' mathematics-related beliefs are relatively stable psychologically held understandings about mathematics in all its aspects and relations that are thought to be true.*

*Mathematics in all its aspects and relations* includes its *products*, *processes*, *epistemology* as well as its *relationships with the surrounding world, including the self which holds the belief*. In the following sections I will clarify how I describe the nature of beliefs and further on describe the sub-organisation of beliefs about mathematics in all its aspect into four main groups appropriate for my study focus.

### *HOW ARE BELIEFS DEVELOPED, MAINTAINED OR CHANGED?*

Students' beliefs about mathematics are continuously developed and maintained through participation in mathematical activities in, and sometimes out of, school. Student's beliefs about mathematics are relatively stable, but not irresistible towards change; some experiences might have have the impact necessary to render possible a change in beliefs (Green, 1971). Two important questions relate to the development and change of beliefs: Firstly, what mechanisms are responsible for the development and change of beliefs, and secondly, would it to some extent be possible to influence these mechanisms? And if so, how could it be useful to teachers to be aware of these things? From literature, we have noticed the suggestion of the self, the social context and the belief objects as constituting elements of students' beliefs (Op t'Eynde et al, 2002), and thus we have a point of departure in our search for candidates for explanations of changes in beliefs.

### *BELIEF OBJECTS AND CONSTITUTING ELEMENTS*

Apart from the *context*, also the *self* is mentioned by Op t'Eynde and his colleagues as a *constituting element* of students' mathematics-related beliefs. In addition to the *context* and the *self*, we have the *belief object*, which these authors conceptualise as *mathematics education* (Op t'Eynde et al., 2002, p. 27). It should be

understood in the sense that (ibid.)

> *"[S]tudents' beliefs about mathematics education are situated in and determined by, the context in which they participate as well as by their individual psychological needs, desires and goals etc."*

This may suggest that both individual factors and contextual factors play a role in the formation of beliefs. Thus, the beliefs are seen as being *situated* in the *social context* and in the *self* and its personal needs, desires and goals. Furthermore, the context and the needs, desires and goals '*determine*' the students' *mathematics-related beliefs*. Moreover, students' mathematics-related beliefs are *constituted* by their "*beliefs* about the classroom context, *beliefs* about the self and *beliefs* about mathematics education" (Op t'Eynde, 2002, p. 27).

It may be complicated to distinguish among whether mathematics-related beliefs are *situated in*, *determined by* or *constituted by* the social context and the self, but the interplay between the three components seems important.

It may be understood in the sense that on the one hand the self and the context determines the beliefs about the object (mathematics education), but on the other hand, the context and the self work as objects themselves.

In this study, the main focus will be on the context and the self as belief *objects* given that they somehow relate to the broad interpretation of mathematics in the study. However, I shall be aware of the possible other features of the context and the self.

### *WHERE AND WHEN DO BELIEFS EXIST?*

In the context of the current study, a student is a young person (typically 15-19 years of age) enrolled in upper secondary education in a Danish 'Gymnasium' or Technical Gymnasium (years 10-12). Students' beliefs about mathematics should

always relate to an object, which can be broad, like mathematics as large, or narrow, like a single mathematical concept.

### *WHAT DO THEY GOVERN?*

Student's beliefs about mathematics are considered to guide, but not determine, their ideas of the role of mathematics in their future life in society. Students' beliefs guide, but do not determine their ideas of what possibilities they have in society in future life in relationship with success, failure or hard work in mathematics and its role in various aspects of life in society.

### *FOUR ASPECTS OF STUDENTS' BELIEFS*

Every aspect of mathematics can work as a belief object (Törner, 2002). And *Mathematics in all its aspects* as formulated in my definition (see p. 43) is defined to include its *products, processes, epistemology* as well as its *relationships with the surrounding world, including the self which holds the belief*.

To organise *students' mathematics-related beliefs* in this study, I have chosen to categorise *mathematics in all its aspects and relations* into four main categories of belief objects. As a whole, I denote these four main categories of belief objects as *aspects of students' mathematics-related beliefs*. They comprise:

1. *Mathematics at School,* including its teaching, learning, assessment, organisation and curriculum,
2. *Mathematics as a Discipline*; including mathematics as a pure science as an applied science.
3. *Mathematics in Society;* mathematics as a tool in professions, trade, technology and societal institutions, and as a system of tools for societal practices.
4. *Mathematics and Me*; concerning oneself as a learner and doer of mathematics including confidence, perceived interest, its role in one's life now and in the future life in society.

This way of categorising student's beliefs about mathematics is chosen because of it's applicability to the research focus of this study. However, other researchers have set up categories differently (Opt'Eynde, 2002). In the following section I will elaborate on the characterisation of the four aspects of beliefs, and compare and contrast with related concepts from mathematics education research.

### BELIEFS ABOUT MATHEMATICS AT SCHOOL

School is the main arena for experiencing mathematical activities, and thus the main arena for the development of beliefs about mathematics. Some distinguished concepts of mathematics education research aim at characterising important features from this context; the concept of *sociomathematical norms* developed by Erna Yackel & Paul Cobb (1996) and the concept of the *didactical contract* developed by Guy Brousseau (Brousseau & Balacheff, 1997). Moreover, the concept of the *task discourse* as suggested by Stieg Mellin-Olsen (1990) also serves as an ingredient in this aspect of students' mathematics-related beliefs. Finally creative versus imitative reasoning, that are parts of a conceptual framework developed by Johan Lithner (2008) are included for illustrating this belief aspect. These constructs are not meant to be an exhaustive list but indicate possible ingredients belonging to this aspect.

#### *SOCIOMATHEMATICAL NORMS*

In their study of teaching experiments Yackel & Cobb (1996) came to realise that the concept of social norms did not suffice for characterising important features of the social environment in the classroom in reform oriented mathematics teaching. For example, in classroom discussions it may be a social norm that the student should contribute with a solution different from what has already been proposed, but what is to be understood as a mathematically different solution is a matter of sociomathematical norms.

> *"[N]ormative understandings of what counts as mathematically different,*

> *mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom are sociomathematical norms. Similarly, what counts as an acceptable mathematical explanation and justification is a sociomathematical norm."(Yackel & Cobb, 1996)*

Sociomathematical norms like the examples given above describe important features of the context in which mathematical experiences are gained by students, and thus may be of importance in the formation of students' beliefs about mathematics.

#### *THE DIDACTICAL CONTRACT*

The concept of the didactical contract is a powerful concept for expressing the tacit mutual expectations between students and teachers in mathematics education. Brousseau defines the didactical contract in the following way:

> *"[T]he set of the teacher behaviours (specific to the taught knowledge) expected by the student and the set of the student behaviours expected by the teacher". (Brousseau, 1980, p. 127)*
>
> *"The DC is the set of reciprocal obligations and "sanctions" which*
>
> - *each partner in the didactical situation imposes or believes to have imposed with respect to the knowledge in question, explicitly or implicitly, on the other, or*
> - *are imposed, or believed by each partner to have been imposed on him/her with respect to the knowledge in question.*
>
> *The DC is the result of an often implicit "negotiation" of the mode of establishing the relationships among a student or group of students, a certain educational environment and an educational system." (EMS, 2013)*

These mutual expectations are also suggested as a possible belief object in the students in this study.

#### *THE TASK DISCOURSE*

Based on twenty interviews with teachers Mellin-Olsen came to point out a travel metaphor as a predominant in the discourse of the teachers, a metaphor having "*the task*" in a dominating role. This involved sequences of tasks as they

appeared in a textbook playing the role of the "*travel plan*" with concepts like "*lagging behind*" or "*being ahead*" deriving from this discourse, as well as an element of competition between the students to be first to finish the sequence of tasks (Mellin-Olsen, 1990). These ways of perceiving mathematics may serve as ideas for elements in students' beliefs about mathematics at school.

#### *CREATIVE AND IMITATIVE REASONING*

A conceptual framework for distinguishing key aspect of reasoning types for as to distinguish imitative from creative reasoning has been developed by Johan Lithner (2008). Within this framework thinking processes, student competencies and learning millieu are linked to reasoning, and these distinctions may serve for explaining facets of students beliefs about mathematics at school in this study.

A few distinctions may be necessary to be pointed out concerning this belief aspect; even though mathematics at school may work as a context for generating beliefs, the concepts and distinctions above are meant as suggestions for belief objects. The belief object 'mathematics education', is described as students' general beliefs about mathematics education, and Op t'Eynde and his colleagues defined this category to include beliefs about mathematics as a subject, mathematics teaching in general and mathematics learning and problem-solving (Op t'Eynde et al., 2002).

Finally, one may easily connote evaluative perspective to these belief objects; e.g. "if this is mathematics at school, then it is not for me". These perspectives are, however, in my definition reserved the belief aspect Mathematics and Me.

### BELIEFS ABOUT MATHEMATICS AS A DISCIPLINE

Mathematics as a discipline will, in this study be defined as mathematics as a *pure science* and as an a*pplied science*. This concerns the kinds of activity characteristic of mathematics; including the rules and the names of these. However, mathematics

as a discipline is a concept which has several interpretations, but it is commonplace to associate many aspects of mathematics. The following ways of making distinctions amongst different facets of the discipline of mathematics are not meant as a means for defining this beliefs aspect, but rather for unfolding a range of perceptions.

Vagn Lundsgaard Hansen (Hansen, 2004) emphasises the duality of mathematics in terms of the interrelatedness between *applied* mathematics and *pure* mathematics, whereas Mogens Niss (1994) describes the five-fold nature of mathematics; a *pure science*, an *applied science*, a *systems of tools* used in society, an *educational subject* and a f*ield of aesthetic* experiences. Based on an analysis of contributions for conceptualising the nature of mathematics offered by Ernest (1989), Dionne (1984) or Grigutsch et al. (1998), Feldbrich, Kaiser & Schmotz (2012) argue that the four fundamental views of mathematics presented by Grigutsch and colleagues: *The formalist-related view*, the *scheme-related view*, the *process-related view* and the *application-related view* essentially represent characterisations of mathematics as either a *static science* (the formalist and the scheme-related view) or as a *dynamic science* (the process-related and the application-related view). Besides they remark that these two views have shown not to be mutually exclusive, neither are the four views. This is neither in contradiction to Niss' five natures of mathematics, nor to Hansen's emphasis on the duality. These are rather just different perspectives on the same underlying discipline.

### BELIEFS ABOUT MATHEMATICS IN SOCIETY

This category concerns the location of mathematics in society; who are doing and using mathematics for what in society? This includes professions, industries, institutions, groups of and individual citizens, aesthetic and cultural use of mathematics, all of which may be represented in students' ideas of the role of

mathematics in professions in society. In this context, one questions is whether mathematics appears to permeate society, another question is whether it is something useful only for getting admittance to further education.

### BELIEFS ABOUT MATHEMATICS AND ME

This category represents the beliefs about the self in relation to mathematics. It contains the self concept and also motivational beliefs. Just as the social context is important for the formation and development of beliefs, so is the self. However, what is represented in this category is the belief object of oneself as a learner and doer of mathematics now and in the perceived future. However, this aspects of students' beliefs relies heavily on Op t'Eynde and his colleagues' interpretation of Mathematics and Self: This dimension includes what is known as "*motivational beliefs*", categorised as: *Self-efficacy beliefs*, *control beliefs*, *task-value beliefs* and *goal-orientation beliefs* (Op t'Eynde et al., 2002, p. 30). The *self-efficacy beliefs* and the *control beliefs* are described as aspects of the motivational construct *expectancy*, whereas *goal-orientation beliefs* and *task-value beliefs* represents aspects of the motivational construct *value* (Op t'Eynde et al., 2002, p. 30). One more motivational construct is mentioned; *affect*, relating to the students' emotional response to the other two motivational constructs (*expectancy* and *value*), but since emotions are not counted as beliefs but rather as reactions to or consequences of beliefs, they do not form an independent category in this study, but appear as nuances of the other two constructs (Op t'Eynde et al., 2002, p. 30). However, it should be noted that from the point of view of Mandler's theory (see also p. 12f) emotional responses are thought of as hot and unstable, but if they are repeated they are thought to influence the more stable and more cognitive constructs of attitudes and beliefs. Thus it is not clear whether emotions are to be counted as a consequence of beliefs or if beliefs are to be seen as a consequence of emotions, or, more specifically, how these constructs work together. Since the focus of this study concerns how some aspects of beliefs relate to other

aspects of beliefs, rather than how beliefs relate to emotions, this ambiguity will not be pursued any further.

## VISUAL MODEL OF THE FOUR ASPECTS

The four aspects of students' beliefs about mathematics may relate to each other in many possible ways. Some aspects may be primary and some may be derivative of each other for each student, and the beliefs at issue may be held with varying degrees of conviction. However, it is not the purpose here to presuppose a fixed a priori organisation. To allow for an open investigation of the mutual relationships between the four aspects of students' beliefs about mathematics, I have set up a visual model of the four aspects by organising them in a tetrahedron.

In this way the mutual relationships of the four aspects of students' beliefs about mathematics may appear symmetrical, which might not be the case either, but it will be up to investigation to establish the dynamics of the interdependency of these aspects over time, as well as the directions and the strengths of each possible relations.

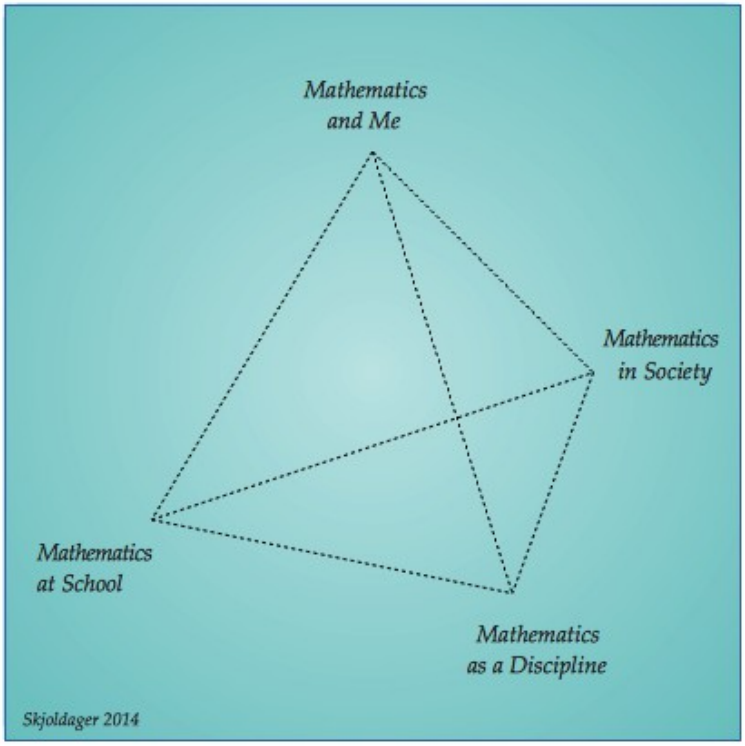



*Illustration 2: Four aspects of 'Students' Beliefs about Mathematics' represented as the four corners of a tetrahedron.*

The edges of the tetrahedron could thus be of different importance and the influence between the aspects could be symmetric, one-directional or not appear at all, which may be illustrated by associated arrows, possibly of different size at each end, or the arrow could be absent, depending on what is meant to be illustrated. The direction representing the quasi-logical structure: what aspect of beliefs appear to be primary to others, and which belief, if any, appear to be derivative to others for a person at a given point of time.

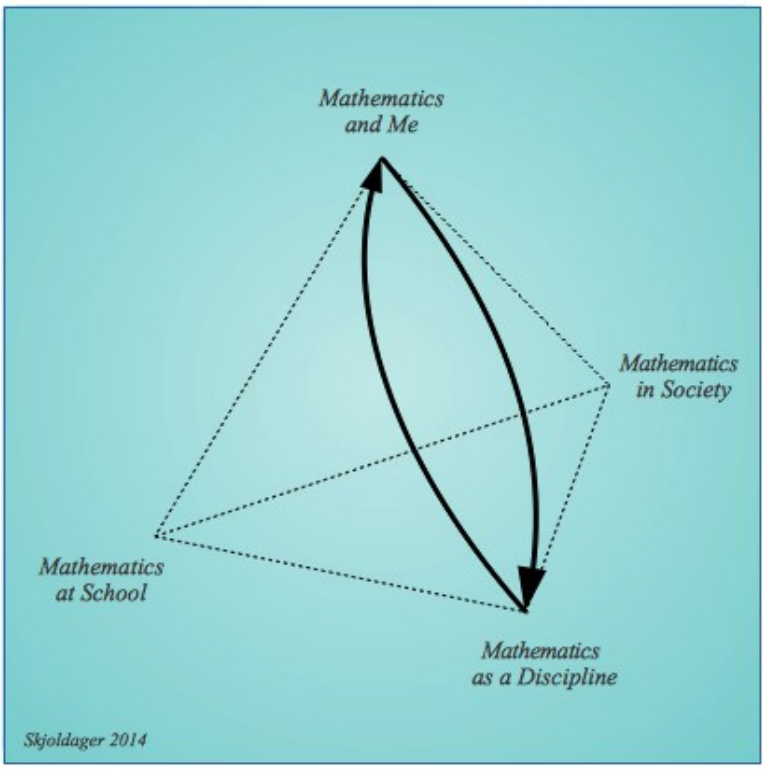



*Illustration 3: An illustration of the possible dynamics between some of the four aspects of 'Students' Beliefs about Mathematics' represented by arrows between two of the four corners of the tetrahedron.*

According to Törner, convincing interpretations of *central* or *peripheral* psychological strength, and *primary* or *derivative* quasi-logical structure have been suggested in a few papers (Törner, 2002, p. 81), e.g. in a dissertation thesis by Jones (1990) reviewed thoroughly by Günter Törner and Erkki Pehkonen (Törner & Pehkonen, 1996). However, as Törner adds (Törner, 2002, p. 81):

> *"An open question is the possible interaction patterns of the accordingly categorized beliefs".*

Thus, organising beliefs in aspects leads to the question of how these aspects

interact. This question may be pursued by applying the four aspects of beliefs in the analysis of the longitudinal study. The next question will then be to investigate possible reasons for the dynamics: *What is responsible for the change of beliefs?* If, for example, beliefs about mathematics at school seem to be primary to other aspects of beliefs, one may wish to seek to understand the background for this. Now since the school context usually is a primary source for mathematical experiences, it is not unthinkable to presume it to play a special role. Also mathematics and self could have an important role to play, since it is characterised by its more evaluative character than are the other categories. However, these questions will be dealt with on the basis of the findings concerning these two questions: *What are beliefs about?*; which has been addressed in this chapter; and the question: *How do they change?;* which is to be investigated by means of the four aspects of beliefs.

# CHAPTER IV: ANALYSIS

At the end of the last year of upper secondary school, what ideas do the students now have about the role of mathematics in their future education? The main aim of this study is to understand students' interpretations of which experiences have had an impact on this issue. In Chapter II, the Methodology of the study was outlined and in Chapter III, the conceptual framework developed for the study was presented. Here, in Chapter IV, the actual collection of the empirical material and the analysis of this material, are on the agenda.

The chapter is built up of the following five main sections:

I. Data Collection
II. Quantitative Analysis
III. Case Selection
IV. Case Analysis
V. An Ideal Typical Student's Beliefs

The *Quantitative Analysis* consists of a descriptive summative analysis of selected questionnaire items, mainly referring to the frequency of types of answers in the first and the third year questionnaire. In the second section, *Case Selection*, one questionnaire item, #1, the so-called *Favourite Subject Scale* question, was explored more thoroughly, which revealed some features that exerted an influence on the set of criteria adopted for the selection of cases for the *Case Analysis*.

The core of the research design is a longitudinal study. The four aspects of beliefs were charted by means of questionnaires consisting of both open-ended and closed items followed up by qualitative semi-structured research interviews. The 1st year interviews were fully transcribed and briefly analysed before the 3rd year interviewing of the same students were carried out.

Nine classes from Zealand participated in the questionnaire study and from four of these classes, six students from each class participated in the interviews (a total of 24 students 1st year and 21 students 3rd year). The nine schools were selected from a strategy of obtaining maximum variation amongst schools in Zealand. The four schools were selected due to the willingness to participate in the project of the principal of the schools. From the 24/21 interviewees, six informants were selected for case analysis. The criteria of the selection for case analysis is described in in the section Case Selection, (pages 129ff).

Questionnaires from147 students form the ground for the analysis of the longitudinal development. These 147 student answered both the 1st year and the 3rd year questionnaires (see Table 1, p. 58).

| All Students | 1st Year | 3rd Year | 1st and 3rd year |
|---|---|---|---|
| N =264 | N1 = 245 | N3 = 189 | N(1& 3) = 147 |

*Table 1: Overview of the number of students answering the questionnaires in the longitudinal study.*

These numbers are partly due to some students dropping out of school or changing school and thus exiting the population, and on the other hand, some students upgrading from intermediate (B-level) to high-level (A-level) mathematics and thus entering the population. Finally, some students decided not to put their name on the questionnaire.

### *INTERVIEWING*

The interviews took place on the interviewees' schools. Their mathematics teacher booked a room for the interview. The interview took place concurrently with mathematics lessons, and the timing was coordinated with the mathematics teachers so as to prevent students from missing important teaching.

The researcher conducted all the interviews. The interviews were semi-structured and followed an interview guide partly based on questionnaire responses. However, there was much room for flexibility. The interviews were recorded digitally and subsequently transcribed. The researcher also took notes during each interview and wrote down reflections on the interviews afterwards.

### *SEQUENCE OF INTERVIEWS*

Each interview was introduced by a briefing to the students about the purpose of the interview and the interviewee was reminded that it was voluntarily to participate and that the interview could be stopped any time. Opening the interview, the first question concerned actual experiences of the interviewee concerning differences between mathematics in compulsory school and now in upper secondary school. From here, we could now proceed to questions pertaining to the four themes: *Mathematics at school*, *Mathematics as a Discipline*, *Mathematics in Society* and *Mathematics & Me*. The sequencing of these four themes varied in order to tune the interview into a natural flow of conversation. At the end of the interview, the interviewees were asked about their ideas for future educational plans after upper secondary school and about the role of mathematics in these.

### *ETHICAL ISSUES*

The study has been carried out living up to the criteria from the Danish Data Protection Agency 'Datatilsynet'. Since no 'sensitive information' (such as economic or health related information) is collected, the study did not have to be reported to the Agency. Anyhow, the study should still live up to certain criteria: The informants

were informed of the purpose of the data collection, they were informed that it was voluntary to participate and that they could withdraw at any time in the process. The interviewee were also promised anonymity.

Jennifer Mason (2002) emphasises that in each stage of a research process the researcher needs to pay attention to research ethics. She accentuates criteria for qualitative interviewing specifically. She encourages the researcher to take the following questions into consideration:

1. What is asked?
2. How is it asked?
3. What are your interviewees 'allowed' to tell you?
4. How can one guarantee confidentiality and anonymity, if it is promised?
5. Power relations in the interview situation

No questions in the interview guide were directed towards evaluating students' teachers; In stead the questions were directed towards their own actions, responsibilities, possibilities and challenges. No trick question were used, and the interview style was inviting for the interviewee to feel comfortable. If issues of challenges in students' personal lives appeared, my focus was on clarifying whether they had resources for dealing with them, rather than on trying to make them reveal more about themselves than appropriate for the interview topic and for their privacy. Aspects of power and responsibility in the interview situation gave me certain obligations towards the interviewees. In setting the agenda and in keeping and processing the data, I have strived to live up to this responsibility.

Before interviewing, and even before sampling, the interviewees were informed in general terms of the purpose of the interviews and were asked to return sheets with their consent. As a consequence of this agreement, I cannot pass on the

rights to the data to others, but I can use the data generated through the interview for research purposes. Before each interview, the interviewee was reminded that he or she could withdraw at any time. On one occasion I also found it appropriate to remind the interviewee of this in the middle of the interview, an offer which the interviewee, however declined. The data have been anonymised as early in the process as possible and stored safely.

# *DATA COLLECTION*

The section DATA COLLECTION consist of a presentation of the 1st year and the 3rd year questionnaires and a justification of the appropriateness of these items in relation to the research aim. Also the interview guide for the 1st year and the 3rd year follow-up interviews is presented and commented upon.

The 1st year and the 3rd year questionnaires have most questions in common. However, a few items were removed and some were added in the 3rd year questionnaire. The items are designed to address issues relevant to the four aspects of beliefs about mathematics defined in Chapter III.

## THE 1ST YEAR QUESTIONNAIRE

The questions in the 1st year questionnaire (Q1-A) are designed to cover the four aspects of beliefs about mathematics defined in chapter III; *Mathematics at School*, *Mathematics as a Discipline*, *Mathematics in Society* and *Mathematics & Me* (p. 46). The questionnaire items address issues relevant to these aspects and invite for answers of both descriptive and evaluative kinds. The question in the tables are translated from the original Danish Questionnaire (See Appendix). In the footnotes the Danish text is provided for comparison.

| Q1-A | TRANSITION |
|---|---|
| # | *Question* |
| **1** | On a scale from 1 to 10, on which 10 stands for your favourite subject, how would you rate mathematics?[6] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching)[7] [8] [9] |
| **3** | Did you like mathematics when you went to lower secondary school?[10] [11] [12] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school?[13] |
| **4b** | Is there anything you liked better before?'[14] |
| **4c** | Is there anything you like better now?[15] |

*Table 2: The 1st year questionnaire, part A – TRANSITION*

### Q1-A TRANSITION

Item #1 invites the student to give mathematics a mark as a way to indicate the appreciation of the subject at the current point in time. The scale is a subjective measure and obviously an ordinal scale (and not a ratio scale). An ordinal scale allows for the use of *mode* and *median* as statistical measures, but not for the *mean value*, since the interpretation of each step of the scale is subjective and there is no common objective distance between the steps. Variations of this item have previously been applied by Jankvist (2009) and by Christensen & Rasmussen (1980).

Items #2 - #4 invite for describing features of *Mathematics at School* and partly

6 På en skala fra 1 - 10 hvor 10 er dit yndlingsfag, hvor ligger matematik?
7 Er der arbejdsformer, du er særligt glad for i matematik?
8 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]
9 [Arbejde alene]; [Arbejde to og to]; [Arbejde i grupper ]; [Hele klassen sammen]; [Projektarbejde]; [Andet:]
10 Kunne du lide matematik, da du gik i folkeskolen?
11 Options: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]
12 [Ja, det var et af mine yndlingsfag] ;[Ja, det var fint] ;[Det var OK] ;[Nej, det var ikke lige mig] ;[Nej, jeg brød mig slet ikke om det] ;[Andet:]
13 Hvordan har matematik ændret sig, fra da du gik I folkeskolen, til nu, hvor du går I gymnasiet?
14 Var der noget du bedre kunne lide før?
15 Er der noget du bedre kan lide nu?

*Mathematics and Me*, by contrasting the experiences now with earlier experiences.

| **Q1-B** | **FOR SCHOOL** |
|---|---|
| # | *Question* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics?[16] |
| **5b** | Is mathematics something you think everybody should learn?[17] |
| **6** | What made you choose a study programme involving A-level mathematics?[18] |
| **7a** | Is mathematics related to your other subjects?[19] |
| **7b** | Please give reasons for your answer[20]: |

*Table 3: The 1st year questionnaire, part B - MATHEMATICS AND SCHOOL*

### Q1-B FOR SCHOOL

These questions (#5-7) address issues concerning mathematics in society as well as the role of mathematics in other disciplines, exemplified by school subjects with which the students' have experiences. Answers to these questions may shed light on the student's beliefs concerning the aspects *Mathematics as a Discipline* and *Mathematics in Society*.

16 Hvorfor tror du, at det er blevet bestemt at alle I Danmark skal lære matematik?
17 Er matematik noget du synes alle bør lære?
18 Hvad fik dig til at vælge en studieretning med matematik på A-niveau?
19 Har matematik noget med dine andre fag at gøre?
20 Begrund dit svar:

| Q1-C | BEYOND SCHOOL |
|---|---|
| # | *Question* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed?[21] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and is then discovered by human beings?[22] |
| **10** | What do you think a professional mathematician in a university is doing?[23] |
| **11** | Would you have to be a genius in order to study mathematics in university?[24] [25] |

*Table 4: The 1st year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

### Q1-C BEYOND SCHOOL

# 8 is intended to give an opportunity for the student to share their ideas of the role of M*athematics in Society*. #9-#11 concern the nature of mathematics and the activities of university mathematicians. Answers to these questions are meant to bring insight into the students' beliefs about M*athematics as a Discipline*.

21 Hvad bliver matematik brugt til her I verden, når det ikke handler om undervisning? Kan du give eksempler på hvor der bliver brugt matematik?

22 Hvordan opstår matematik? - er matematik noget mennesker har opfundet? - eller findes matematik i forvejen og så bliver den opdaget af mennesker?

23 Hvad tror du en professionel matematiker på et universitet laver?

24 Skal man være geni for at læse matematik på et universitet?

25 Options:[Yes]; [No]; [I do not know]

| Q1-D | IMRPOVING |
|---|---|
| # | *Question* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics?[26] |
| **12b** | What do you think is the best means for improving in mathematics?[27] |
| **12c** | What do you do to improve in mathematics?[28] |
| **13a** | What do you do if you get stuck on a task at school?[29] |
| **13b** | What do you do if you get stuck on your homework?[30] |

*Table 5: The 1st year Questionnaire, part D – STRATEGIES FOR IMPROVING*

### Q1-D IMPROVING

Items #12-13 refer to hindrances and means for improving in mathematics and students' strategies for overcoming hindrances. Both students' beliefs about *Mathematics at School* and *Mathematics & Me* are expected to be elucidated by the answers to these questions.

26 Hvad mener du er den største *hindring* for at du kunne blive endnu bedre til matematik?
27 Hvad mener du er det bedste *middel* til at blive bedre til matematik?
28 Hvad *gør* du for at blive endnu bedre til matematik?
29 Hvad gør du, hvis du går i stå med en opgave i en matematiktime på gymnasiet?
30 Hvad gør du, hvis du går i stå med dine lektier i matematik?

| Q1-E | CHALLENGES & SUPPORT | |
|---|---|---|
| # | *Question*[31] [32] | |
| **14** | What issues involve more challenges to you?[33] | a) Remembering |
| | | b) Computing |
| | | c) Figuring out the purpose of a task |
| | | d) Finding a way to solve a task |
| | | e) Reading and understanding the textbook |
| **15** | Where can you find support for mathematical activities? [34] [35] | |
| **16** | Did you parents take the Upper Secondary School Leaving Certificate?[36] [37] | |

*Table 6: The 1st year Questionnaire, part E – CHALLENGE AND SUPPORT*

### Q1-E CHALLENGES & SUPPORT

In item #14 a few specific activities in dealing with mathematics are mentioned as possible challenges. Students' answers to these questions can give way to insight onto their perception of both the level of challenge involved in the activity, and their images of themselves as mathematics learners, depending on their answers. Items #15-16 address their (perceived) resources for support of Mathematical activities. These items address issues concerning the belief aspect *Mathematics & Me*.

31 Svar: [1]=Flest; [2]=Flere; [3]=Midt imellem [4]=Få; [5]=Færrest

32 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

33 Hvilke ting har du flest udfordringer med?

34 Hvor kan du hente støtte til matematiske aktiviteter?

35 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From friends]; [Other places] If other places, from where or from whom?

36 Har dine forældre taget studentereksamen?

37 [Yes, my mother did]; [Yes, my father did]; [Both my parents did]; [None of them did]

| Q1-F | IN CLASS |
|---|---|
| # | *Question* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson?[38] |
| **17b** | Do you think that you ask questions more frequently than other students in class?[39] |
| **17c** | Are you content with that?[40] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson?[41] |
| **18b** | Do you think that you answer questions more frequently than other students in class?[42] |
| **18c** | Are you content with that? |
| **19a** | In your class, is it okay to be good at mathematics?[43] |
| **19b** | In your class, is it okay to have difficulties in mathematics?[44] |

*Table 7: The 1st year Questionnaire, part F - MATHEMATICS IN CLASS*

### Q1-F IN CLASS

Items #17-19 concern questions relating to classroom norms and individual norms. Answers to these questions may give insight into aspects concerning the didactical contract in class. Both *Mathematics at School* and *Mathematics and Me* could be addressed by way of answers to these questions.

38 Hvor tit rækker du typisk hånden I vejret for at *spørge om noget* I løbet af en matematiktime?
39 Synes du at du spørger om noget oftere end andre fra din klasse?
40 Er du tilfreds med det?
41 Hvor tit rækker du typisk hånden I vejret for at *svare på noget* I løbet af en matematiktime?
42 Synes du, at du svarer på noget oftere end andre fra din klasse?
43 Er det, i din klasse, OK at være god til matematik?
44 Er det, i din klasse, OK at have svært ved matematik?

| Q1-G | PLANS |
|---|---|
| # | *Question* |
| **20** | What are your educational plans so far after upper secondary school?[45] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics?[46] |
| **21b** | Comments: |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics?[47] |
| **22b** | Comments: |

*Table 8: The 1st year Questionnaire, part G – PLANS*

### Q1-G PLANS

Students' plans for their future are central to the research question, and especially whether the student would deselect or actively go for tertiary studies involving mathematics. Changes in these answers are therefore central as indicators of the student's experiences as well as interpretations of these that relate to these changes. Students plans are part of the belief aspect *Mathematics & Me*.

### *SUMMARY OF ITEMS & BELIEF ASPECTS*

All in all, the seven groups of Questionnaire items in the 1st Year Questionnaire cover issues that relate to the four aspects of beliefs, as intended. An overview is provided in Table 9 (p. 71).

45 Hvilke planer har du om uddannelse efter gymnasiet?

46 Kan du forestille dig, at du vil gå efter en uddannelse, der indeholder en del matematik?

47 Kan du forestille dig, at du vil forsøge at undgå en uddannelse, der indeholder en del matematik?

| Group of Items | | Belief Aspects |
|---|---|---|
| Q1-A | TRANSITION | *Mathematics at School*<br>*Mathematics & Me* |
| Q1-B | FOR SCHOOL | *Mathematics in Society*<br>*Mathematics as a Discipline* |
| Q1-C | BEYOND SCHOOL | *Mathematics in Society*<br>*Mathematics as a Discipline* |
| Q1-D | IMPROVING | *Mathematics at School*<br>*Mathematics & Me* |
| Q1-E | CHALLENGES & SUPPORT | *Mathematics & Me* |
| Q1-F | IN CLASS | *Mathematics at School*<br>*Mathematics & Me* |
| Q1-G | PLANS | *Mathematics & Me* |

*Table 9: Belief Aspects related to groups of Questionnaire Items in the 1st year.*

## THE 3RD YEAR QUESTIONNAIRE

| Q3-A | TRANSITION |
|---|---|
| # | *Question* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching)[48] [49] [50] |
| **3** | Did you like mathematics when you went to lower secondary school?[51] [52] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school?[53] |
| **New 4d** | In which ways has mathematics changed from first year to third year of upper secondary school? |

*Table 10: The 3rd year questionnaire, part A – TRANSITION*

### Q3-A TRANSITION

Items in Q3-A Transition still covers the belief aspects *Mathematics at School* and *Mathematics & Me*. Item #4b (is there anything you liked better before) and #4c (is there anything you like better now) have been left out and a new item #4d, concerning the change from the 1st to the 3rd year, has been added. But the questions are still intended for making the students describe features of school mathematics, which may concern both descriptive and evaluative facets. The descriptive facets may then shed light primarily on the belief aspect *Mathematics at School,* while the more evaluative facets of students' answers could be more likely to enlighten the belief aspect *Mathematics & Me*.

48 Er der arbejdsformer, du er særligt glad for i matematik?

49 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

50 [Arbejde alene]; [Arbejde to og to]; [Arbejde i grupper ]; [Hele klassen sammen]; [Projektarbejde]; [Andet:]

51 Kunne du lide matematik, da du gik i folkeskolen?

52 Options: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

53 Hvordan har matematik ændret sig, fra da du gik I folkeskolen, til nu, hvor du går I gymnasiet?

| Q3-B | FOR SCHOOL |
|---|---|
| # | *Question* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics?[54] |
| **5b** | Is mathematics something you think everybody should learn?[55] |
| **7a** | Is mathematics related to your other subjects?[56] |
| **7b** | Please give reasons for your answer[57]: |

*Table 11: The 3rd year questionnaire, part B - MATHEMATICS AND SCHOOL*

**Q3-B FOR SCHOOL**

Item # 6 (What made you choose a study programme involving A-level mathematics) has been left out, but this group of items (Q3-B) still concerns both *Mathematics as a Discipline* and *Mathematics in Society*.

54 Hvorfor tror du, at det er blevet bestemt at alle I Danmark skal lære matematik?

55 Er matematik noget du synes alle bør lære?

56 Har matematik noget med dine andre fag at gøre?

57 Begrund dit svar:

| Q3-C | BEYOND SCHOOL |
|---|---|
| # | *Question* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed?[58] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and is then discovered by human beings?[59] [60] [61] |
| **10** | What do you think a professional mathematician in a university is doing?[62] |
| **11** | Would you have to be a genius in order to study mathematics in university?[63] [64] |

*Table 12: The 3rd year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

### Q3-C BEYOND SCHOOL

There has been no changes to this group of items.

58 Hvad bliver matematik brugt til her I verden, når det ikke handler om undervisning? Kan du give eksempler på hvor der bliver brugt matematik?

59 Hvordan opstår matematik? - er matematik noget mennesker har opfundet? - eller findes matematik i forvejen og så bliver den opdaget af mennesker?

60 Svar: [Opfundet]; [Opdaget]; [Begge dele]; [Ingen af delene}; [Ved ikke]

61 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]

62 Hvad tror du en professionel matematiker på et universitet laver?

63 Skal man være geni for at læse matematik på et universitet?

64 [Yes]; [No]; [I do not know]

| Q3-D | IMRPOVING |
|---|---|
| # | *Question* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics?[65] |
| **12b** | What do you think is the best means for improving in mathematics?[66] |
| **12c** | What do you do to improve in mathematics?[67] |
| **13a** | What do you do if you get stuck on a task at school?[68] |
| **13b** | What do you do if you get stuck on your homework?[69] |
| **13c** | What do you do if you get stuck on your written assignments?[70] |

*Table 13: The 3rd year Questionnaire, part D – STRATEGIES FOR IMPROVING*

**Q3-D**

Item #13c has been added, since the students now in the 3rd year have experiences with written assignments to a greater extent than in the beginning of the 1st year.

65 Hvad mener du er den største *hindring* for at du kunne blive endnu bedre til matematik?
66 Hvad mener du er det bedste *middel* til at blive bedre til matematik?
67 Hvad *gør* du for at blive endnu bedre til matematik?
68 Hvad gør du, hvis du går i stå med en opgave i en matematiktime på gymnasiet?
69 Hvad gør du, hvis du går i stå med dine lektier i matematik?
70 Hvad gør du, hvis du går i stå med dine afleveringer i matematik?

| Q3-E | CHALLENGES & SUPPORT | |
|---|---|---|
| # | *Question*[71] [72] | |
| **14** | What issues involve more challenges to you?[73] | a) Remembering |
| | | b) Computing |
| | | c) Figuring out the purpose of a task |
| | | d) Finding a way to solve a task |
| | | e) Reading and understanding the textbook |
| **15** | Where can you find support for mathematical activities? [74] [75] | |
| **New 16a** | Do you find that you meet some special challenges related to succeeding in mathematics compared to your classmates?[76] | |
| **New 16b** | Do you find that you have some some special strengths related to succeeding in mathematics compared to your classmates?[77] | |

*Table 14: The 3rd year Questionnaire, part E – CHALLENGE & SUPPORT*

### Q3-E CHALLENGES & SUPPORT

Students are no longer asked about their parents' upper secondary education. Instead, new item #16 a & b ask whether the student experiences special strengths or challenges compared to others in class. Answers to these questions relate to one's idea of oneself as a mathematics learner and belong to the belief aspect *Mathematics & Me*.

71 Svar: [1]=Flest; [2]=Flere; [3]=Midt imellem [4]=Få; [5]=Færrest

72 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

73 Hvilke ting har du flest udfordringer med?

74 Hvor kan du hente støtte til matematiske aktiviteter?

75 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From friends]; [Other places] If other places, from where or from whom?

76 Synes du, at du møder nogle særlige udfordringer i forhold til at klare dig godt i matematik sammenlignet med dine klassekammerater?

77 Synes du, at du har nogle særlige styrker i forhold til at klare dig godt i matematik sammenlignet med dine klassekammerater?

| Q3-F | IN CLASS |
|---|---|
| # | *Question* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson?[78] [79] |
| **17b** | Do you think that you ask questions more frequently than other students in class?[80] |
| **17c** | Are you content with that?[81] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson?[82] [83] |
| **18b** | Do you think that you answer questions more frequently than other students in class?[84] |
| **18c** | Are you content with that? |
| **19a** | In your class, is it okay to be good at mathematics?[85] |
| **19b** | In your class, is it okay to have difficulties in mathematics?[86] |

*Table 15: The 3rd year Questionnaire, part F - MATHEMATICS IN CLASS*

### Q3-F IN CLASS

This group of items is unchanged compared to the 1st year.

78 Hvor tit rækker du typisk hånden I vejret for at *spørge om noget* I løbet af en matematiktime?
79 Options: [0]; [1-3]; [4-8]; [More than 8 times]
80 Synes du at du spørger om noget oftere end andre fra din klasse?
81 Er du tilfreds med det?
82 Hvor tit rækker du typisk hånden I vejret for at *svare på noget* I løbet af en matematiktime?
83 Options: [0]; [1-3]; [4-8]; [More than 8 times]
84 Synes du, at du svarer på noget oftere end andre fra din klasse?
85 Er det, i din klasse, OK at være god til matematik?
86 Er det, i din klasse, OK at have svært ved matematik?

| Q3-XA | UNDERSTANDING |
|---|---|
| # | *Question* |
| **X.1.** | Have you recently experienced to *understand* what you worked with in mathematics? [87] [88] |
| | If yes, on what occasion?[89] |
| **X.3.** | Have you, during upper secondary school experienced that you *understood* what you worked with and then s*ubsequently learned it by heart*?[90] [91] |
| | If yes, on what occasion? |
| **X.5.** | Have you during upper secondary school experienced *understanding something* but *never learning it by heart?*[92] [93] |
| | If yes, on what occasion? |

| Q3-XB | LEARNING BY HEART |
|---|---|
| # | *Question* |
| **X.2.** | Have you recently experiences having to *learn something by heart*?[94] [95] |
| | If yes, on what occasion? |
| **X.4.** | Have you, during upper secondary school experienced that you had to *learn something by heart*, and then, *subsequently understood it*?[96] [97] |
| | If yes, on what occasion? |
| **X.6.** | Have you during upper secondary school experienced *learning something by heart without ever understanding it*?[98] [99] |
| | If yes, on what occasion? |
| **X.7.** | Additional comments on understanding or rote learning in mathematics |

87 Har du på det seneste haft oplevelsen af at forstå det I arbejdede med i matematik?

88 Options: [Yes], [No] or [I do not know]

89 Hvis ja, ved hvilken lejlighed?

90 Har du i matematik i gymnasiet oplevet at du forstod det, i arbejdede med, og derefter lærte det udenad, for at kunne huske det?

91 Options: [Yes], [No] or [I do not know]

92 Har du i matematik i gymnasiet oplevet, at du har forstået det, I arbejdede med, men aldrig har lært det uden ad?

93 Options: [Yes], [No] or [I do not know]

94 Har du på det seneste haft oplevelsen af at være nødt til at læ¦re noget uden ad i matematik?

95 Options: [Yes], [No] or [I do not know]

96 Har du i matematik i gymnasiet oplevet, at du først lærte noget uden ad, og først senere forstod det, du arbejdede med?

**Q3-XA UNDERSTANDING & Q3-XB LEARNING BY HEART**

These questions were added due to the 1st year interviews, in which the issue of understanding certain parts of mathematics or learning them by heart came up frequently. In the original 3rd year questionnaire in Danish, these items were grouped in order X.1.; X.2.; X.3. and so on, but I decided to reorganise the order in the presentation in the thesis, because it seemed more logical to group the questions as nuances of *Understanding* and nuances of *Learning by heart* in stead of the original order.

However, students' answers to these items seemed less enthusiastic compared to most other items. This may relate to the fact that the students' did not see the point in dividing up the question of understanding versus learning by heart in so many sub-aspects, or just to the fact that the questionnaire was given during their Study Programme Project and they may have felt that the questionnaire was becoming too long for them to answer.

However, even though the answers to these questions seemed sparse in general, they did provide a reasonable point of departure for a dialogue in the subsequent 3rd year interviews.

97 Options: [Yes], [No] or [I do not know]

98 Har i du matematik i gymnasiet oplevet at lære noget uden ad, som du aldrig har forstået?

99 Options: [Yes], [No] or [I do not know]

| Q3-XC | A-LEVEL EXAMINATION |
|---|---|
| **X.8.** | *Topic*[100 101] |
| **(a)** | Parabola[102] |
| **(b)** | Exponential[103] |
| **(c)** | Pythagoras[104] |
| **(d)** | Sine and cosine relations[105] |
| **(e)** | Definition of differentiability[106] |
| **(f)** | Sum and product of differentiable functions[107] |
| **(g)** | Indefinite integral[108] |
| **(h)** | Volume of solid of revolution[109] |
| **(i)** | Differential Equations and their solutions[110] |
| **(j)** | Vectors in the plane, including scalar product[111] |
| **(k)** | Lines and planes[112] |
| **X.9.a.** | Which topic is your favourite? - and why? |
| **X.9.b.** | Which topic would you rather avoid? - and why? |

*Table 16: Typical topics for oral and written examinations in A-level Mathematics for Upper Secondary School Programmes in Denmark 2013[113]. The English translation is somewhat abbreviated compared to the original Danish formulation of these items.*

100 Options: [Readily], [Okay], [Rather not], [I do not know]

101 Svar: [Meget gerne]; [OK], [Helst ikke], [Ved ikke]

102 Parablen, herunder toppunktsformlen

103 Eksponentialfunktionen, herunder at bestemme forskriften ud fra to punkter på dens graf

104 Beregning af sider og vinkler i retvinklede trekanter: Pythagoras' sætning, definition af sinus og cosinus

105 Beregning af sider og vikler i vilkårlige trekanter: Sinusrelationerne og cosinusrelationerne

106 Differentialregning: Definition af differentiabilitet

107 Differentialregning: Regneregler for sum og produkt af to differentiable funktioner

108 Integralregning: Bestemmelse af stamfunktion

109 Integralregning: Volumen af omdrejningslegeme (omdrejning af funktion 360 grader om en af akserne)

110 Differentialligninger og deres løsninger. Eksempel: y'=ky med løsningen f(x) = c*exp(ax)

111 Vektorer i planen, herunder skalarprodukt

112 Linjer og planer i rummet: Planens ligning, afstand mellem punkt og plan

113 Topics common to both the Technical (htx) and the General (stx) upper secondary programme examinations are suggested.

### Q3-XC A-LEVEL EXAMINATIONS

These questions were formed taking as the point of departure a typical set of exam questions for the oral exam in A-level mathematics in both the general and the technical upper secondary programmes.

Answers to these questions worked well as point of departure for a dialogue in the subsequent 3rd year interviews, but their contribution to the general picture of the population has not been of major importance.

| Q3-G | PLANS |
|---|---|
| # | *Question* |
| **20** | What are your educational plans so far after upper secondary school?[114] |
| **20b** | What kind of influence would your final marks in mathematics have on your plans (or hope for plans) after upper secondary school? |
| **20c** | What kind of influence would your experiences with mathematics in upper secondary school have on your plans after upper secondary school?[115] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics?[116] |
| **21b** | Comments: |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics?[117] |
| **22b** | Comments |

*Table 17: The 3rd year Questionnaire, part G – PLANS*

### Q3-G PLANS

Two items have been added compared to the 1st year questionnaire; Items #20b & #20c. These questions concern the influence of the final marks in mathematics on students future plans and the influences of their experiences with A-level

114 Hvilke planer har du om uddannelse efter gymnasiet?

115 Hvilken betydning har dine oplevelser I matematik I gymnasiet for dine planer efter gymnasiet?

116 Kan du forestille dig, at du vil gå efter en uddannelse, der indeholder en del matematik?

117 Kan du forestille dig, at du vil forsøge at undgå en uddannelse, der indeholder en del matematik?

mathematics on the same. The influence of the final grades relates to mathematics as a means for admission to certain study programmes in tertiary education, which can be associated with an instrumental rationale for learning, whereas the influence of their experiences may relate to intrinsic values held by the students. Both issues belong to the belief aspect *Mathematics & Me*.

### INTERVIEW GUIDES

The interview guide for the first year interviews followed the themes in the questionnaire quite closely. In the third year interviews, however, the four aspects of beliefs and students' interpretations of experiences leading to insight into the possible interrelationships between these aspects, were in focus. This approach was in some sense guided by the idea of the belief tetrahedron with special emphasis to the edges.

# PRACTICALITIES OF DATA COLLECTION

*On the next pages, information relevant for the data collection for the longitudinal study is listed in tables, including dates and duration for the various parts of the empirical work in the longitudinal study and details from managing the Questionnaires:*

| Date of Distribution to Schools | |
|---|---|
| **1st Year Questionnaire** | **3rd Year Questionnaire** |
| 12 November 2010<br>+ supplementary questions<br>12 December 2010 | 23 November 2012 |

| NAME of School | 1st year Questionnaire | 1st year Interviews | 3rd year Questionnaire | 3rd year Interviews |
|---|---|---|---|---|
| **ALFA** | 24 November 2010<br><br>+ *10 January 2011* | 4 May 2011<br>6 interviews | 28 November 2012 | 4 March 2013<br>5 interviews |
| **BETA** | 25 November 2010<br>+ *16 December 2010* | 11 April 2011<br>6 interviews | 6 December 2012<br>+ *additional students' answers*<br>*7 December 2012* | 6 March 2013<br>5 interviews |
| **GAMMA** | 29 November 2010 | 6 April 2011<br>3 interviews<br><br>13 April 2011<br>2 interviews<br>20 April<br>1 interview | 11 December 2012<br><br>+ *additional students' answers*<br>*15 January 2013* | 14 March 2013<br>4 interviews |
| **DELTA** | 24 November 2010<br><br>+ *16 December 2010* | 24 March 2011<br>3 interviews<br><br>31 March 2011<br>3 interviews | 17 December 2012 | 13 March 2013<br>6 interviews |

*Table 18: Dates for Interviews and Questionnaires Answers from Schools*

| INTERVIEWS | | | | | |
|---|---|---|---|---|---|
| **School** | **Name** | **1st Year Interview** | | **3rd year Interview** | |
| **(Alias)** | **(Alias)** | **Date** | **Length** | **Date** | **Length** |
| **ALFA** General Upper Secondary School | Andrea | 4.5.2011 | 26:31 | 4.3.2013 | 21:12[118] |
| | Adele | 4.5.2011 | 33:58 | 4.3.2013 | 21:12 |
| | Amy | 4.5.2011 | 27:04 | 4.3.2013 | 21:41 |
| | (Adam) | 4.5.2011 | 28:45 | - | - |
| | Alan | 4.5.2011 | 29:21 | 4.3.2013 | 19:35 |
| | Andrew | 4.5.2011 | 35:55 | 4.3.2013 | 24:37 |
| **BETA** General Upper Secondary School | (Bianca)[119] | 11.4.2011 | 23:53 | - | - |
| | Brooke | 11.4.2011 | 24:04 | 6.3.2013 | 21:58 |
| | (Brenda)[120] | 11.4.2011 | 27:33 | - | - |
| | Ben | 11.4.2011 | 26:16 | 6.3.2013 | 16:07 |
| | Bryan | 11.4.2011 | 30:22 | 6.3.2013 | 22:13 |
| | Brandon | 11.4.2011 | 24:55 | 6.3.2013 | 22:49 |
| **GAMMA** Technical Upper Secondary School | Grace | 6.4.2011 | 28:23 | 14.3.2013 | 16:27 |
| | Gwen | 14.4.2011 | 49:20 | 14.3.2013 | 36:41 |
| | Gary | 6.4.2011 | 23:01 | 14.3.2013 | 20:38 |
| | (George)[121] | 13.4.2011 (11:59 AM) | 23:46 | - | - |
| | (Glenn)[122] | 13.4.2011 | (ca. 19 min.) | - | - |
| | Gordon | 13.4.2011 (12:43 PM) | 37:29 | 14.3.2013 | 15:36 |
| **DELTA** Technical Upper Secondary School | Denise | 24.3.2011 | 16:57 | 13.3.2013 | 30:37 |
| | Donna | 24.3.2011 | 15:40 | 13.3.2013 | 25:50 |
| | Donna | 31.3.2011 | 17:59 | 13.3.2013 | 23:06 |
| | Duncan | 24.3.2011 | 16:59 | 13.3.2013 | 37:15 |
| | Dylan | 31.3.2011 | 26:33 | 13.3.2013 | 45:30 |
| | David | 31.3.2011 | 27:23 | 13.3.2013 | 24:57 |

*Table 19: Time, date and duration of interviews.*

118 Andrea and Adele recorded were on the same slot of length 42:24 (mm:ss)

119 Bianca transferred to another Upper Secondary School after first year

120 Brenda did not attend school the day for the interview. The interview was announced to the students in advance.

121 George did not attend school the day for the interview. The interview was not announced to the students in advance.

122 Glenn left school before it was his turn to be interviewed.

# *QUANTITATIVE ANALYSIS*

## SELECTED QUESTIONNAIRE ITEMS

The questions listed below have been selected for the quantitative analysis of the longitudinal development of A-level students' belief about mathematics from the 1st to the 3rd year of upper secondary school. The items have been selected due to their seemingly significant contributions to answering the research questions:

1. *On a scale from 1 to 10, on which 10 stands for your favourite subject, how would you rate mathematics?*[123]
2. *Is mathematics something you think everybody should learn?*[124]
3. *How does mathematics develop?- Is it invented by human beings? - Or does it exist already, and then it is discovered by human beings?*[125] [126] [127]
4. *Would you have to be a genius in order to study mathematics in university?*[128]
5. *What issues involve more challenges to you? [a) Remembering; b) Computing; c) Figuring out the purpose of a task; d) Finding a way to solve a task; e) Reading and understanding the textbook]*[129]
6. *Where can you find support for mathematical activities?*
7. *What are your plans so far?*
8. *Could you imagine opting for an education involving a good deal of mathematics?*[130]
9. *Could you imagine trying to avoid an education involving a good deal of mathematics?*[131]

The answers to these eight questionnaire items are presented and commented

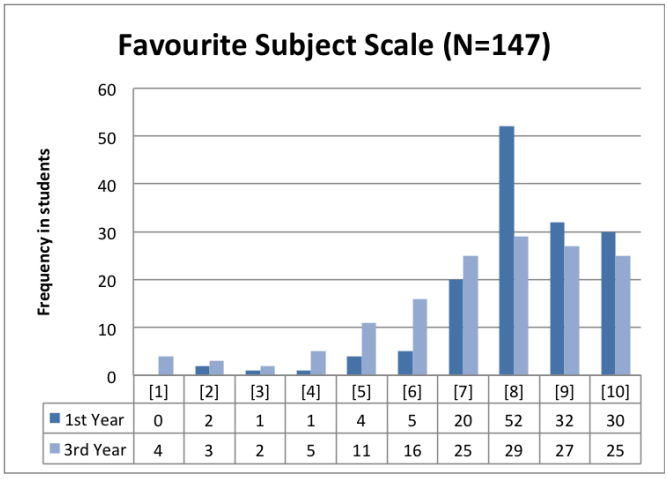
123 På en skala fra 1 - 10 hvor 10 er dit yndlingsfag, hvor ligger matematik?
124 Er matematik noget du synes alle bør lære?
125 Hvordan opstår matematik? - er matematik noget mennesker har opfundet? - eller findes matematik i forvejen og så bliver den opdaget af mennesker?
126 Svar: [Opfundet]; [Opdaget]; [Begge dele]; [Ingen af delene}; [Ved ikke]
127 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]
128 Skal man være geni for at læse matematik på et universitet?
129 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest
130 Kan du forestille dig, at du vil gå efter en uddannelse, der indeholder en del matematik?
131 Kan du forestille dig, at du vil forsøge at undgå en uddannelse, der indeholder en del matematik?

on in the following sections. The counting is aggregated which means that the development in single students are not traced in this part of the analysis.

# TOTAL POPULATION

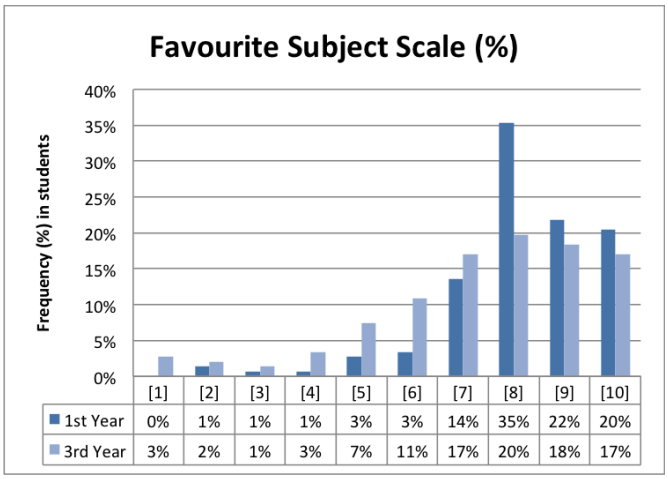
## *1. FAVOURITE SUBJECT SCALE (FSS)*

From the 1st to the 3rd year of upper secondary school, an overall considerable decline in the rating in the Favourite Subject Scale is shown in the answers from the 147 A-level mathematics students.




| | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |
|---|---|---|---|---|---|---|---|---|---|---|
| 1st Year | 0 | 2 | 1 | 1 | 4 | 5 | 20 | 52 | 32 | 30 |
| 3rd Year | 4 | 3 | 2 | 5 | 11 | 16 | 25 | 29 | 27 | 25 |

*Illustration 1: Descriptive Analysis: Item 1. Favourite Subject Scale. Frequency in numbers.*




| | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |
|---|---|---|---|---|---|---|---|---|---|---|
| 1st Year | 0% | 1% | 1% | 1% | 3% | 3% | 14% | 35% | 22% | 20% |
| 3rd Year | 3% | 2% | 1% | 3% | 7% | 11% | 17% | 20% | 18% | 17% |

*Illustration 2: Descriptive Analysis: Item 1. Favourite Subject Scale. Frequency in percent.*

In the 1st year, the *mode* of the students' rating of mathematics on the favourite Subject Scale (FSS) is [8]. The same is the case in the 3rd year, but as it is shown in the table and graph in Illustration 2 (p.87), the proportion of students giving the rating [8] declines from more than one third (35%) to one fifth of the students (20%). In the levels of rating below [8], there is in general an increase from the 1st to the 3rd year, and in the ratings above [8] there is in general a decline from the 1st to the 3rd year.

These trends around the rating [8] stand out even more in the graphs of grouped ratings below.

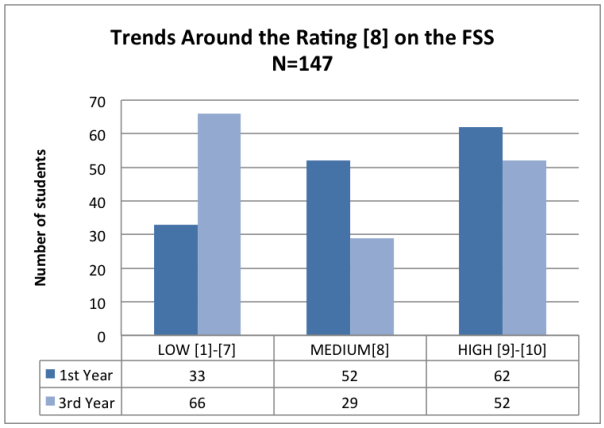



*Illustration 3: Rating around [8] N=147*

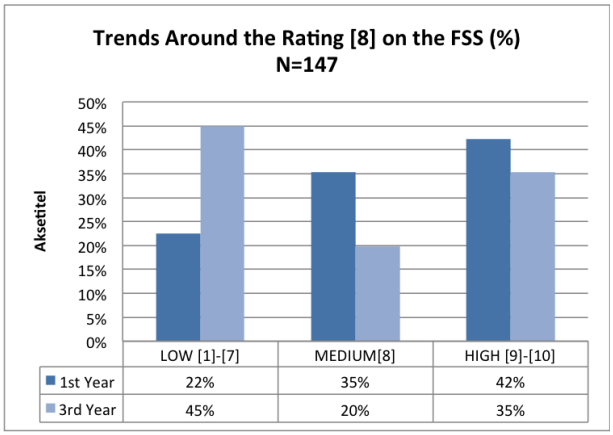



*Illustration 4: Rating around [8] (in %)*

The increase in the lower part of the favourite subject scale ([1]-[7]) is clear; twice as many students in the 3rd year place their rating of mathematics in this end of the scale. The most popular rating in the 1st year, [8], also suffers; more than one third of the students in the 1st year gave this rating, but only one fifth of the students in the 3rd year. The proportion of students having mathematics on one of the top two possible level of rating, [9] and ]10], decreases from more than two fifths (42%) to around one third of them (35%). Overall, 77% of the students in the 1st year and 55% of the students in the 3rd year rate mathematics as one of the top three categories. This is in line with findings by Michelsen & Sriraman (2009), who found that the majority of 255 grade 11 students in technical and general upper secondary school in other region of Denmark had mathematics as one of their three most interesting subjects in upper secondary school.

A further investigation of the same partitioning in the group of female versus male students, shows that a higher proportion of the female student (45%) rate mathematics in the highest and of the scale compared to male students (40%). Also the proportion of female students giving the highest rating seems more stable, since the decline consists of only 2% of 51 students, which is one student less in the 3rd year. In the group of male students, the fall in the proportion of those giving the highest rating to mathematics is more dramatic than in the female group; the 40% of the male students giving the highest rating in the 1st year reduces to 31% of them in the 3rd year.

In the group of students rating mathematics on the lower part of the scale, the proportion in this group is doubled over the three years. In the female group, the 20% in the 1st year increases to 41% in the 3rd year, and in the male group the 24% in the 1st year increases to 47% in the 3rd year. Even though the trend of doubling the percentage of students in this category is common for both genders, lower ratings

are even more prevalent amongst the males, with almost half of them rating mathematics this low in the 3rd year, but also the proportion of female students giving these ratings is considerable.

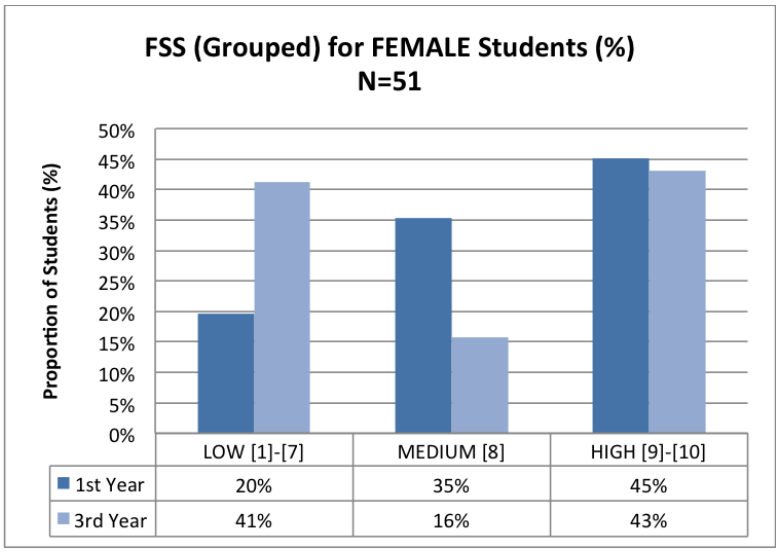



*Illustration 5: Rating around [8] – FEMALE students.*

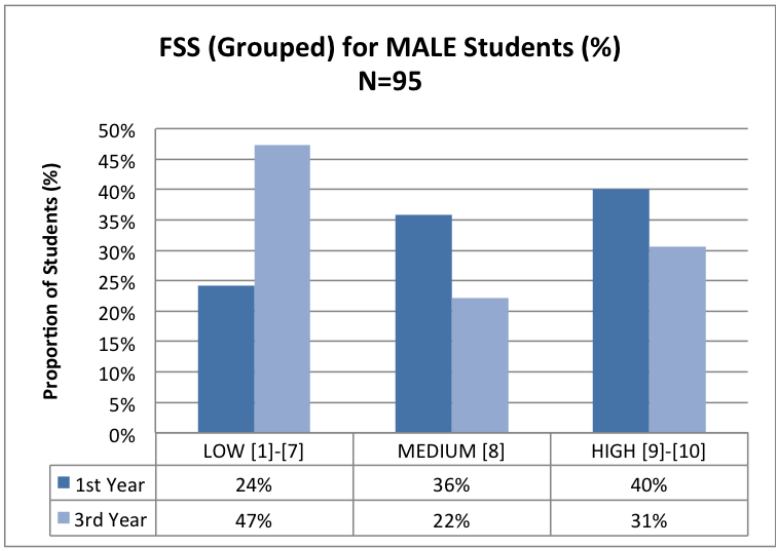



*Illustration 6: Rating around [8] – MALE students.*

The general trend; a decline in high rating, and a rise in the lower levels of rating, is remarkable. Possible reasons for this trend will be examined further in the case analysis.

### 2. *MATHEMATICS FOR ALL?*

One perhaps surprising finding in the analysis of the questionnaire items is that there is considerable agreement amongst upper secondary A-level mathematics students that mathematics is something everybody should learn. This is remarkable, since it means that in the general A-level population mathematics is seen as useful outside a school setting and perceived as providing knowledge and skills relevant for the life of citizens in society in general.

| Mathematics for everybody? | 1st Year | 3rd Year |
|---|---|---|
| Yes | 138 | 139 |
| No | 1 | 2 |
| Do not know | 5 | 5 |
| Yes and no | 3 | 1 |

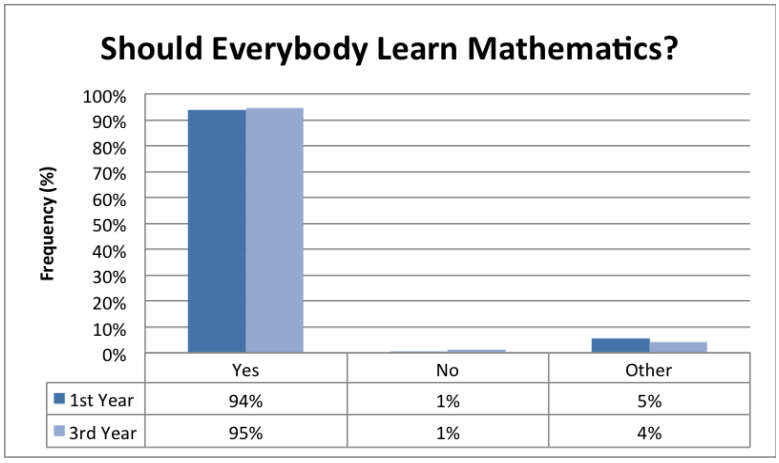



*Illustration 7: Descriptive Analysis: Item 2. Mathematics for all. Frequency in percent.*

The fact that this answer is stable suggests that the experiences with A-level mathematics teaching in Danish upper secondary school does not intervene with this particular facet of the aspect of '*Mathematics in Society*' in the students' beliefs about mathematics.

### *3. HOW DOES MATHEMATICS DEVELOP?*

The question of whether Mathematics is *invented* or *discovered* relates to a classical discussion concerning the philosophy of Mathematics. For the purpose of this study this discussion is merely touched upon, but interested readers may consult Hersh (1997) or Maddy (1990) for thorough unfolding of this discussion.

What is on the agenda for this study is primarily to make an inquiry into students' beliefs concerning one facet of this; the *interrelatedness* of *discovering* and *inventing*, when contributing to mathematical theory, and in line with this, that mathematical relations may be seen as already existing and that the application of names, symbols and definitions is invented by human beings. However, when students' answers to this item is pursued in the follow-up interviews, their ideas of mathematics as either a dynamic or a static discipline may also come up (see the section “Beliefs about Mathematics as a Discipline“ p. 49).

The typical answer from A-level students to the question of whether mathematics is invented or discovered, is that it is both (38% in the 1st year and 37% in the 3rd year). This part of the development is stable. Beside this trend, there is an increase in students seeing mathematics as invented (25% in the first year and 36% in the 3rd year) and consequently a decline in students seeing mathematics as discovered (28% in the 1st year and 18% in the 3rd year).

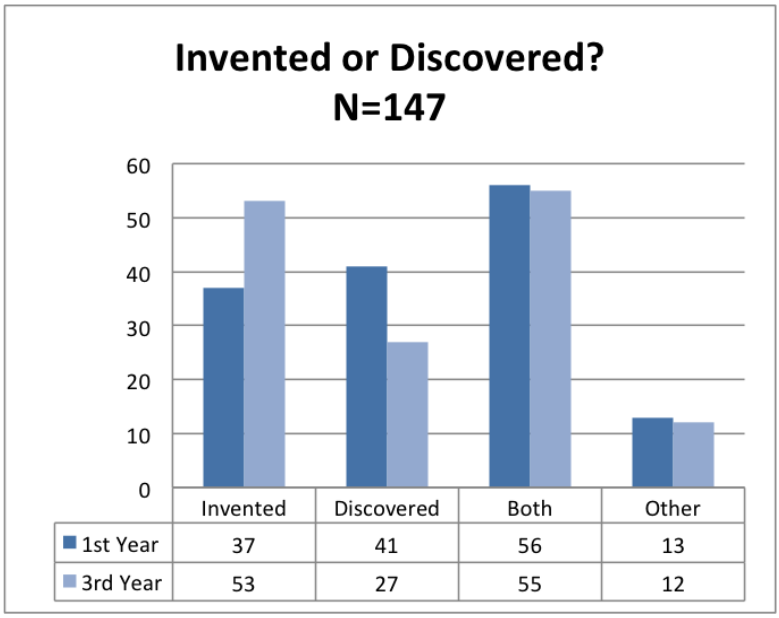



*Illustration 8: Descriptive Analysis: Item 3. Invented or discovered? Frequency in numbers.*

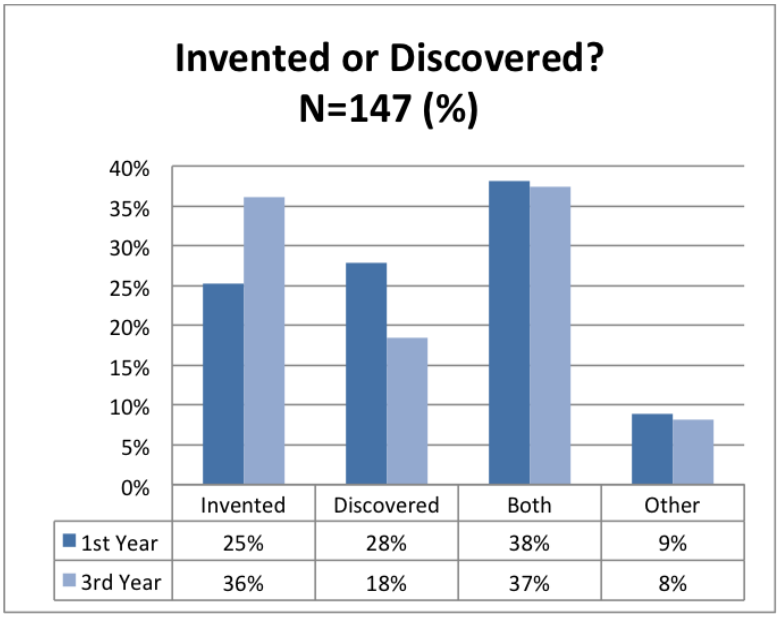



*Illustration 9: Descriptive Analysis: Item 3. Invented or discovered? Frequency in percent.*

A consequence of seeing mathematics as *invented*, only, is that it not seen as existing

already independently of human beings. Consequently, seeing mathematics as something human beings *discover* may relate to an idea of mathematics existing already, independently of human beings. The first of these two options may be seen as a dynamic view of mathematics and the latter as a static view.

The development in the students' answers suggests that learning A-level mathematics in Danish upper secondary schools influences the students' beliefs about the nature of mathematics in a direction towards a less static and more dynamic view.

### *4. SHOULD ONE BE A GENIUS TO STUDY MATHEMATICS?*

This questionnaire item investigates the question of whether the capability for studying mathematics in university is believed to be reserved for people of a certain type to which you either belong or do not, and if you do not, then there is nothing to do about it.

| Genius | 1st Year | 3rd Year |
|---|---|---|
| Yes | 20 | 14 |
| No | 96 | 97 |
| Do not know | 29 | 33 |
| No answer | 2 | 3 |
| **Total** | **147** | **147** |

It is remarkable that two thirds of the students in the population in the 1st and in the 3rd year of A-level mathematics in upper secondary school agree on the belief that one does not necessarily have to be a genius in order to study mathematics at university.

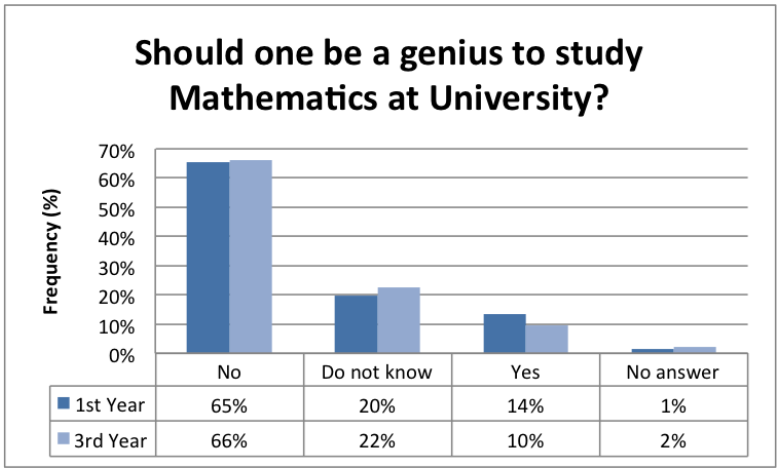



*Illustration 10: Frequency in answers to the item: "Should one be a genius to study mathematics in university?" N=147.*

This could be an indication of a general belief amongst the students that mathematics is something you can learn if you invest a sufficiently large effort.

| Answer | Comments |
|---|---|
| **[No]** | 1. *But you need to be on top of the basics*[132]. |
| | 2. *No, but good at it – have an understanding of it and be interested in it*[133]. |
| | 3. *But you need a good memory for remembering all the proofs and theorems*[134]. |
| | 4. *But having an interest - and being a genius is definitely a huge advantage*[135] |
| | 5. *But it will not harm*[136]. |

*Table 20: Comments to the option [No] to the question of one should be a genius in order to study mathematics in university.*

However, amongst those student discarding the necessity of being a genius in order to study mathematics at university, the least "exclusive" comments suggest

132 Men man skal have styr på det grundlæggende

133 Nej men god til det – have en forståelse for det og interessere sig for det

134 Men skal have en god hukommelse for at huske alle beviser og sætninger

135 Men have interessen og det at være et geni er klart en stor fordel

136 Men det skader ikke

that you should be on top of the basics, or just be good at it and interested in it (Table 20, p. 95). One student suggests that you need a good memory to remember all the theorems and proofs, and two students write that being a genius is a "*huge advantage*" or "*it will not harm*", suggesting that studying mathematics in university may not require genes of a genius, but those having them would have an advantage that the ordinary student would not.

Amongst the students ticking off [I do not know] as an answer to whether one should be a genius in order to study mathematics in university, (20% in the 1st year, 22% in the 3rd year) some students commented by suggesting that one should be clever, rather than necessarily a genius, and one student writes that it is easy to be admitted, but it is a hard study programme (Table 21, p. 96).

| Answer | Comments |
|---|---|
| **[Do not know]** | *Clever, but not a genius*[137]. |
| | *Easy to get admitted – HARD STUDY PROGRAMME*[138] |
| | *Yes and no*[139] |
| | *Yes and no, but you should feel for it and be dedicated to it*[140] |
| | *A little*[141] |

*Table 21: Comments to the option [I do not know] to the question of one should be a genius in order to study mathematics in university.*

137 Klog men ikke geni
138 Let at søge ind – HÅRDT STUDIE
139 Både og
140 Ja og nej, men skal i hvert fald have lyst og være indstillet til det
141 Lidt

| Answer | Comments |
|---|---|
| **[Yes]** | *If a genius is defined as a person who has a flair for mathematics, then yes*[142] |
| | *Logician*[143] |
| | *At any rate, one needs to remember a lot, maybe not a genius, but good at mathematics and having a good understanding*[144] |

*Table 22: Comments to the option [Yes] to the question of one should be a genius in order to study mathematics in university.*

For those choosing [Yes] as an answer to whether one should be a genius to study mathematics in university, some comments suggest that one should have a flair for it, be good at logical thinking or even be good at remembering and understanding (Table 22, p. 97).

142 Hvis man definerer geni som en der har anlæg for mat, så ja

143 Logiker

144 Man skal i hvert fald huske meget, måske ikke geni, men være god indenfor matematikken og have en god forståelse

### *5. WHAT ISSUES INVOLVE MORE CHALLENGES TO YOU?*

In item #14, the students were asked to rate five *issues [ a) Remembering; b) Computing; c) Figuring out the purpose of a task; d) Finding a way to solve a task; e) Reading and understanding the textbook]*[145] concerning mathematical activity, in terms of the level of challenges involved in that issue, by ticking off amongst these options: [1] if this issue involved [The most challenges]; [2] if this issue involved [Several challenges]; [3] if the issue in question involved a [Moderate] level of challenges; [4] if that particular issue only involved [Few] challenges and finally [5] if this issue involved [The fewest] challenges.

Most students find "computing" to involve either [few] or [the fewest] challenges (70%), and this proportion stays stable from the 1st to the 3rd year questionnaire. This answer may relate to the fact that "computing" is a more simple activity compared to the other issues, but it may also be due to the wide-spread use of technological aids for computing applied in the teaching in upper secondary school.

The issue most frequently indicated to involve [several] or [the most] challenges, is "remembering". In the 1st year 27% of the students indicate this issue to involve [several] or [the most] challenges, which is much more than those indicating one of the other four issues to involve these levels of challenges: Only 7% indicate this for "computing", for "figuring out the purpose of a task" it is 19% and for "finding a way to solve a task" these ratings are given by 21%, and finally "reading and understanding the textbook" receive these levels of rating by 18% of the students. In the 3rd year "remembering" is still the issue involving the most challenges to the most students, and the proportion indicating these levels increases to 38%.

145 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

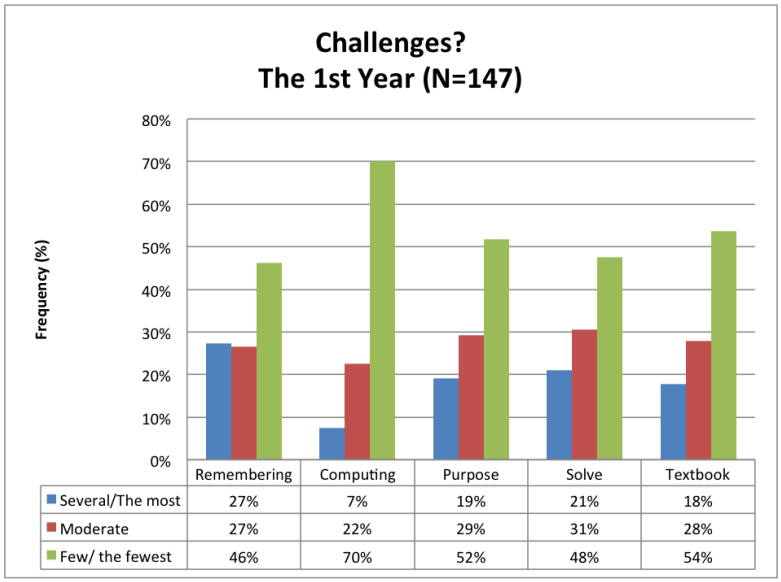



*Illustration 11: The 1st Year: Students' rating of five issues concerning mathematical activity, but the level of challenges involved in dealing with this issue. (in %) N=147*

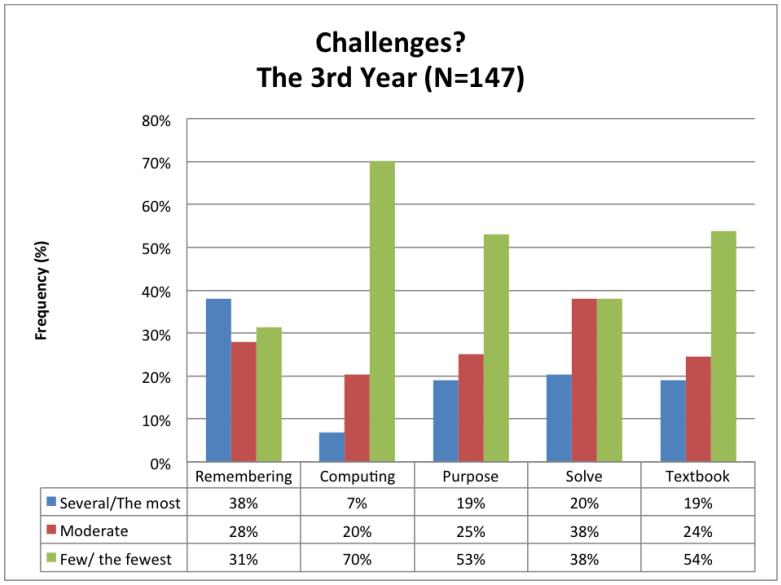



*Illustration 12: The 3rd Year: Students' rating of five issues concerning mathematical activity, but the level of challenges involved in dealing with this issue. (in %) N=147*

This means that "*remembering*" seems to be the hardest challenge for A-level mathematics students in upper secondary school amongst the suggested issues, and that the pressure on this challenge increases for the students from the 1st to the 3rd year. The fact that this challenge seems more important than the others, may have implications for the belief aspect *Mathematics at School* for the A-level mathematics student: "*Being good at mathematics at school involves being good at remembering*" would be likely to relate to these answers.

The proportion of students ticking off [several] or [the most] challenges for the other four issues: *b) Computing; c) Figuring out the purpose of a task; d) Finding a way to solve a task; e) Reading and understanding the textbook,* stays stable from the 1st to the 3rd year.

For the issue "*Finding a way to solve a task*", the proportion of students indicating this to involve [Few] or [The Fewest] challenges decreases from 48% in the 1st year to 38% in the 3rd year. This could be an indication pointing to the increasing demands from the 1st to the 3rd year for the A-level students, but also an example of the ways in which mathematics has become harder from the 1st to the 3rd year.

As one last comment to this item, it should be noted that the A-level mathematics students did not distribute their rating evenly on the levels of challenges. The level [Few] challenges is much more common than the others, being applied in 37% of the ratings of the issues of challenges, which in general is skewed to "the right" meaning that the students prefer to indicate the lower or the moderate levels of challenges as opposed to the higher levels of challenges.

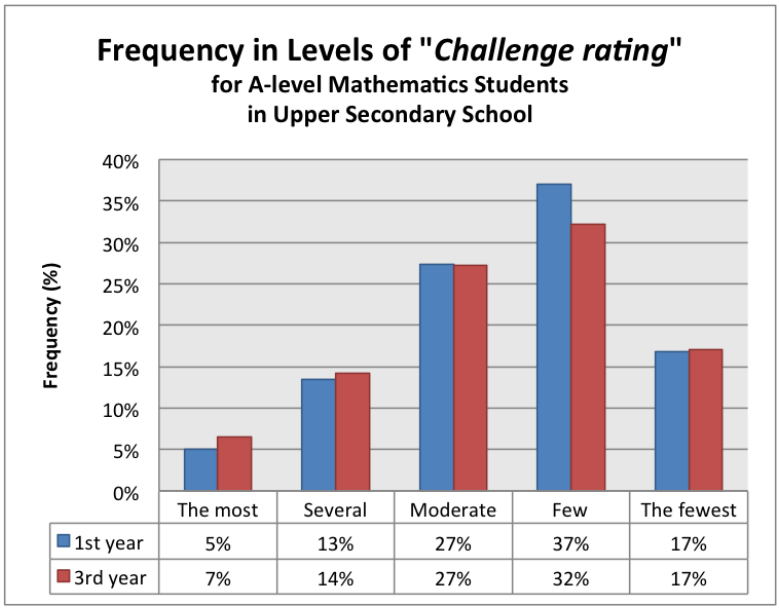



*Illustration 13: Frequency in the five different levels for challenges in Item #14: "*What issues involve more challenges to you?*". If all the students had chosen an equal distribution of the five levels for challenges, each level would have received 20% of the ratings. However,* [Few Challenges] *is more typical than the other possible levels of rating.*

### *6. SUPPORT FOR MATHEMATICAL ACTIVITIES*

Both in the 1st year and in the 3rd year, classmates are the main source of support (possibly amongst several sources) for mathematical activities (75% of the A-level students tick off this option in the 1st year questionnaire, and 88% of the A-level students tick off this option in the 3rd year questionnaire). More than half of the students (54%) can get help from their parents in the 1st year, but in the 3rd year this has declined to less than a third of the students (29%). Siblings are a resource to 30% of the students in the 1st year, but in the 3rd year this has declined to 17% of the students. Beside classmates, parents, siblings and other relatives, typically, the A-level students can also get help from the *teacher*, the "*Homework Café*" at the school, they find help in *books*, on the *Internet* or maybe from a friend of the family or a boyfriend or a girlfriend.

The general trend in the population of A-level mathematics students in upper secondary school is that there is a decline in the possibility of getting help from family members, and you become more dependent of your peers. Some students mention in their questionnaire or in the interview that since mathematics has become much harder in upper secondary school compared to lower secondary school, they need to cooperate with their peers to a much greater extent than they did in lower secondary school.

To quite a few of these students, this phenomenon is very welcome; it is appreciated that they are no longer the only student in class interested in and good at mathematics. Now, in an A-level programme, they are teamed up with all the other students that were good at mathematics in lower secondary school, and there are more possibilities for getting help. However, being one of those students who cannot get help at home, is mentioned as a disadvantage.

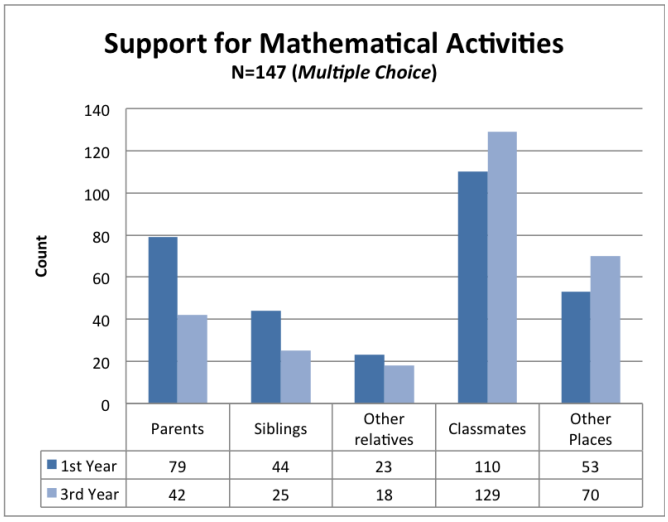



*Illustration 15: Descriptive Analysis: Item 5. Support. Frequency in answers.*

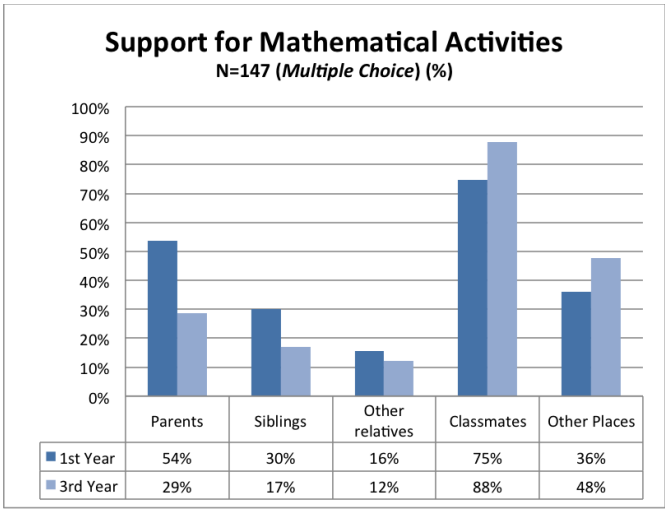



*Illustration 16: Descriptive Analysis: Item 5. Support. Frequency in percent.*

### *7. PRELIMINARY PLANS*

The answers to the question of preliminary plans after upper secondary school are diverse, because the question was open-ended. The answers have been grouped as shown in the table below (Table 23, p. 104).

| | |
|---|---|
| **STEM** | Science-Technology-Engineering-Mathematics |
| **MED-VET** | Medicine/ Veterinarian/ Nursing/ Health/ Physical Education Studies |
| **ART-SOC-BUSI-EDU** | Arts/ Humanities/ Social Science/ Law/ Educational Studies/ Business |
| **POL-MILI** | Police/ Army/ Airforce/ Navy/ Emergency Management Agency |
| **SOME-EDU** | Some kind of unspecified tertiary education |
| **NONE-YET** | No clear plans yet |

*Table 23: Categories for preliminary plans.*

***STEM**: Any kind of education concerning physics, chemistry, biology, biotechnology, engineering, technology or mathematics falls under this category. There is no differentiation between bachelor of engineering and Ph.D. in physics, just to mention two answers. The criterion for classifying answers into this category has been whether there was a STEM element as part of the student's plan or not, since this is central to the research aim of the study.*

***MED-VET**: In this category I have placed both "coach in nutrition", medical doctor, veterinarian nurse, and anything health-related for humans or animals.*

***ART-SOC-BUSI-EDU**: In this category studies within the arts, such as conservatory of music, architecture and design are grouped together with studies like lawyer, teacher, social worker and business studies, despite their diversity.*

***POL-MILI**: These educational tracks have in common that they involve wearing a uniform. They encompass the military in general, including the Defence Force, the Navy and the Airforce as well as the Emergency Management Agency and the Police.*

***SOME-EDU**: In this category the quite common answer "university" is placed together with any variation of the answer "some kind of education".*

***NONE-YET**: In this category I have placed both answers resembling "I do not know*

*yet" or just writing a question mark "?".*

In the 1st year questionnaire 142 students answered this question and 146 in the 3rd year questionnaire. The reason why it was not 147 students in the first year, as in some of the other items, is that the item of plans for future education was sent in a separate supplementary questionnaire and some students may not have answered both questionnaires. In the third year questionnaire, one student has left the item unanswered.

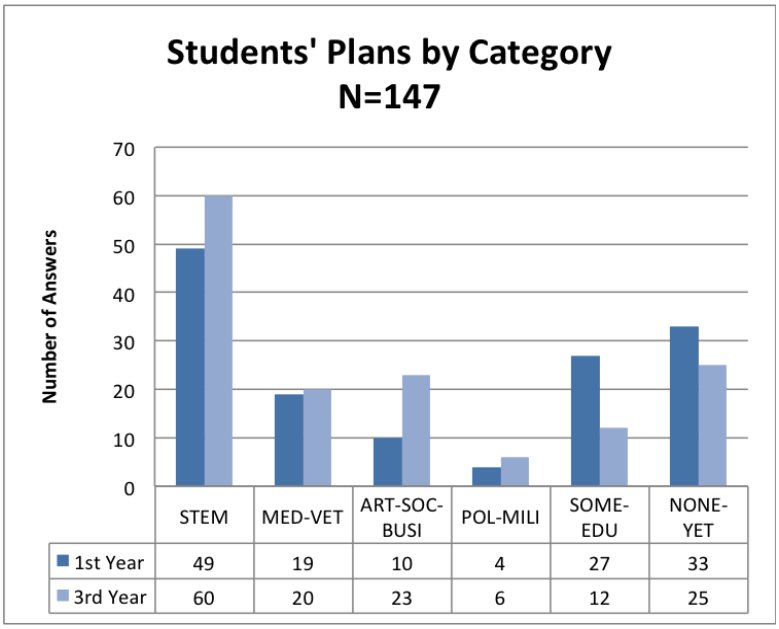
| | STEM | MED-VET | ART- SOC- BUSI- EDU | POL-MILI | SOME- EDU | NONE- YET | Total |
|---|---|---|---|---|---|---|---|
| **1st Year** | 49 | 19 | 10 | 4 | 27 | 33 | 142 |
| **3rd Year** | 60 | 20 | 23 | 6 | 12 | 25 | 146 |
| **Change** | *11* | *1* | *7* | *2* | *-15* | *-8* | *4* |

*Table 24: Number of students in each category of educations for future plans after upper secondary school.*

| | STEM | MED-VET | ART- SOC- BUSI- EDU | POL-MILI | SOME- EDU | NONE- YET | Total |
|---|---|---|---|---|---|---|---|
| **1st Year** | 35% | 13% | 7% | 3% | 19% | 23% | 100% |
| **3rd Year** | 41% | 14% | 16% | 4% | 8% | 17% | 100% |
| **Change** (pct. points) | *(+7 pp)* | *(0 pp)* | *(+9 pp)* | *(+1 pp)* | *(-11 pp)* | *(-6 pp)* | |

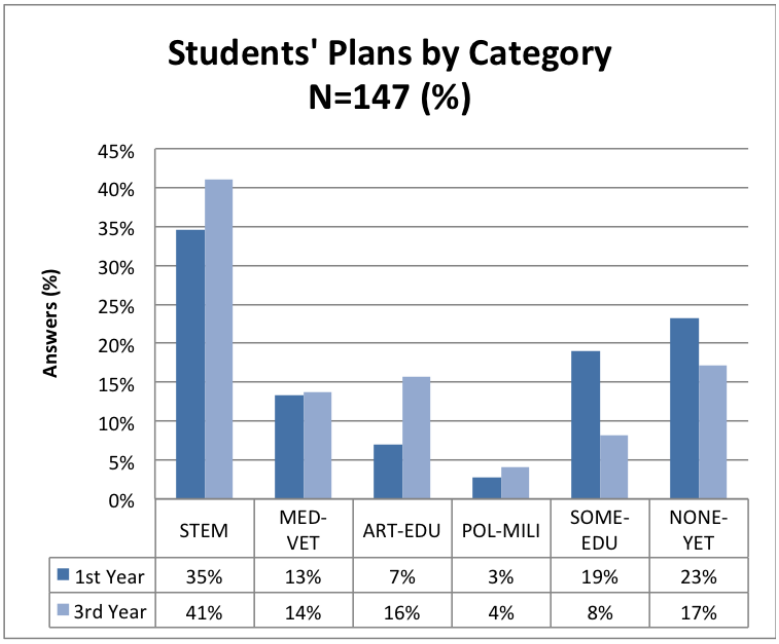
*Table 25: Percentage of students in each category of educations for future plans after upper secondary school. (N=142 in the 1st year, N= 146 in the 3rd year)*

The STEM category includes all studies that may draw on mathematics to a lesser or greater extent. Biology may not involve just as much mathematics as physics or statistics, but it has not been deselected completely.




| | STEM | MED-VET | ART-SOC-BUSI | POL-MILI | SOME-EDU | NONE-YET |
|---|---|---|---|---|---|---|
| 1st Year | 49 | 19 | 10 | 4 | 27 | 33 |
| 3rd Year | 60 | 20 | 23 | 6 | 12 | 25 |

*Illustration 17: Descriptive Analysis: Item 6. Plans. Frequency in numbers.*




| | STEM | MED-VET | ART-EDU | POL-MILI | SOME-EDU | NONE-YET |
|---|---|---|---|---|---|---|
| 1st Year | 35% | 13% | 7% | 3% | 19% | 23% |
| 3rd Year | 41% | 14% | 16% | 4% | 8% | 17% |

*Illustration 18: Descriptive Analysis: Item 6. Plans. Frequency in percent.*

The category for Medicine and Veterinarian educations and the like, have in

common that most of them, at least medicine, which is the most common study aim in this group, demand A-level mathematics for getting admission, but it may not involve that much mathematics in the actual content. The other categories do not demand A-level mathematics, neither for admission nor for the content.

The general trends in the development from the 1st to the 3rd year in the A-level students' plans are that the group planning to study medicine stays stable in size. Some students realise after a short while in upper secondary school , that they will not be able to get the average of grades needed for getting admission to medical school, but they may have realised that even before answering the first questionnaire. In this way, they may have left this group even before this study "begins to count". Another option is that the same student changes plans into a less selective – in terms of average of grades for admission – tertiary education but still in the MED-VET category.

*UNDECIDED STUDENTS*

There is a decline in the number of undecided students from the 1st to the 3rd year, which cannot be ascribed to the mathematics education in upper secondary school itself. It is a development that could be expected no matter what. Those who are considering an unspecified education or those who have no plans yet, decline from sixty people all in all in the 1st year to thirty-seven in the third year, which is twenty-three people less. Half of that number seem to end up in the STEM-category and the other half seem to choose the ART-SOC-BUSINESS category, still leaving close to a fourth of the A-level students undecided of their future education when they are half the way through their last year of upper secondary school. Many students plan to take a year off and work before they decide if and what they want to study. Some students may even decide to take secondary education again, but in a vocational programme.

***STEM***

The most popular category in the students' plans is the STEM educations, counting more than one third of the students in the 1st year and more than two fifth in the 3rd year. This is a broad category, and students may change from considering a master programme in engineering to a bachelor programme in engineering, or they may change from one branch of engineering to another. This is not the main point in the analysis, which is strictly to consider whether STEM subjects stay as an idea in the student's plans for her- or himself in the future, which is one of the main aims of the study.

## *7. CHOOSE OR AVOID MATHEMATICS?*

The questions of whether one would actively choose an education involving a good deal of mathematics after upper secondary school is related to the question of whether one would try to avoid an education involving a good deal of mathematics.

### POSITIVELY CHOOSE MATHEMATICS

The general trend in the 147 students from the nine classes participating in the questionnaire study, is a decline in the number of students indicating to be willing to choose mathematics positively in their tertiary education after upper secondary school. In the 1st year, 112 of the 147 students (76%) would actively choose an education involving a good deal of mathematics for tertiary education, but only 83 students (56%) in the 3rd year; a decrease consisting of 38% of the 112 students positively choosing mathematics for tertiary education in the 1st year questionnaire. (Which is 20% of the student population N=147).

| Positively Choose Mathematics? | 1st Year | 3rd Year |
|---|---|---|
| Yes | 112 | 83 |
| No | 4 | 25 |
| Do not know | 22 | 31 |
| No answer | 9 | 8 |
| **Total** | **147** | **147** |

Consequently, there is an increase in the number of students not willing to positively choose mathematics in their tertiary education, from 4 students (3% of 147) in the 1st year to 25 students (17% of 147) in the 3rd year and also in those students who indicate not to know whether they would positively choose mathematics or not.

This is an indication that the experiences in upper secondary school A-level mathematics leads to a deselection of mathematics-related tertiary studies for 20% of those students initially indicating a positive attitude to such education programmes.

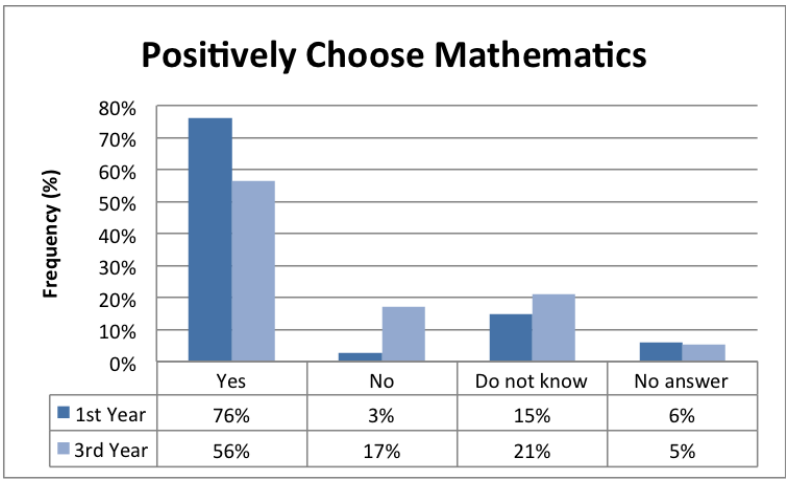



*Illustration 19: Frequency in answers to the item: "Could you imagine positively choosing an education involving a good deal of mathematics?" N=147.*

Possible reasons for this trend will be pursued in the qualitative analysis of the interviews of case informants.

### AVOID MATHEMATICS

As to the question of whether the student would try to avoid an education involving a good deal of mathematics, 80% refuse that option in the 1st year, but in the 3rd year, only 60% of the A-level mathematics students refuse that they would try to avoid an education involving a good deal of mathematics.

| Avoid Mathematics? | 1st Year | 3rd Year |
|---|---|---|
| Yes | 4 | 24 |
| No | 117 | 88 |
| Do not know | 15 | 27 |
| No answer | 11 | 8 |

The group of students confirming that they would try to avoid an education involving a good deal of mathematics increases from 3% in the 1st year to 16% in the 3rd year. Students indicating that they do not know whether they would try to avoid an education involving mathematics increases from 10% in the 1st year to 18% in the 3rd year.

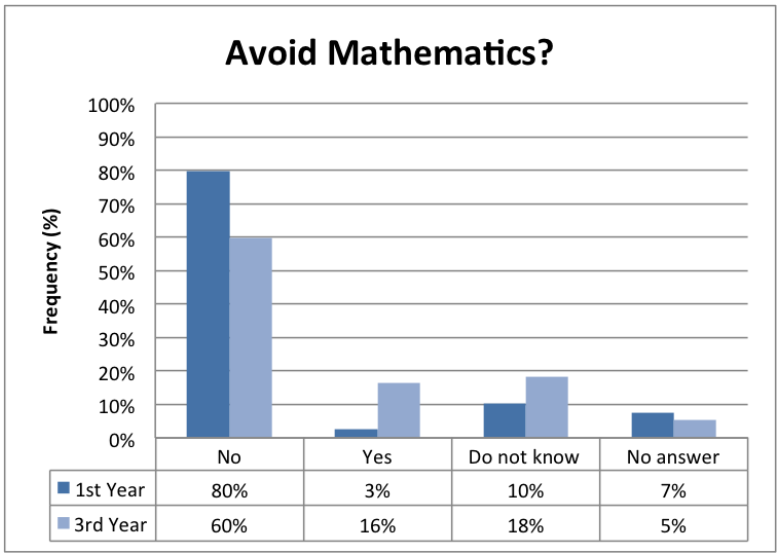



*Illustration 20: Frequency in answers to the item: "Could you imagine avoiding an education containing a good deal of mathematics?" N=147.*

All in all 40% of the students do not directly reject the question of avoiding mathematics. Amongst those not rejecting it, we would find students who may positively want to do something that does not involve mathematics, such as music related education programmes, as well as students who do not know whether for example medical school would involve mathematics or not.

## *GENERAL* VERSUS *TECHNICAL* UPPER SECONDARY SCHOOL

In the following subsections, indications of differences between the answers of the students in the general versus the technical upper secondary school programmes are pursued. A first indication was found in item #1, which involves the so-called "Favourite Subject Scale": "*On a scale from 1 to 10, on which 10 stands for your favourite subject, how would you rate mathematics?".* As a point of departure, the differences in this item will be explored, but afterwards we shall turn to some of the other items, to see if we can find indications of explanations to this phenomenon.

### *FAVOURITE SUBJECT SCALE*

The general decrease in rating from the 1st to the 3rd year of upper secondary school, seemed to be less prevalent in STX (the general upper secondary school programme) compared to HTX (the technical upper secondary school programme). In the group of STX A-level students in this study (N=89), there is an increase in those students rating mathematics on [9] or [10] on the scale, but there is a decrease in those rating it [8] and an increase in the low end (from 16% of the students in the 1st year to 26% of the student in the 3rd year).

In HTX (the technical upper secondary school programme), the decrease in the rating amongst the 58 students participating in this study seems much more severe; In the low end (from [1] to [7]) there is an increase from 33% in the 1st year to 74% in the 3rd year. In the middle, the rating [8] received 31% in the 1st year, but only 16% in the 3rd year, and the high ratings ([9] and [10]) received 36% in the 1st year, but decreased to 10% in the 3rd year.

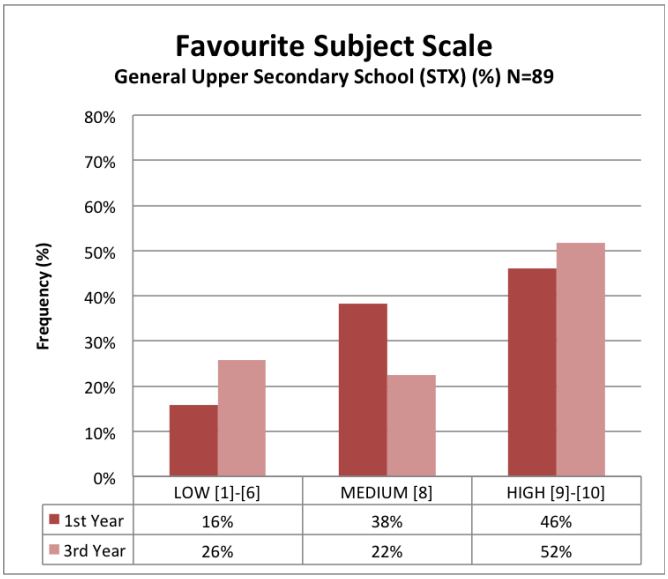



*Illustration 21: STX: (Item #1)*

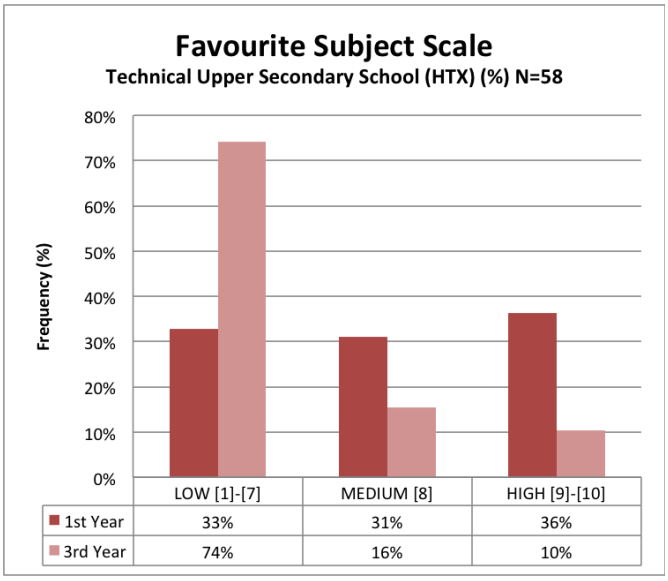



*Illustration 22: HTX: (Item #1)*

This means that the initial level in the HTX group of students was lower compared to the rating in the STX group, but then the considerable decrease in rating in the HTX group makes the difference between these two groups appear even more dramatic.

It should be kept in mind that neither the STX group nor the HTX group were selected randomly, nor did the response rate reach any impressive level. The two groups can therefore not be taken as representative for STX or HTX students in general. However, even though the material does not allow for generalisation from the sample to the whole population, the indications apparent in this material suggests that these differences deserve to be explored further.

In the following sections, items involving *the possibilities for support for mathematical activities;* the question of whether one considers *choosing mathematics positively in further education;* the question of whether one considers *avoiding mathematics in further education* and the question of *future plans after upper secondary school* (grouped by category) serve as means for adding to the understanding of these differences.

### *SUPPORT*

The two groups of A-level mathematics students, the STX and the HTX students, have in common that peers are the main resource for support for mathematical activities; in STX more than 80% of the students mention this resource, while two thirds or 66% of the HTX students mention this resource in the 1st year, while it increases to 90% in the 3rd year. In both groups close to half of the students mention their parents as a resource for support for mathematical activities in the 1st year (56% of the STX students and 50% of the HTX students), and in both groups this proportion reduces to less than 30% in the 3rd year (29% in STX and 28% in HTX. The differences in the support from siblings or other relatives are small, and so they are not too likely to provide indications of explanations of the differences on the favourite subject scale between the two groups.

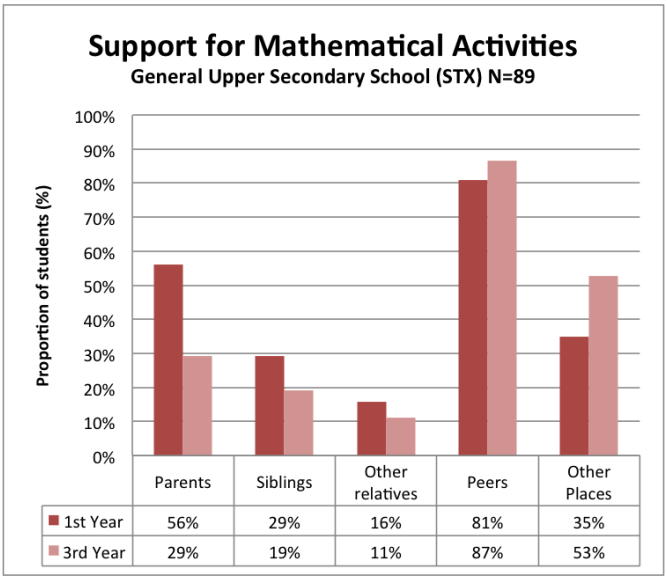



*Illustration 23: STX: Where can you find support for mathematical activities?(Item #15)*

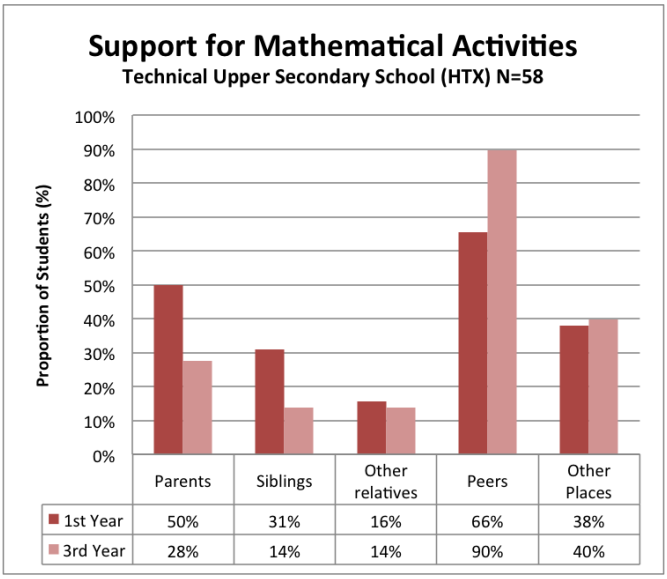



*Illustration 23: HTX: Where can you find support for mathematical activities?(Item #15)*

***POSITIVELY CHOOSE MATHEMATICS?***

*"Could you imagine opting for an education involving a good deal of mathematics?"*

Concerning the matter of plans after upper secondary school, some differences between the STX and the HTX group appear.

More STX students choose [Yes] to the question of positively choosing an education involving a good deal of mathematics compared to the HTX group in both the 1st year and the 3rd year questionnaire; In the 1st year slightly more than 80% of the students choose [Yes] in the STX group compared to slightly less than 70% in the HTX group. In the 3rd year questionnaire both the STX group and the HTX group show a decline in the proportion of students choosing [Yes] to the question of positively opting for an education involving mathematics by close to 20 percentage points; now slightly more than 60% of the STX students and half of the HTX students choose this option. Consequently, the proportion of students in each of the two groups answering [No] to this question increases; in STX from 1% in the 1st year to 13% in the 3rd year and in the HTX group from 5% in the 1st year to 22% in the 3rd year.

It may come as a surprise that the students who chose the HTX programme apparently are less interested in continuing dealing with mathematics after upper secondary school compared to the general upper secondary programme.

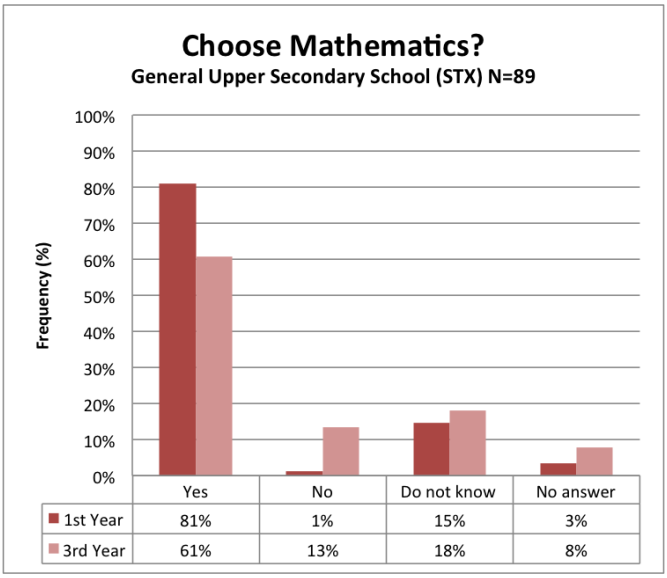



*Illustration 25: STX: Could you imagine opting for an education involving a good deal of mathematics?(Item #21a)*

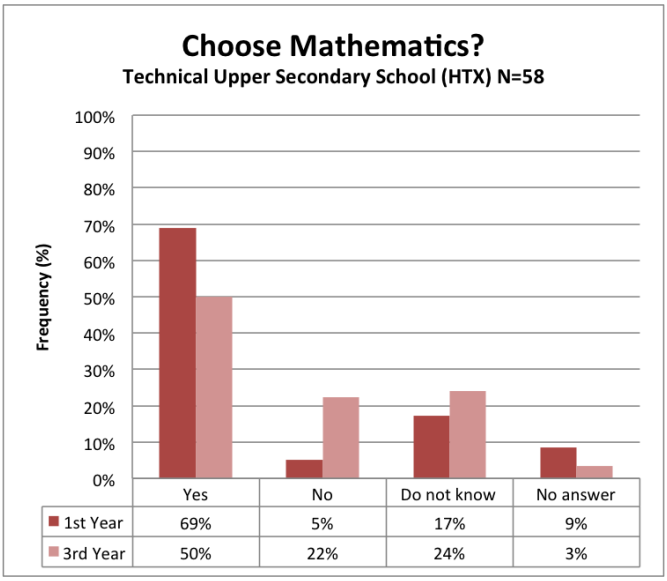



*Illustration 26: HTX: Could you imagine opting for an education involving a good deal of mathematics?(Item #21a)*

### *AVOID MATHEMATICS?*

*"Could you imagine trying to avoid an education involving a good deal of mathematics?"*

The trends in the answers to the question of trying to avoid mathematics after upper secondary school correspond to the answers to the question of choosing mathematics positively after upper secondary school; also here we see a decline of close to 20 percentage points from the 1st to the 3rd year in both the STX and the HTX group. This means that the proportion of students answering [No] to the question of whether they would try to avoid mathematics in tertiary education decreases considerably from the 1st year questionnaire to the 3rd year questionnaire answers:

In the 1st year more than 80% of the STX students refuse that they would try to avoid mathematics, declining to 64% in the 3rd year, and in the HTX group three fourths of the students (76%) in the 1st year and close to half of the students (53%) in the 3rd year answer [No] to the question of trying to avoid mathematics in tertiary education. This means that one third of the STX students and close to half of the HTX students are not rejecting the idea of trying to avoid mathematics after upper secondary school.

In the STX group the proportion of students answering [Yes] to the question of considering avoiding mathematics increases from 3% only in the 1st year to 13% in the 3rd year; an increase from hardly any students choosing this answer to a proportion close to one out of every eight students. In the HTX group this trend is even more prevalent; in the 1st year only 2% of the HTX students answered [Yes] to the question of whether they would consider trying to avoid mathematics in tertiary education. This proportion corresponds to 1 students in that population, but in the 3rd year more than 20% or one out of five students confirm that they would try to

avoid mathematics in tertiary education.

Again, it may come as a surprise that the technical upper secondary school students are less open towards mathematics than the students in the general programme. Also the fact than there is a clear decline in the answers from both programmes is rather intriguing.

Apparently, something happens during upper secondary school that influences on the ideas for educational plans after upper secondary school in the students in both programmes, but the HTX students are apparently less inclined already in the first year and this difference is maintained over the years in upper secondary school.

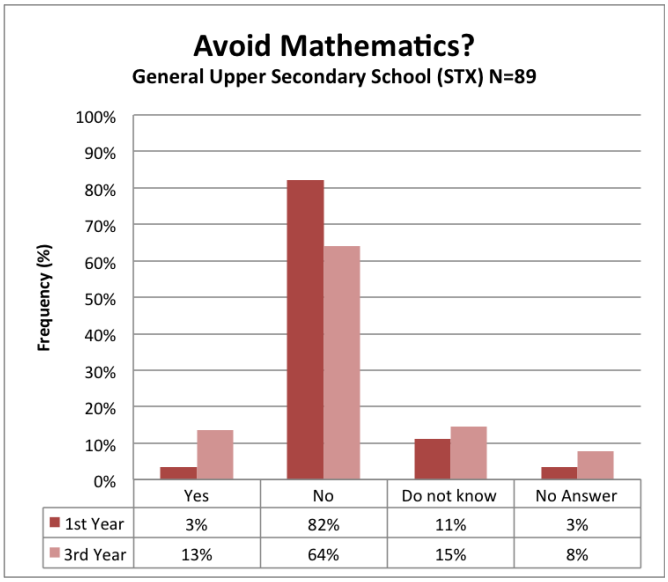



*Illustration 27: STX: Could you imagine trying to avoid an education involving a good deal of mathematics?(Item #22a)*

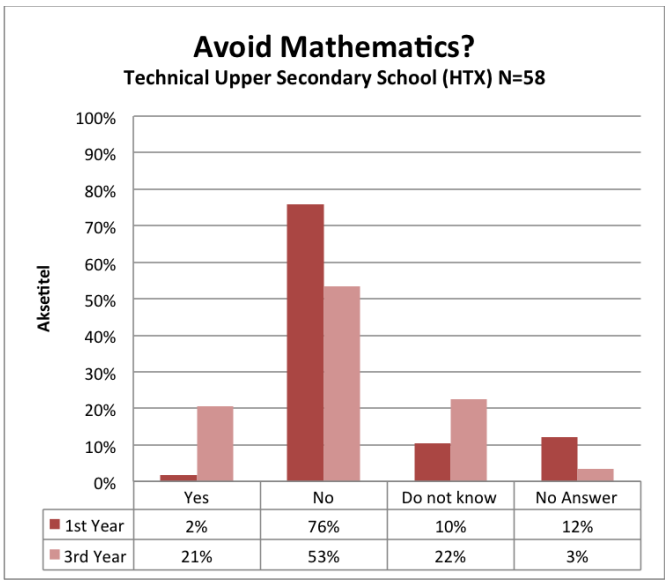



*Illustration 28: HTX: Could you imagine trying to avoid an education involving a good deal of mathematics?(Item #22a)*

### *PLANS BY CATEGORY*

Despite the fact that more HTX students than STX students indicate to try to deselect mathematics after upper secondary school, this does not influence on the direction of the plans of the HTX students; in the HTX programme the proportion of students opting for a tertiary education within the STEM is higher in the 1st year (41% of the HTX students versus 29% of the STX students) and stays slightly higher even though the STX students catch in over time.

In some sense the HTX students stay true to the STEM category for tertiary education, even though they become more disinclined to the mathematical content of them. This may be interpreted as a trend towards STEM study programmes less demanding in terms of the involvement of mathematics as an option for these students. However, even though the STEM category stays stable over the years for the HTX students, the NON-STEM category (ARTS/SOCIAL SCIENCES/BUSINESS) increases from 2% only in the 1st year to 16% in the 3rd year for these students.

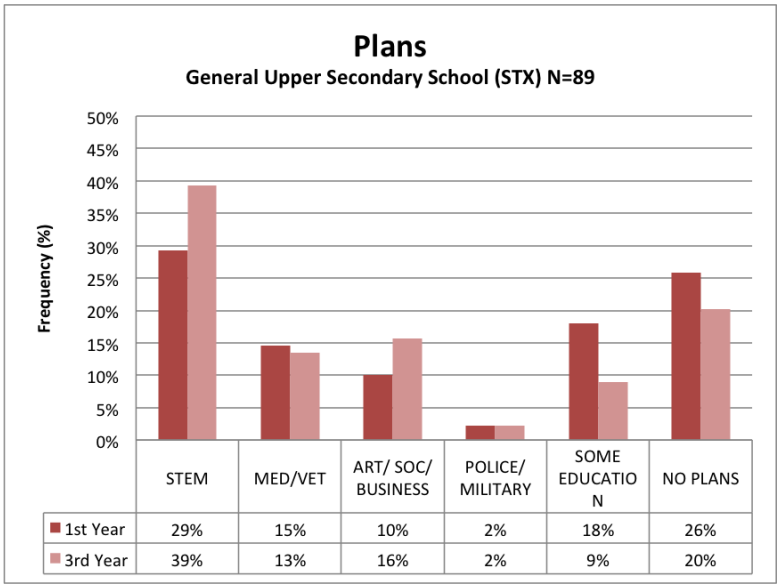



*Illustration 29: STX: What are your educational plans so far after upper secondary school?(Item #20). Open-ended question; researchers categorisation.*

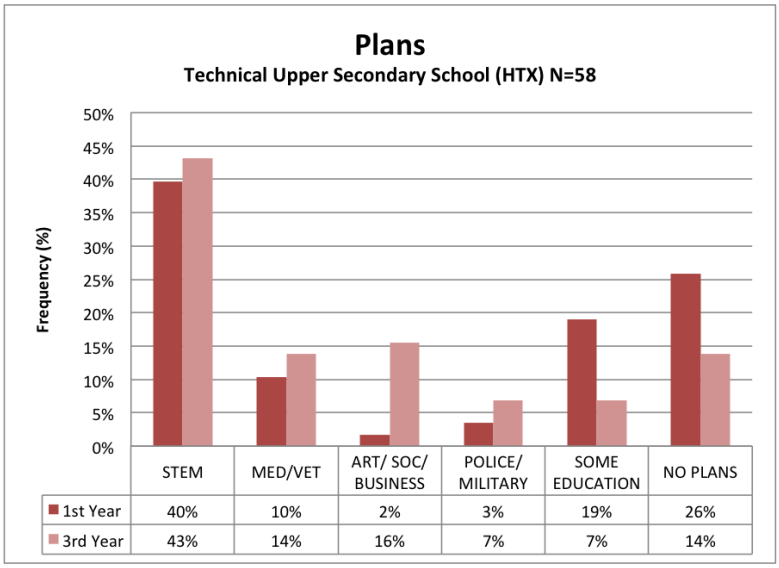



*Illustration 30: HTX: What are your educational plans so far after upper secondary school?(Item #20). Open-ended question; researchers categorisation.*

### *HAS MATHEMATICS SOMETHING TO DO WITH YOUR OTHER SUBJECTS?*

There is a clear trend in the answers to the question of whether mathematics relates to the other subjects for both groups of A-level mathematics students in science study programmes; it does for more than 90% of both the HTX and the STX students and stays high in both the beginning and at the end of upper secondary school.

This item was meant to explore the idea of the role of mathematics in other disciplines and this aspects of students mathematics-related beliefs is generally highly visible to the A-level students in science study programmes in upper secondary school.

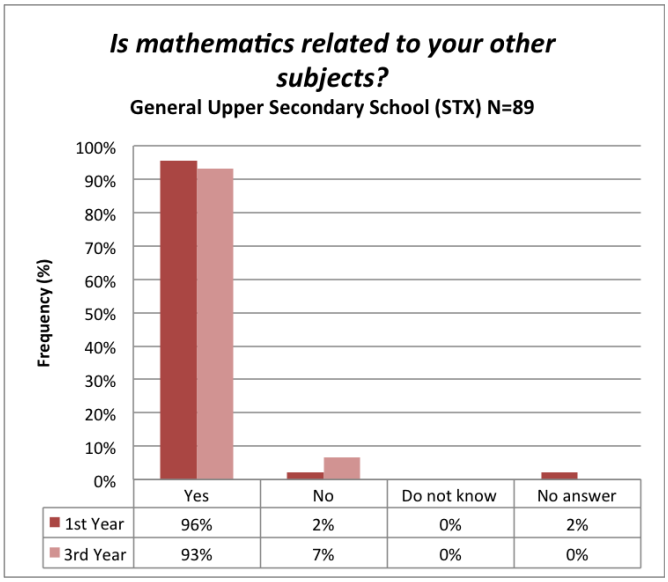



*Illustration 31: STX: Item #7a*

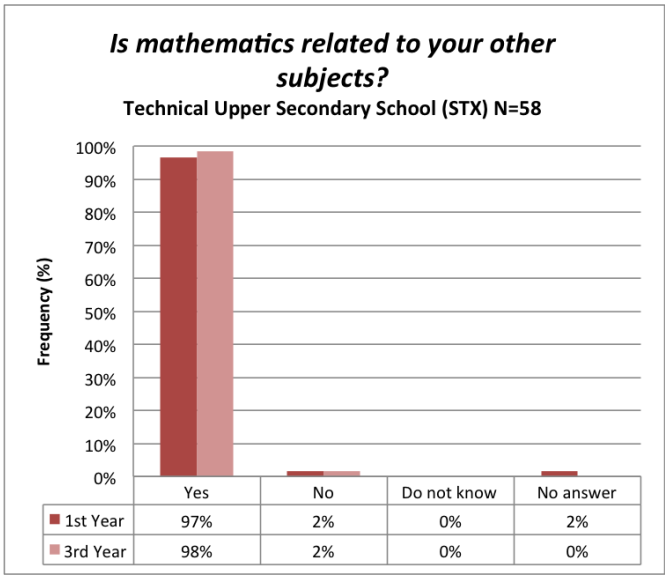



*Illustration 32: HTX: Item #7a*

### *MATHEMATICS IN LOWER SECONDARY SCHOOL*

As one last attempt to get to understand some of differences between the mathematics-related beliefs of the STX students and those of the HTX students, we shall now explore the answers to Item #3 in the questionnaires:

*Did you like mathematics when you went to lower secondary school?*

The students could choose between the following options:

1. [Yes, it was one of my favourite subjects];
2. [Yes, it was fine];
3. [It was okay];
4. [It was not really me];
5. [No, I did not like it at all];
6. [Other:_____]

In the graphs for the STX and the HTX students' answers to this question, we see a small difference; In the 1st year the tail of the graph is slightly longer in the HTX group, and the proportion of students in the top category [1] has a slightly lower level than in the STX group. In the STX group, the proportion of students answering the best category is around 70% (it is 69%, but the 1% answering [Other] added: "AWESOME"(sic.)). Together with the group of the 25% STX students ticking off the option [Yes, it was fine], 95% of the STX students are found in these two top categories. In the HTX group, the 62% indicating mathematics to have been one of their favourite subjects together with the 22% that chose the answer [Yes, it was fine] composes 84% of the group of A-level mathematics HTX students in this study. These numbers give some kind of indication in terms of a starting point, in which the HTX group has a take-off view in which a larger proportion of the students do not have mathematics in their top-category compared to the STX group.

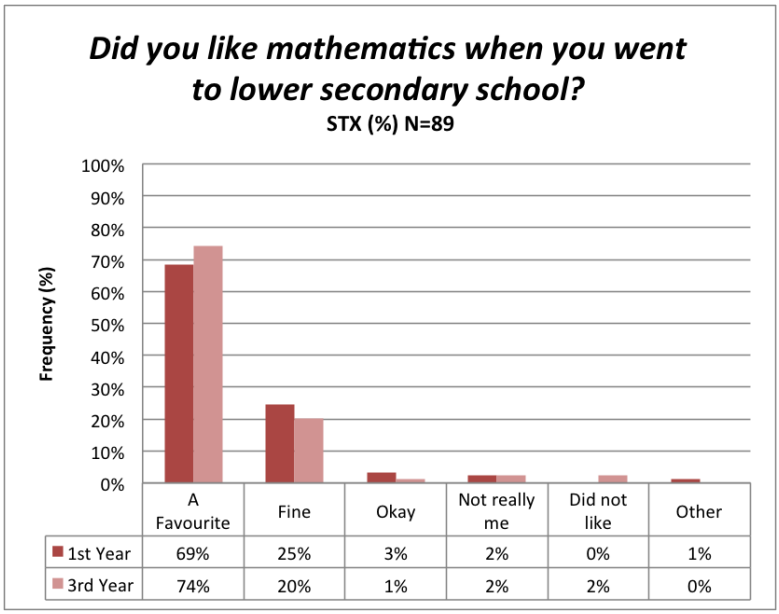



*Illustration 33: STX: Did you like mathematics in lower secondary school? Item #3. N=89.*

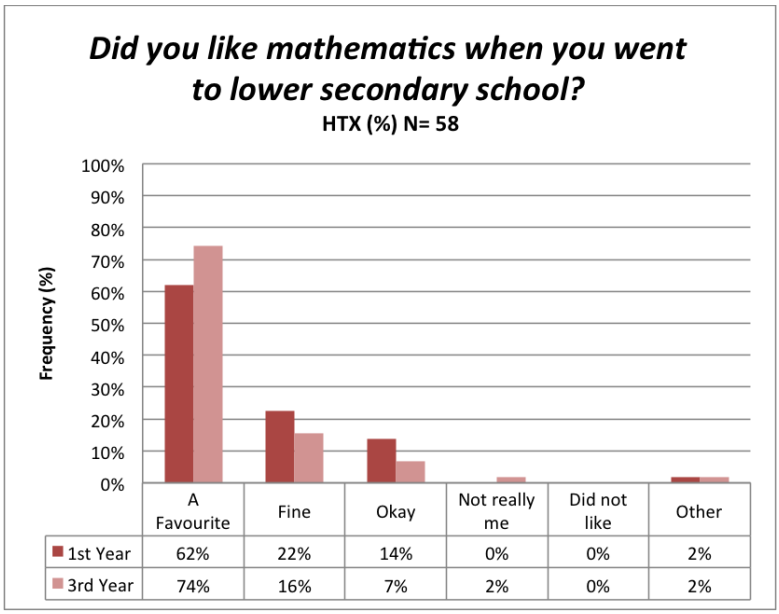



*Illustration 34: HTX: Did you like mathematics in lower secondary school? Item #3. N=58.*

In the 3rd year, a slightly larger group of students think back on mathematics in

lower secondary school a one of their favourite subjects compared to the 1st year answers; Now 74% in both the STX and the HTX group of students think back on mathematics in this light, but where 25% of the STX students select the next option [Yes, it was fine], only 16% of the HTX students choose this category. This means that 94% of the students in the STX group versus 90% of the HTX group refer to mathematics in lower secondary school in one of the two top categories. This is not a major difference, but still, it means that we find 9% of the HTX students compared to 5% of the STX students choosing the lower range of categories.

The development in the answers to this item over time may not say too much about the differences between the two groups in the 3rd year. But as an indication of the initial differences between the two groups, it may serve as a tool for the interpretations and attempts to evaluate influences that stems from experiences in mathematics education in upper secondary school understanding of the fact that not all aspects of development over the three years of upper secondary school happens due to the actual impact from these settings.

# *CASE SELECTION*

Initially, 24 students from four schools volunteered to participate in the interview part of the longitudinal study, beside answering the questionnaires. Of these 24 students, one student changed school after the first year, and thus exited the group. Three students did not meet in school on the day for the third year interviews and thus their trajectories are not fully covered till the end of third year. This means that 20 out of the 24 students participated fully in the extended version of the longitudinal study.

The cases are described by tracking the empirical material from the longitudinal study: Beginning with the 1st Year Questionnaire (from late November 2010 to early January 2011 first year in upper secondary school), and then the Interview Transcripts from the 1st Year (from late March to early May 2011 the first year), then proceeding to the 3rd Year Questionnaire (late November 2012 to early January 2013 during their last year of upper secondary school) and finally the Interview Transcripts from Third Year (March 2013, just before teaching ends and the final examinations begins).

I have chosen a strategy for the case analysis that involves an extensive display of empirical material of the case informants for the reader in order to make the process of analysis transparent. As a consequence of this strategy a set of six case informants have been selected in order to keep the analysis in a manageable format. The criteria for the selection of these six case informants is to be explained in the following sections.

## GROUPS OF RATING

As an approach to selecting cases for further analysis, a criterion for the selection has been to line up the changes in the rating of mathematics on the favourite subject scale from the questionnaires.

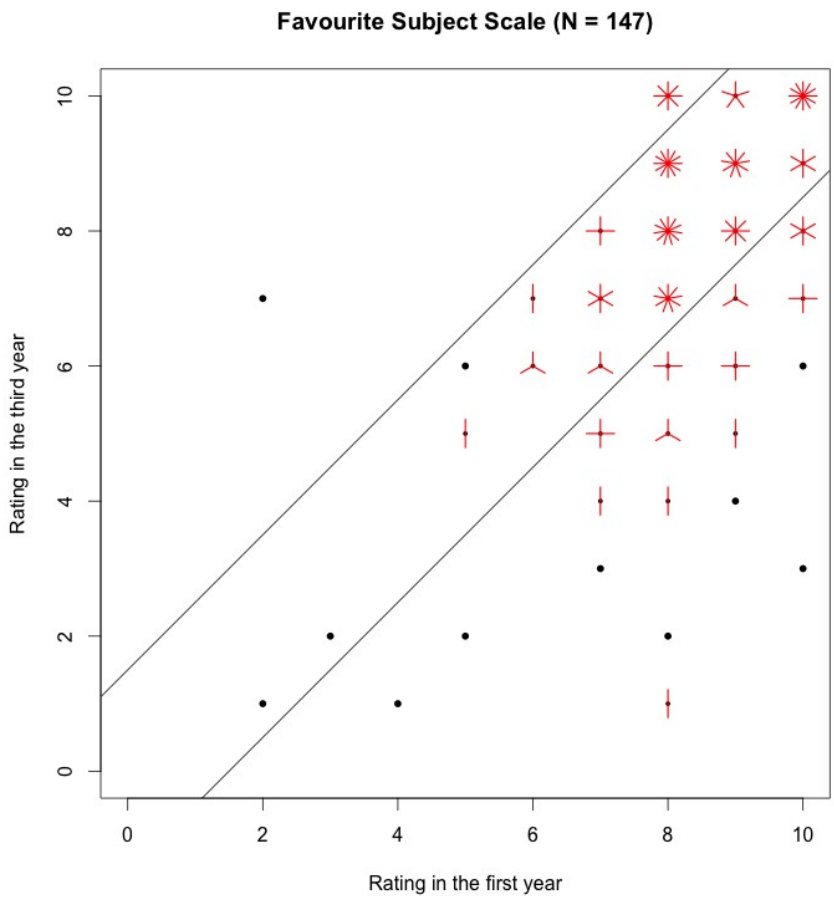



*Illustration 35: Sunflower plot (R Core Team, 2012) of the development in the rating on the favourite subject scale (FSS) from the 1st to the 3rd year in upper secondary school in the 147 respondents.*

*A dot without petals represent one student with that type of development, two petals means two students, three petals means three students, and so on.*

*UPPER LEFT REGION: INCREASE in rating on the FSS*

*REGION AROUND DIAGONAL: STABLE development in rating on the FSS*

*LOWER RIGHT REGION: DECREASE in rating on FSS*

As an illustration, the development in those respondents, who answered the question of their rating of mathematics in the scale and also wrote the same name on the questionnaires in both 1st year and 3rd year (N=147) has been plotted in a so-called SUNFLOWER PLOT in R (R Core Team, 2012) (see Illustration 1, p. 157).

This works as a first way of indicating the development from first to third year in their appreciation of mathematics as a subject in upper secondary school. The Item #1 ratings (see p. 87) were given by the students in the first year questionnaire and in the third year questionnaire. In the third year questionnaire the first year rating of the student was not revealed.

Note that the 'Favourite Subject Scale' (FSS) is an ordinal scale and thus allowing the statistic measures of *mode* and *median* as measures of *central tendency*, and not, as for the interval scale or the ratio scale, mean or standard deviation. The favourite subject scale is meant as a subjective measure, indicating the 'grade' the student would give the school subject mathematics at a certain point in time.

It can be read in the SUNFLOWER PLOT that one student rates mathematics [2] in the 1st year and increases the rating to [7] in the 3rd year (a dot with no petals in this point) and that two students decreased their rating of mathematics from [8] in the 1st year to [1] in the 3rd year (a dot with two petals in this point).

The general picture is that more students drop their rating of mathematics from the 1st to the 3rd year, compared to those increasing their rating of mathematics from the 1st to the 3rd year.

### MAIN TYPES OF DEVELOPMENT

Taking this illustration as my point of departure, three main types of development can be identified:

1. INCREASE
2. SAME LEVEL
3. DECREASE

However, one should be aware that due to the properties of an ordinal scale, the size of the steps in rating of different respondents cannot be compared, as these steps are not to be interpreted as equal in size by any means. However, it is a pragmatic criterion to consider that a third year rating close to the first year rating might not be a considerable change. It should also be kept in mind that the scale is quite sensitive to the students' changes on even short time scales.

| 3RD YEAR RATING ON FAVOURITE SUBJECT SCALE | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |
|---|---|---|---|---|---|---|---|---|---|---|
| [10] | | | | | | | | 8 | 5 | 12 |
| [9] | | | | | | | | 12 | 9 | 6 |
| [8] | | | | | | | 4 | 11 | 8 | 6 |
| [7] | | 1 | | | | 2 | 6 | 9 | 3 | 4 |
| [6] | | | | | 1 | 3 | 3 | 4 | 4 | 1 |
| [5] | | | | | 2 | | 4 | 3 | 2 | |
| [4] | | | | | | | 2 | 2 | 1 | |
| [3] | | | | | | | 1 | | | 1 |
| [2] | | | 1 | | 1 | | | 1 | | |
| [1] | | 1 | | 1 | | | | 2 | | |
| N = 147 | 1ST YEAR RATING ON FAVOURITE SUBJECT SCALE | | | | | | | | | |

*Illustration 36: Favourite Subject Scale Development from 1st to 3rd year. GREEN: INCREASE. BLUE: SAME LEVEL. YELLOW: DECREASE. N=147. DARK GREY: the six students selected for case analysis.*

Due to these properties, SAME LEVEL is defined to mean either the exact same

level or a one-step increase or decrease, INCREASE is defined to mean an increase of two steps or more, and DECREASE means a decrease of two steps ore more. These three main types of development are emphasised by colour coding in Illustration 36 (p. 132). *GREEN* represents an INCREASE, *BLUE* represents the SAME LEVEL and *YELLOW* represents a DECREASE. From this table it can be seen that only nine out of 147 A-level mathematics students increase their rating of mathematics by two steps or more, and that the number of students decreasing their rating is much higher. The six *DARK GREY fields* indicates the development for the six students selected for case analysis, whereas the NUMBER in each field refers to the number of students in the full population with exactly this type of development in their rating of mathematics from the first to the third year of upper-secondary school.

The numbers and percentages of students in each level of development is displayed in a table (Table 26, p. 134). It should be emphasised that the selection of classes that participated in the questionnaire answering was not random, but they were selected from a strategy of obtaining as much variation as possible.

| Main types of development | Number of students | Part of students (in percent) |
|---|---|---|
| **INCREASE** *Students increasing their rating of mathematics with two steps or more* | 9 | 6 % |
| **SAME LEVEL** *Students keeping their rating of mathematics on the same level or give it a one-step increase or decrease* | 95 | 65 % |
| **DECREASE** *Students decreasing their rating of mathematics with two steps or more* | 43 | 29 % |
| **Total** | **147** | **100 %** |

*Table 26: Number and percentage of students representing increasing their rating, maintaining it, or decreasing their rating on the favourite subject scale (FSS). N=147.*

The students participating in the interview part of the longitudinal study were, as far as possible, chosen to represent the same range in values on the favourite subject scale in the 1st year as the total population. Nevertheless, the development in rating amongst the interviewees only partly resembles that of the whole population.

Amongst the interviewees, also only close to 5% (1 student) represents an increase of two steps or more on the favourite subject scale. However, keeping the rating on the same level from the 1st to the 3rd year is less common amongst the interviewees than amongst the general population. Just around half of the interviewees keep the same level of rating (12 out of 23 students, or 52%) whereas in the general population almost two thirds show a stable development.

| Main types of development | Number of students (Interviewees) | Part of students (in percent) |
|---|---|---|
| INCREASE | 1 | 4% |
| SAME LEVEL | 12 | 52% |
| DECREASE | 10 | 43 % |
| **Total** | **23** | **100**[146] **%** |

*Table 27: Number and percentage of the INTERVIEWEES, representing increasing their rating, maintaining it, or decreasing their rating on the favourite subject scale (FSS).N=23*

Accordingly, 43% of the interviewees decrease their rating, whereas 29% of the students in the general population do the same.

*In the next sections, there will be given an account of the criteria for selecting case informants for the case analysis amongst the interviewees.*

146 4,35% + 43,48 % + 52,17 % = 100 %

## THE GROUP OF INTERVIEWEES

It is thought-provoking that the majority of this sample (A-level mathematics, volunteering for interviews) seems to have ended up decreasing their rating of the subject mathematics in their final year of upper secondary school. However, one must keep in mind that the 24 students were not randomly selected. First, all students in each of the four classes were asked to fill out a form indicating whether they would be willing to participate in the interviews. From each of these four groups, six students were selected with a first criterion being to achieve a gender distribution comparable to the that of the whole class. The second criterion was to achieve a certain distribution between the ratings on the favourite subject scale.

However, in BETA Upper Secondary school, this instruction was misinterpreted, and the teacher instead *selected* six volunteers, whereas the gender criterion was fulfilled. The rationale behind this recruitment method was to maximise the probability of the volunteers to hang on to school for all three years. The teacher was rather successful in this strategy. One student among the volunteers changed school after the first year, but since almost half of the class left school for good after the second year, and none of the interviewees, the teacher's sense of judgement for that purpose seemed to work.

Another mechanism was in play in DELTA Technical Gymnasium. In this class, according to the teacher, the group of more ambitious students was not represented among the volunteers, since they were afraid it would take up too much time and leave too little for learning.

In ALFA and GAMMA, the selection based on criterion 2 was done pragmatically, according to which students attended school on the days of the interviews and of course obtaining a decent distribution in the ratings.

| School (Alias) | Name (Alias) | "Favourite Subject Scale" Year 1 | "Favourite Subject Scale" Year 3 | Development from first to third year |
|---|---|---|---|---|
| **ALFA** General Upper Secondary School | Andrea | 10 | 7 | - 3 |
| | ADELE | 7 | 7 | 0 |
| | Amy | 8 | 9 | 1 |
| | (Adam) | 7 | 6 | - 1 |
| | Alan | 9 | 7 | - 2 |
| | Andrew | 9 | 9 | 0 |
| **BETA** General Upper Secondary School | (Bianca) | 9 | - | |
| | Brooke | 8 | 8 | 0 |
| | (Brenda) | 7 | 5 | - 2 |
| | Ben | 9 | 8 | - 1 |
| | Bryan | 6 | 7 | 1 |
| | BRANDON | 10 | 10 | 0 |
| **GAMMA** Technical Upper Secondary School | GRACE | 8 | 4 | - 4 |
| | Gwen | 7 | 8 | 1 |
| | Gary | 5 | 5 | 0 |
| | (George) | 10 | 8 | - 2 |
| | (Glen) | 8 | 5 | - 3 |
| | Gordon | 9 | 7 | - 2 |
| **DELTA** Technical Upper Secondary School | Denise | 7 | 4 | - 3 |
| | DONNA | 2 | 7 | 5 |
| | Diana | 10 | 6 | - 4 |
| | Duncan | 7 | 7 | 0 |
| | DYLAN | 8 | 6 | - 2 |
| | David | 7 | 8 | 1 |

*Table 28: The interviewee's rating of mathematics on the "Favourite Subject Scale" in the 1st and the 3rd year. Names in UPPERCASE: Selected for case analysis. Names in parenthesis: the student did not participate in all activities (interviews). Colour coding for types of development of the rating from 1st to 3rd year: GREEN: an INCREASE of 2 steps or more; BLUE: the SAME LEVEL or a 1-step increase or decrease; YELLOW: a DECREASE of 2 steps or more.*

## SELECTING CASES FOR CASE DESCRIPTIONS

The development of the rating on the favourite subject scale for the students in the project classes gives one indication of their interpretations of their experiences with A-level mathematics. The measure is subjective as it will be influenced by interpretations of episodes in both first year and third year mathematics. The rating has turned out to be an instrument for discussing which experiences have led to changes in the views of mathematics. However, due to its summative character, it is also a means for achieving an overview of interpretations with the students. Taking the latter as point of departure, I have decided to incorporate it in the selection criteria for the case descriptions. A set of criteria for selecting cases for further analysis has been set up. These criteria are:

1. All four classes
    i. Alfa
    ii. Beta
    iii. Gamma
    iv. Delta
2. Both gender
    i. Female
    ii. Male
3. All three kinds of development in rating on Favourite Subject Scale:
    i. Increased by 2 steps or more,
    ii. Stable or a one-step de- or increase
    iii. Decreased by two step or more)

- should be represented by the case selection.

| 3RD YEAR RATING ON FAVOURITE SUBJECT SCALE | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| [10] | | | | | | | | 8 | 5 | 12 |
| [9] | | | | | | | | 12 | 9 | 6 |
| [8] | | | | | | | 4 | 11 | 8 | 6 |
| [7] | | 1 | | | | 2 | 6 | 9 | 3 | 4 |
| [6] | | | | | 1 | 3 | 3 | 4 | 4 | 1 |
| [5] | | | | | 2 | | 4 | 3 | 2 | |
| [4] | | | | | | | 2 | 2 | 1 | |
| [3] | | | | | | | 1 | | | 1 |
| [2] | | | 1 | | 1 | | | 1 | | |
| [1] | | 1 | | 1 | | | | 2 | | |
| N = 147 | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |
| | 1ST YEAR RATING ON FAVOURITE SUBJECT SCALE | | | | | | | | | |

*Illustration 37: Development in Favourite Subject Scale Rating in the total population of students (N=147). Types of development represented by the 21 interviewees in the four project classes (LIGHT GREY) and the types of development represented in the six students selected for case analysis (DARK GREY). The numbers refer to the number of students in the total population representing this specific development in rating.*

### *STABLE OR WITH A 1-STEP INCREASE OR DECREASE*

Adele, Amy, Adam, Andrew, Brooke, Ben, Bryan, Brandon, Gwen, Gary, Duncan and David all kept their rating of mathematics close to their initial rating of the subject after 3 years. These twelve students out of twenty-four, represent half of the population of interviewees, or fifty percent.

From this group Brandon, Adele and Gary's responses have been chosen for

case analysis; Gary keeps the rating relatively low (1st year: [5], 3rd year: [5]) compared to the rest of the group.

### *DECREASE OF 2 STEPS OR MORE*

Andrea, Alan, Brenda, Grace, George, Glen, Gordon, Denise, Diana and Dylan lowered their rating of mathematics by 2 steps or more in the 3rd year questionnaire compared to the 1st year questionnaire. These ten students out of twenty-four represent close to forty percent.

Grace and Dylan's responses have been chosen for case analysis; Grace is one of two students decreasing the rating of mathematics the most on the favourite subject scale, by a four steps decrease. Dylan gives the subject a two-step decrease and represents a less dramatic decrease.

### *INCREASE OF 2 STEPS OR MORE*

One student among the 24 interviewees gives the rating of mathematics a lift of more than one step in the 3rd year compared to the 1st year. This one representative out of 24, represents less than 5 percent of the interviewees.

Donna's responses have been chosen for case analysis; Donna raised the rating by five steps, starting out giving it [2], indicating mathematics to be quite far from her favourite subject, and ending up giving it a [7], which is in the better end of the scale without indicating it to be a profound favourite.

**In 'cells': Number or name of interviewees with this type of development in rating**

| Third Year Rating on the FSS | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |
|---|---|---|---|---|---|---|---|---|---|---|
| [10] | | | | | | | | | | BRANDON |
| [9] | | | | | | | | 1 | 1 | |
| [8] | | | | | | | 2 | 1 | 1 | 1 |
| [7] | | DONNA | | | | 1 | ADELE 1 | | 2 | 1 |
| [6] | | | | | | | 1 | DYLAN | | 1 |
| [5] | | | | | GARY | | 1 | 1 | | |
| [4] | | | | | | | 1 | GRACE | | |
| [3] | | | | | | | | | | |
| [2] | | | | | | | | | | |
| [1] | | | | | | | | | | |

**the 1st Year Rating on the Favourite Subject Scale (FSS)**

*Table 29: The interviewees' development in rating on the Favourite Subject Scale from 1st to 3rd Year. The cells with colour coding represent those students participating in all activities (both questionnaire response and interview in both first and third year). This means that the colour coded cells represents those interviewees selectable for CASE analysis. The number in each cell represents the number of students with this development in rating.*

Since Donna is the only student in the group of interviewees to represent an increase by two steps or more on the favourite subject scale, she is chosen for the case description sample of the interviewees (see Table 28, p. 137). Diana and Grace are the two students indicating the largest decrease in rating, Diana from [10] to [6]

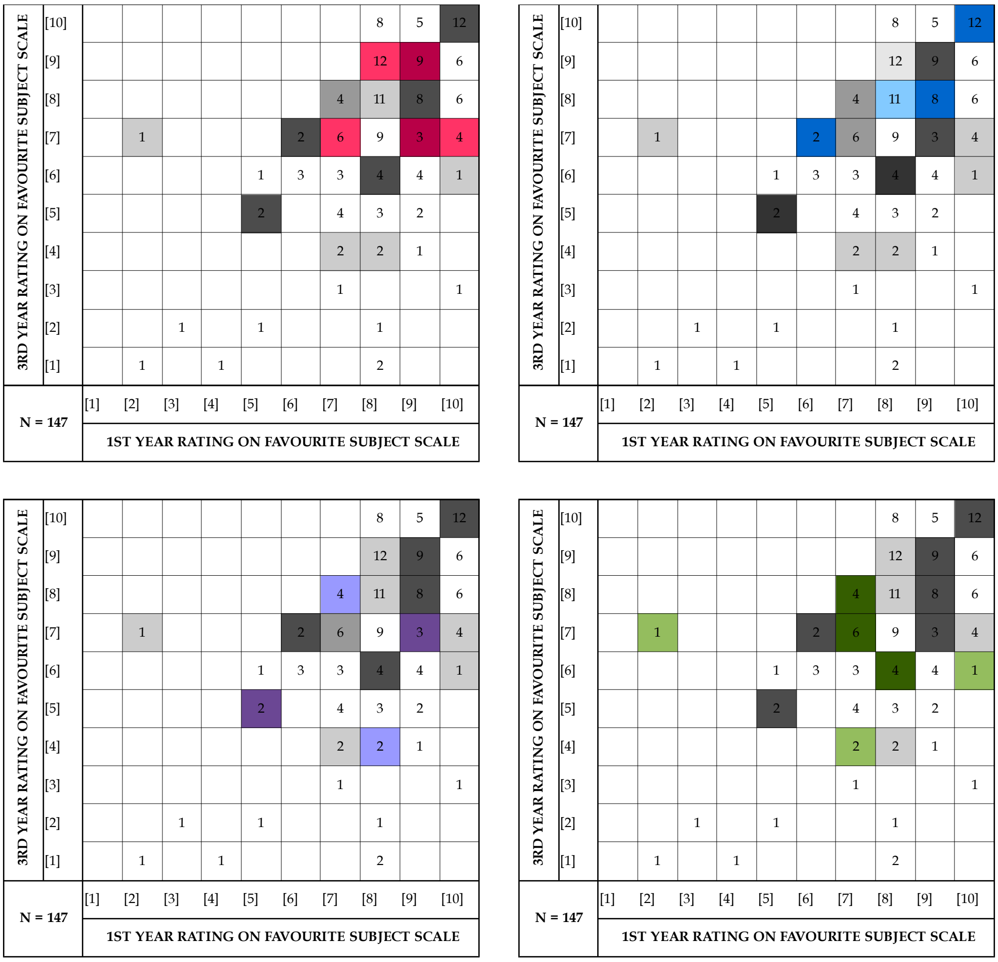
and Grace from [8] to [4]. I have selected Grace for the case description sample, since I already had selected one female student from DELTA.

Amongst the BETA students, only the stable development is represented, since Brenda, who indicated a decrease in the rating, did not turn up on the day of the interviews. The fact that the teacher helped to recruit students so as to optimise the chance of the students not leaving school before the third year, which almost half of the class did, may also have had some influence on this. I have selected Brandon, since he is the only student in the group of 24 interviewees who ends on a [10], even though this third year rating is quite common in the full sample of students in the longitudinal study.

| | **INCREASE** | | **STABLE** | | **DECREASE** | |
|---|---|---|---|---|---|---|
| | **Female** | **Male** | **Female** | **Male** | **Female** | **Male** |
| **ALFA** | - | - | **Adele**<br>Amy | Andrew | Andrea | Alan |
| **BETA** | - | - | Brooke | Ben<br>Bryan<br>**Brandon** | - | - |
| **GAMMA** | - | - | Gwen | **Gary** | **Grace** | Gordon |
| **DELTA** | **Donna** | - | - | Duncan<br>David | Denise<br>Diana | **Dylan** |

*Table 30: Overview of selected students (underlined) from each class, ALFA, BETA, GAMMA & DELTA, and how the three types of development are covered by female and male students.*

In some sense Donna, Grace and Brandon represent extreme cases; Donna indicating the maximum increase in rating, Grace representing the maximum decrease and Brandon representing the highest stable development. Criteria 2 and 3 are satisfied, since now both gender and all three kinds of Favourite Subject Scale Development are represented.



3RD YEAR RATING ON FAVOURITE SUBJECT SCALE

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| [10] | | | | | | | | 8 | 5 | 12 |
| [9] | | | | | | | | 12 | 9 | 6 |
| [8] | | | | | | | 4 | 11 | 8 | 6 |
| [7] | | 1 | | | | 2 | 6 | 9 | 3 | 4 |
| [6] | | | | | 1 | 3 | 3 | 4 | 4 | 1 |
| [5] | | | | | 2 | | 4 | 3 | 2 | |
| [4] | | | | | | | 2 | 2 | 1 | |
| [3] | | | | | | | 1 | | | 1 |
| [2] | | | 1 | | 1 | | | 1 | | |
| [1] | | 1 | | 1 | | | | 2 | | |
| N = 147 | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |

1ST YEAR RATING ON FAVOURITE SUBJECT SCALE

3RD YEAR RATING ON FAVOURITE SUBJECT SCALE

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| [10] | | | | | | | | 8 | 5 | 12 |
| [9] | | | | | | | | 12 | 9 | 6 |
| [8] | | | | | | | 4 | 11 | 8 | 6 |
| [7] | | 1 | | | | 2 | 6 | 9 | 3 | 4 |
| [6] | | | | | 1 | 3 | 3 | 4 | 4 | 1 |
| [5] | | | | | 2 | | 4 | 3 | 2 | |
| [4] | | | | | | | 2 | 2 | 1 | |
| [3] | | | | | | | 1 | | | 1 |
| [2] | | | 1 | | 1 | | | 1 | | |
| [1] | | 1 | | 1 | | | | 2 | | |
| N = 147 | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |

1ST YEAR RATING ON FAVOURITE SUBJECT SCALE

3RD YEAR RATING ON FAVOURITE SUBJECT SCALE

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| [10] | | | | | | | | 8 | 5 | 12 |
| [9] | | | | | | | | 12 | 9 | 6 |
| [8] | | | | | | | 4 | 11 | 8 | 6 |
| [7] | | 1 | | | | 2 | 6 | 9 | 3 | 4 |
| [6] | | | | | 1 | 3 | 3 | 4 | 4 | 1 |
| [5] | | | | | 2 | | 4 | 3 | 2 | |
| [4] | | | | | | | 2 | 2 | 1 | |
| [3] | | | | | | | 1 | | | 1 |
| [2] | | | 1 | | 1 | | | 1 | | |
| [1] | | 1 | | 1 | | | | 2 | | |
| N = 147 | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |

1ST YEAR RATING ON FAVOURITE SUBJECT SCALE

3RD YEAR RATING ON FAVOURITE SUBJECT SCALE

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| [10] | | | | | | | | 8 | 5 | 12 |
| [9] | | | | | | | | 12 | 9 | 6 |
| [8] | | | | | | | 4 | 11 | 8 | 6 |
| [7] | | 1 | | | | 2 | 6 | 9 | 3 | 4 |
| [6] | | | | | 1 | 3 | 3 | 4 | 4 | 1 |
| [5] | | | | | 2 | | 4 | 3 | 2 | |
| [4] | | | | | | | 2 | 2 | 1 | |
| [3] | | | | | | | 1 | | | 1 |
| [2] | | | 1 | | 1 | | | 1 | | |
| [1] | | 1 | | 1 | | | | 2 | | |
| N = 147 | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] |

1ST YEAR RATING ON FAVOURITE SUBJECT SCALE

*Illustration 38: Favourite Subject Scale development in the four project classes.*

*Number in each cell: Number of students in the full population of N=147 showing that particular development in rating on the FSS from the 1st to the 3rd year.*

*Grey cells: Interviewee(s) with this FSS development: Dark grey cells: Male interviewee(s) with this FSS development . Light grey cells: Female interviewee(s) with this FSS development.*

*Upper left: ALFA: Dark red cells: Male interviewees in ALFA. Light red cells: Female interviewees in ALFA.*

*Upper right: BETA: Dark blue cells: Male interviewees in BETA. Light blue cell: Female interviewee in BETA.*

*Lower left: GAMMA: Dark purple cells: Male interviewees in GAMMA. Light purple cells: Female interviewees in GAMMA.*

*Lower right: DELTA: Dark green cells: Male interviewees in DELTA. Light green cells: Female interviewees in DELTA.*

### *SUMMARY OF CASE SELECTION*

The overall objective for the selection of students is to obtain a maximum of variation on the FSS within a minimum number of cases. More specifically:

1. *All three kinds of development (increase, stable, decrease) should be represented in each gender, if possible.*
2. *EACH OF THE four classes (ALFA, BETA, GAMMA, DELTA) should be represented by at least one student*

There is no unique way of obtaining that in the selection, so these are the reasons that has determined the actual selection:

1. *I chose Donna (2,7) because she is the only interviewee to represent an increase in the FSS development by 2 steps or more.*
2. *I wanted one more female student from the technical programme, since Donna's FSS development was so unique.*
3. *I preferred the other female student to be from GAMMA in order to represent an additional school.*
4. *I chose Grace (8,4) rather than Gwen (7,8), since Grace represents a considerable decrease in FSS, and additionally, Gwen had given some special answers that I found less likely to represent a larger class of people. Grace represents a decrease on the FSS.*
5. *The (10,10)-development on the FSS is the highest possible stable development. This type of development is represented by Brandon.*
6. *In the general programme, I chose Adele (7,7) to represent the stable development for female students. I chose her over Brooke from BETA, because Brooke's FSS development (8, 8) would provide less variation from Brandon (10,10) in the same class.*
7. *I chose Dylan (8,6), because he represents a decrease on the FSS for a male student.*
8. *As one last student, I chose Gary, because he represents a low, stable development (5,5), which is different from the high, stable development in Brandon and also the stable development in Adele (7,7).*

| | FSS Development | | | Gender | | Schools | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Increase | Stable | Decrease | Female | Male | Alfa | Beta | Gamma | Delta |
| **Donna** | X | | | X | | | | | X |
| **Brandon** | | X | | | X | | X | | |
| **Adele** | | X | | X | | X | | | |
| **Gary** | | X | | | X | | | X | |
| **Dylan** | | | X | | X | | | | X |
| **Grace** | | | X | X | | | | X | |

*Table 31: The six selected informants for case analysis: DONNA, BRANDON, ADELE, GARY, DYLAN & GRACE, and an overview of the distribution in the FSS development, gender and schools.*

Before heading to the actual case analysis, an overview of the development on the favourite subject scale for each case informant is given in Table 32 (p. 145). The final mark for the national written exam in A-level mathematics has been included in the table for reference.

| **Type of Development** | **INCREASING** | **DECREASING** | | **STABLE** | | |
|---|---|---|---|---|---|---|
| **Name** | Donna | Dylan | Grace | Brandon | Adele | Gary |
| **FSS** | 2 → 7 | 8 → 6 | 8 → 4 | 12 → 12 | 7 → 7 | 5 → 5 |
| **Final Mark in written exam** | A | C | D | A | C | - |

*Table 32: Overview of selected Case Informants; Favourite Subject Scale development.*

# CASE ANALYSIS

*First I will remind the reader of the aims of the case analysis, then introduce the strategy and the tactics of it, and then each of the six case informants will be analysed according to these ideas.*

## AIMS OF THE CASE ANALYSIS

What I aim at in the case analysis, is to provide insight into the interpretations of experiences of six upper secondary school students in A-level mathematics study programmes in order to characterise their beliefs about mathematics in the first year and in the third year of upper secondary school, respectively, and to detect the development of these beliefs about mathematics as well possible reasons for these changes. Moreover, special attention will be given to matters influencing students' sensitivity to mathematics in their plans for tertiary education.

## STRATEGY OF THE CASE ANALYSIS

The presentation of the cases will consist of some main sections of descriptive nature, each followed by sections of more interpretive nature and finally sections of synthetic-analytical character. The descriptive sections are organised around the questionnaires and the interview transcripts and the more interpretive sections are organised as a summing-up section after the first year and after the third year materials, respectively.

In the synthetic-analytical section, the Four Aspects of Students Beliefs about Mathematics; *Mathematics at School*, *Mathematics as a Discipline*, *Mathematics in Society*

and *Mathematics and Me*, as well characteristics of belief systems, are to be employed.

| Aspect of Beliefs about Mathematics | Clues |
|---|---|
| *Mathematics at School* | Mathematics as it is met in school:<br>The transposition of the discipline to its school curriculum as encountered by the students, the social context in class around mathematics learning.<br>The reciprocal expectations of the teacher and the students.<br>The social environment in class in terms of sociomathematical norms. |
| *Mathematics as a Discipline* | Mathematics as a science in its own right and as a tool for other scientific disciplines |
| *Mathematics and Society* | Mathematics as it is employed by professions, institutions, groups and individuals in society. |
| *Mathematics and Me* | Myself as a mathematics learner and doer.<br>Self-efficacy beliefs and motivational beliefs.<br>The image of the role of mathematics in ones life now and in the future. |

*Table 1: Four Aspects of Students' Beliefs about Mathematics, an overview.*

The process of analysis of each case can be summed up in an abbreviated manner as follows:

To keep track of the researcher's interpretations and to enable the reader to critically examine these, the interpretations have been divided into three stages:

1. Reference (R)
2. Thematic Analysis (A)
3. Synthetic Analysis (S)

For the presentation of each case, a sequence consisting of these three stages will be applied to the empirical material from the first and the third year, respectively:

For the 1st Year Questionnaire Responses and the 1st Year Interview Transcripts, which together are denoted by the 1st Year Material ($m_{y1}$), the process can be represented in a table in an abbreviated form.

| **First year material – Stages of interpretation** | | |
|---|---|---|
| *Step* | *Interpretation Stage* | *Abbreviated notation* |
| 1 | Reference | $R(m_{y1})$ |
| 2 | Thematic Analysis | $A(m_{y1}, R(m_{y1}))$ |
| 3 | Synthetic | $S(m_{y1}, R(m_{y1}), A( m_{y1}, R(m_{y1})))$ |

And for the 3rd Year Questionnaire Replies and the 3rd Year Interview Transcripts, which together are denoted by the 3rd Year Material ($m_{y3}$), it can be abbreviated like this:

| **Third year material – Stages of interpretation** | | |
|---|---|---|
| *Step* | *Interpretation Stage* | *Abbreviated notation* |
| 1 | Reference | $R(m_{y3})$ |
| 2 | Thematic Analysis | $A(m_{y3}, R(m_{y3}))$ |
| 3 | Synthetic | $S(m_{y3}, R(m_{y3}), A(m_{y3}, R(m_{y3})))$ |

After each of these processes, a synthesis of the transposition (T) from the first to the third year will be provided with its point of departure in the synthetic-analytical analysis of the material of the two years:

$$T(S(m_{y1}, R(m_{y1}), A( m_{y1}, R(m_{y1}))),\ S(m_{y3}, R(m_{y3}), A(m_{y3}, R(m_{y3}))))$$

This means that process of analysis may loop between the three stages of

analysis, however, before drawing the final conclusions (Miles & Huberman, 1994).

## TACTICS OF THE CASE ANALYSIS

A *rich account* of data is given *first*, then, *second*, an interpretation in terms of groups of themes representing different facets of the four aspects of the case informant's beliefs about mathematics in that year, for the 1st year and the 3rd year material respectively is presented and thirdly, a summary of the main trends in that case informant's beliefs that year is given. As a final step, for each case informant a suggestion of the main elements in the transition of beliefs from the first year to the year year is presented, in terms of elements that have shown to stay stable, and elements, that have been subject to change.

### *OVERVIEW OF THE MATERIAL*

*For each Case Respondent these are the materials that are displayed:*

- The 1st Year Questionnaire Answers
- The 1st Year Interview Transcript Excerpts
- The 3rd Year Questionnaire Answers
- The 3rd Year Interview Transcript Excerpts

These materials will enable the reader to critically evaluate the interpretations by the researcher in the analysis and provide insight into the many facets of each case informant.

### THE 1ST YEAR QUESTIONNAIRE

For each case informant, answers to all questions in the *1st Year Questionnaire* are displayed in tables organised by these themes:

Q1-A TRANSITION

Q1-B FOR SCHOOL

Q1-C BEYOND SCHOOL

Q1-D IMPROVING

Q1-E CHALLENGES & SUPPORT

Q1-F IN CLASS

Q1-G PLANS

In these tables, answers in brackets [] were ticked off by the informant from a selection of choices, either multiple or single choice option, whereas answers without brackets are answers to open-ended questions. "No reply" is signified by "-"

### THE 1ST YEAR INTERVIEW TRANSCRIPTS

The interviews were conducted in Danish and transcribed in Danish. For each case informant, the excerpts from the *1st Year Interview* transcripts are displayed in the order they appear in the original transcript. The excerpts are *numbered* (1.A., 1.B., 1.C. etc.) and each excerpt is given a *headline*. The excerpts do not include all details; repetitions have been left out and also passages of "warming up" for asking or answering a question. The quotes from the interviews have been selected based on whether they concerned beliefs about *mathematics* or not. Quotes of too general a nature have been left out. This criterion can be compared to that of distinguishing between socio-mathematical norms and general norms (Yackel & Cobb, 1996). If the norms or beliefs are not related to mathematics in a broad sense, they have been left out (Törner, 1999). The excerpts have subsequently been translated into English, but their Danish counterparts are displayed in footnotes.

In the following, Q stand for the Question (posed by the interviewer, the researcher) (Q1:) The number following the letter designates position in the sequence of the quotes selected from the interview, Q1 being the first quoted question. The same goes for the initial of the Interviewee, e.g. D for Donna and D1 representing Donna's answer to the first question. Square brackets [ ] indicate

comments or corrections by the author.

### THE 3RD YEAR QUESTIONNAIRE AND INTERVIEW

The same type of system in the notation is used for the for the 3rd Year Questionnaire and the 3rd Year Interview, except that the questionnaire in the 3rd year included some extra categories of questions, while a few items were left out. An overview of the tables for the 3rd Year Questionnaires are given here:

Q3-A TRANSITION
Q3-B FOR SCHOOL
Q3-C BEYOND SCHOOL
Q3-D IMPROVING
Q3-E CHALLENGES & SUPPORT
Q3-F IN CLASS
*Q3-XA UNDERSTANDING*
*Q3-XB LEARNING BY HEART*
*Q3-XC A-LEVEL EXAMINATION*
Q3-G PLANS

Also, the 3rd Year interview transcript excerpts are numbered 3.A., 3.B., 3.C. etc.

### *OVERVIEW OF CONTENT IN THE CASE ANALYSIS*

Both the 1st year material and the 3rd year material will each be analysed by means of the three steps just mentioned:

### REFERENCE

For each case informant, the *1st year questionnaire answers*, the *1st year interview excerpts*, the *3rd year questionnaire answers* and the *3rd year interview excerpts* respectively, are included in the analysis sections in the dissertation, as part of the 1st

year and the 3rd year analysis respectively.

### THEMATIC ANALYSIS

An *interpretation* of what this material tells us about the informant and the informants' relation to mathematics is given. Emphasis is placed on incidents and issues demonstrating an interrelationship, or lack of the same, between each of the four aspects of beliefs about mathematics. Special attention is given to incidents, issues and interpretations in terms of belief aspects that seems to have an impact on the role of mathematics in the future plans of the case informant.

So as to follow the track of the research questions, special attention will be paid to the student's ideas for further education after upper secondary school, and the role of mathematics in relation to these, and to the student's interpretations of her or his experiences from mathematics education and the trajectory of these through the four probes into them. Comparing the questionnaire answers to those from the follow-up interviews it should be kept in mind whether the students' interpretation is an elaboration to their initial response or whether it has developed subsequently.

### SYNTHETIC ANALYSIS

A summary of the strongest trends in the beliefs analysis of the actual year provides an overview of these features. The predominant trends in the beliefs dynamics of this informant also provides a source for comparison of the development from the first to the third year:

### 1ST YEAR TO 3RD YEAR TRANSPOSITION

Beside the *REFERENCE* in terms of the presented data, the *THEMATIC INTERPRETATIONS* of the informant's relation with mathematics seen with regard to the *Four Aspects of Beliefs*, and the *SUMMARY* of the students' *Four Aspects of Beliefs* in that year (the 1st and the 3rd) as a final step, a characterisation of the

*TRANSPOSITION* from the *1st Year* to the *3rd Year Beliefs* is presented and discussed in terms of the possible factors responsible for that change.

## ORDER OF CASE ANALYSIS

Now we will proceed to the analysis of the beliefs and belief development of the six selected case informants. They will be presented in the following order:

- First **Donna**, representing an *increase* on the favourite subject scale. Donna is given special attention in the analysis, since she stands out in her development on the Favourite Subject Scale (FSS).

- Then the two students representing a *decrease* on the favourite subject scale: **Dylan** and **Grace**.

- And finally **Brandon**, **Adele** and **Gary** representing three types of *stable development* on the favourite subject scale: Brandon a high level, Adele a relatively moderate level and Gary a relatively low level.

| Name | Type of FSS Development | FSS Development | Page no. |
|---|---|---|---|
| 1. Donna | INCREASE | [2] → [7] | p. 157 |
| 2. Dylan | DECREASE | [8] → [6] | p. 209 |
| 3. Grace | | [8] → [4] | p. 259 |
| 4. Brandon | STABLE | [10] → [10] | p. 309 |
| 5. Adele | | [7] → [7] | p. 367 |
| 6. Gary | | [5] → [5] | p. 423 |

*Table 2: Overview of Case informants, incl. page for case analysis*

# *THE CASE OF DONNA*

*Donna is a female student from Delta Technical Upper Secondary School, in a biotechnology study programme which involves studying A-level mathematics.*

*The case of Donna is treated thoroughly, since she is a unique case, not only in the population of interviewees, but in the general population of A-level mathematics students, since she is the only student increasing her rating of mathematics to the extend she does (from [2] in the 1st year to [7] in the 3rd year).*

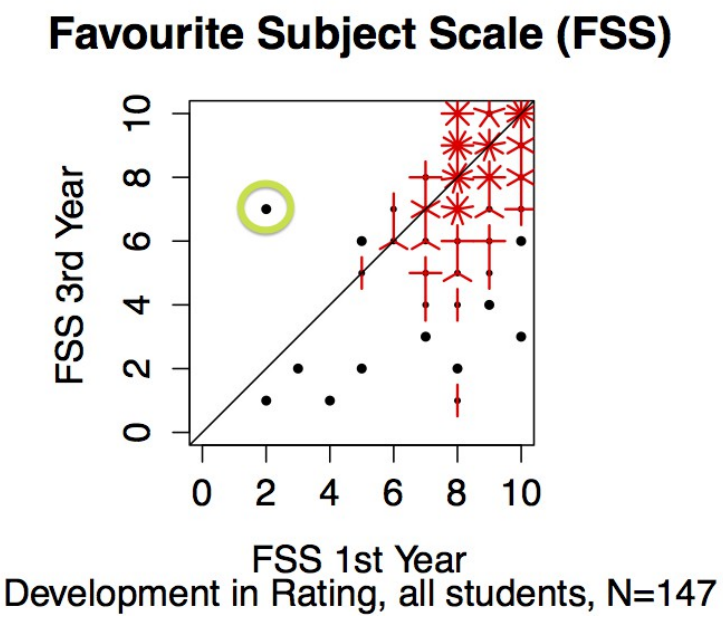



*Illustration 1: Sunflowerplot (R Core Team, 2012) of full population of 147 students. Donna is emphasised by a green circle (2,7)*

*Initially, Donna had to struggle to keep up in mathematics. However, at a certain point in the second year, when calculus was introduced, she said she began to understand not only what she was supposed to do in mathematics, but also why. All along, Donna had an idea of her future; in a white coat and safe glasses, working for a biotechnological company. In her spare time, Donna is a member of an elite team in a branch of sports.*

| Donna | Date for Questionnaire | Date for interview |
|---|---|---|
| the 1st Year | 24 November 2010<br>(+ supplement<br>16 December 2010) | 24 March 2011 |
| Third year | 17 December 2012 | 13 March 2013 |

*Table 3: Dates for Questionnaires and Interviews*

In the next two sections, Donna's answer to the first year questionnaire and interview transcripts from the first year interviews are displayed. Then an account of Donna's 1st year in upper secondary school is given in a narrative form. Finally an analysis in terms of the four aspects of beliefs; *Mathematics at School*, *Mathematics as a Discipline*, *Mathematics in Society*, and *Mathematics & Me* is given, leading to an account of the transition in her beliefs from the first to the third year of upper secondary school.

## DONNA'S 1ST YEAR QUESTIONNAIRE

| Q1-A | TRANSITION | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [2] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [147] | [Working on your own] |
| **3** | Did you like mathematics when you went to lower secondary school?[148] | [Yes, it was fine] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | It has become much harder which makes it hard for me to keep up. Then mathematics is not fun any more.[149] |
| **4b** | Is there anything you liked better before?' | The level (of mathematics, red.), the amount of homework[150] |
| **4c** | Is there anything you like better now? | I get more challenges[151] |

*Table 4: Donna's 1st year questionnaire, part A – TRANSITION*

147 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

148 Options: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

149 Det er blevet langt sværere, og det gør at jeg får svært ved at følge med. Så er matematik ikke sjovt mere.

150 Niveauet, mængden af lektier

151 Jeg får mere udfordring

| Q1-B | FOR SCHOOL | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | It is important. Otherwise people can cheat you[152] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **6** | What made you choose a study programme involving A-level mathematics? | I will be going to university after (graduation) so I need A-level mathematics[153] |
| **7a** | Is mathematics related to your other subjects? | [No] |
| **7b** | Please give reasons for your answer: | Chemistry and biology do not relate much to mathematics[154] |

*Table 5: Donna's 1st year questionnaire, part B - MATHEMATICS AND SCHOOL*

152 Det er vigtigt. Ellers kan folk jo snyde en.

153 Jeg skal læse på universitetet bag efter, så jeg har brug for mat A.

154 Kemi og biologi har ikke så meget med mat at gøre

| Q1-C | BEYOND SCHOOL | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | In a company mathematics is employed a lot, for example when wages are to be paid.[155] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? [156] | [Mathematics is invented] |
| **10** | What do you think a professional mathematician at a university is doing? | Trying to find new formulae and ways to calculate[157] |
| **11** | Would you have to be a genius in order to study mathematics in university? | [I do not know] |

*Table 6: Donna's 1st year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

| Q1-D | IMRPOVING | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | That I blank out, when I cannot work it out[158] |
| **12b** | What do you think is the best means for improving in mathematics? | To get a task, I should be able to explain how to solve, to the class [159] |
| **12c** | What do you do to improve in mathematics? | I keep up with the lesson and do my homework[160] |
| **13a** | What do you do if you get stuck on a task at school? | I solve the task[161] |
| **13b** | What do you do if you get stuck on your homework? | (I) ask my brother[162] |

*Table 7: Donna's 1st year Questionnaire, part D – STRATEGIES FOR IMPROVING*

155 I en virksomhed bliver der brugt meget matematik. Når der skal gives løn, fx
156 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]
157 Prøver at finde nye formler og måder at regne på
158 At jeg lukker af, når jeg ikke kan finde ud af det.
159 At få et stykke som jeg skal kunne forklare til klassen hvordan man løser
160 Følger med og laver mine ting
161 Jeg løser opgaven
162 Spørger min bror

| Q1-E | CHALLENGES & SUPPORT | | DONNA |
|---|---|---|---|
| # | *Question* | | *Answer* [163] |
| **14** | What issues involve more challenges to you? | a) Remembering | [4] Few challenges |
| | | b) Computing | [3] Moderate challenges |
| | | c) Figuring out the purpose of a task | [1] The most challenges |
| | | d) Finding a way to solve a task | [2] Several challenges |
| | | e) Reading and understanding the textbook | [5] The fewest challenges |
| **15** | Where can you find support for mathematical activities? | | [From siblings] |
| **16** | Did you parents take the Upper Secondary School Leaving Certificate? | | [None of them did][164] |

*Table 8: Donna's 1st year Questionnaire, part E – CHALLENGE AND SUPPORT*

163 Options: [1]= The most; [2]= Several; [3]= Moderate; [4]= Few; [5]=The fewest

164 [Yes, my mother did]; [Yes, my father did]; [Both my parents did]; [None of them did]

| Q1-F | IN CLASS | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? | [4-8][165] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [Yes] |
| **17c** | Are you content with that? | It is not fun[166] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? | [1-3][167] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | - |
| **19a** | In your class, is it okay to be good at mathematics? | The majority are good, so yes.[168] |
| **19b** | In your class, is it okay to have difficulties in mathematics? | It is okay[169] |

*Table 9: Donna's 1st year Questionnaire, part F - MATHEMATICS IN CLASS*

165 Options: [0]; [1-3]; [4-8]; [More than 8 times]

166 Det er ikke sjovt

167 Options: [0]; [1-3]; [4-8]; [More than 8 times]

168 Det er flertallet der er gode, så ja

169 Det er fint nok

| Q1-G | PLANS | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | I want to go to university[170] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [I do not know] |
| **21b** | Comments: | - |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [No] |
| **22b** | Comments | I just want to study chemistry, but I hope it does not involve any mathematics[171] |

*Table 10: Donna's 1st year Questionnaire, part G – PLANS*

170 Vil læse på universitetet

171 Vil bare læse kemi, men håber ikke der er noget matematik

# DONNA'S 1ST YEAR INTERVIEW

### *1A. TRANSITION TO UPPER SECONDARY SCHOOL: LESS FUN — A LITTLE BEHIND*[172]

Q1: How has mathematics as a subject changed from lower secondary to upper secondary school?

D1: It has become harder, that is for sure. Well, and since you have to work with it so much it also becomes a little less fun, I think. Then it is not like you are the best any more, or anything, because then you are like, a little behind, or I am at least.

Q2: How about in lower secondary school, were you the best back then?

D2: Yes

### *1B. UPPER SECONDARY SCHOOL: YOU HAVE TO THINK MUCH MORE*[173]

Q4: But what about the teaching, has it also changed? Do you think other aspects are emphasised now?

D4: Not really. I think it is more or less the same as in lower secondary school, also concerning handing in homework every week and such, that was more or less the same.

Q5: So it is the content that has changed, or the amount of it?

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172

Q1: Og det første det er det der med, hvordan synes du faget matematik har ændret sig fra du gik i folkeskolen, og så til nu, hvor du går på HTX?

D1: Det er blevet sværere, det er helt sikkert. Ja, også fordi man skal knokle så meget med det, det bliver det også sådan lidt mindre sjovt, synes jeg. Så det er ikke sådan, at man er den bedste længere eller noget som helst, fordi der er man sådan, der halter man lidt, eller jeg gør i hvert fald.

Q2: Ja. Hvad med i folkeskolen, var du den bedste der?

D2: Ja

173

Q4: Men er det også, er undervisningen også anderledes? Er det nogle andre ting, der bliver lagt vægt på synes du?

D4:, Ikke rigtigt. Jeg synes, det er sådan nogenlunde som jeg også havde det i folkeskolen, også omkring sådan hjemmeopgaver hver uge og sådan nogle ting, det var nogenlunde det samme.

Q5: Ja. (…) Så det er indholdet i det, og det der med at der er meget mere af det, der er den store ændring?

D5: Det er bare at det er sværere synes jeg. Det er noget mere kompliceret og man skal tænke meget meget mere. Og det er ikke sådan, at man direkte får at vide hvad det er for nogle ting man skal bruge. Man skal ligesom selv prøve at se hvor det er, hvor tingene skal være, og hvor vinkler sidder.

D5: It is just harder, I think. It is much more complicated and you have to think much more. And you are not told what tools to use. You have to figure out yourself, where everything should be and where the angles are placed.

***1C. CHOICE OF STUDY PROGRAMME***[174]

Q6: When you chose this study programme, was it Biotechnology which was of interest to you?

D6: Yes, definitely, it was the Biotechnology that was decisive. I also considered choosing [the study programme with] Chemistry, [A-level] Biology and Mathematics as B [-level]. But...well I find Biotechnology to be very interesting. And I know, that if I should go to university, I would have to take Mathematics at A-level anyway. So I might as well just take it.

***1D. RESOURCES FOR OVERCOMING CHALLENGES***[175]

Q7: Concerning all these challenges in Mathematics, where do you find resources for overcoming these obstacles? I mean, where can you find help, or how can you help yourself?

D7: Well, my brother also went to this school, and he also studied A-level Mathematics, so that was a help, at least. So, if I were completely lost at home, he would put on his graduation hat and pretend to be a teacher. That is actually quite nice, but yes. And then I think it is cool [because]

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174

Q6: (…) Når du så valgte denne her linje, altså var det bioteknologien der interesserede dig eller? Ja?

D6: Ja det var det helt sikkert, det var bioteknologien der gjorde det. Jeg overvejede også at tage den der hedder (utydeligt) med kemi, biologi og matematik som B. Men… ja, jeg synes bioteknologi lyder meget spændende. Og jeg ved, at hvis jeg skulle læse på universitetet så ville jeg være nødt til at have matematik A alligevel. Så jeg kunne lige så godt tage det.

175

Q7: (…) når der nu er alle de her store udfordringer, i matematikken, altså, hvor kan man så få støtte til så at overvinde nogle af de her forhindringer? Altså, hvor kan du få hjælp til det, eller hvordan kan du hjælpe dig selv med det?

D7: Ja altså min bror, han har gået her før, han havde også matematik A, så det hjalp i hvert fald. Sådan så, hvis jeg var fuldstændigt lost derhjemme, så tager han altid studenterhuen på og leger lærer. Det er egentlig meget rart. (…) Men ja, (…) Og så synes jeg det er fedt, sådan det der gør, at jeg bliver ved med at være med i matematiktimerne, det er sådan, jeg går til sport på eliteniveau, det vil sige at man er sammen med ens holdkammerater hele tiden, og det er sådan nogle STX'ere, og de tænker: matematik, ej det går ikke. Og sådan, når jeg så alligevel er en del bedre end alle de andre så tænker jeg sådan, så er det sådan noget som jeg er god til. Og jeg bliver sådan lidt kaldt "*nørden*" og sådan nogle ting, men det kan jeg godt lide. Så det er sådan, det er ligesom der hvor jeg tænker "*yes*", jeg skal bare følge med, jeg skal bare… Det er ligesom det, der gør, at jeg ligesom "*yes, matematik er fedt*".

this has been vital for my keeping up in the lessons … I do this sport at an elite level, and you hang out with your team mates all the time, and they are from the general upper secondary school [STX[176]], and they think, Mathematics, no, it will not work, and since I am somewhat better than all the others anyway, then I am thinking, then this is something I am good at. And they sometimes call me 'the nerd' and things like that, but I rather like it. It makes me think: "*Yes, I have to keep up, I really have to*". This is kind of what makes me like (saying, red.): "*Yes, mathematics is great*".

***1E. THE PREFERRED ROLE OF MATHEMATICS IN FUTURE EDUCATION[177]***

Q9: Well, since you need A-level Mathematics anyway if you are going to university, then it is necessary in order for you to study what you have in mind. Do you already now have thoughts about what you would like it to become?

D9: Well, I do have a few... I would prefer not to deal with too much mathematics. Because, yes, my brother is in university now and he also studies Mathematics. And it looks extremely difficult. And I am more into Chemistry or Pharmacology or something like that. Then I would be happy, anyway.

Q10: But I suppose these, too, involves a bit of mathematics?

D10: Well, yes. I kind of know, it is.. (unintelligible)...

Q11: But it does not matter if there is a bit of it?

D11: No, a little is all right, and also, as soon as you have learned it, it is quite simple. It is just something about getting it.

176 Dana is in the *Technical* Upper Secondary School Programme (HTX).

177

Q9: (...) altså, du skal alligevel bruge matematik på A-niveau hvis du skal læse videre på universitetet, så det er sådan ligesom, altså, det er i hvert fald nødvendigt for at lære det du gerne vil. Tænker du allerede nu på, hvad du godt kunne tænke dig at det skulle være eller har du sådan nogle forskellige idéer om det?

D9: Altså jeg, jeg har da nogle få… altså det… jeg har det sådan lidt, at jeg vil helst ikke have alt alt for meget matematik. Fordi ja, min bror går på universitetet nu og også på matematik ikke? Og det ser jo bare sygt svært ud. Og så er jeg mere til sådan, kemi. Så ja, noget kemiker eller farmakolog eller noget i den stil. Så ville jeg i hvert fald være glad.

Q10: Ja. (…) Men de har vel også lidt matematik?

D10: Ja, det er det. Og det ved jeg så, det er (utydeligt)…

Q11: Men det gør ikke noget, at der er lidt af det?

D11: Nej, der må godt være lidt, også sådan, så snart man ligesom har lært det, så er det jo rimeligt simpelt. Det er bare lige at få det ind.

*1F. ASKING IN CLASS AND UNDERSTANDING*[178]

Q12: ... If I may ask about being active during the lessons? Well, about daring to ask questions and such (...) you are not really content about it, it is not fun asking, but you do it anyway - you do ask questions?

D12: Well, definitely, since it is quite hard for me to get it in the first instance. It is difficult for me to learn it from the beginning. So I have to ask all the time, and all the time, and all the time, and it gets a little annoying in the long run, to be the one who asks questions all the time. Even though the others do not understand things, they do not ask about them. It is like this, I have to understand things, otherwise I would not be able to use them in calculations, if I do not understand it. So, well, yes, I do ask.

Q13: But that must be quite demanding, actually, since you say that a lot of other students also would need to ask, they do not, but you actually face it and say, now I will do it?

D13: Yes. I am kind of used to it. Well.. it is just fundamental, if there is something you do not get, then you have to say (to yourself, red.): "*That is just how it is*!" And then you have to, well, learn it...

178

Q12: … Jo, hvis jeg må spørge til det der med at være aktiv i timerne? Altså, med at få stillet nogle spørgsmål og være med i diskussionerne og sådan noget. (...) Det er ikke fordi du som sådan er tilfreds med det, det er ikke sjovt at spørge, men du gør det alligevel - du får spurgt?

D12: Jamen helt sikkert, det er jo fordi jeg har rimelig svært ved at få det ind, sådan i første omgang. Jeg har svært ved ligesom at lære det lige fra starten af. Så jeg er ligesom nødt til at spørge hele tiden og hele tiden og hele tiden, og det bliver lidt træls til sidst, at være den, der hele tiden spørger. Også selv om de andre ikke forstår det, så spørger de ikke mere til det. Det er sådan, jeg er nødt til at forstå det, ellers så vil jeg ikke kunne regne med det, hvis jeg ikke forstår det (…) Så ja, jeg spørger.

Q13: ... Jamen det må jo også kræve noget af dig, egentlig, (utydeligt) siden (...) du siger, at der faktisk er en masse andre der også havde brug for at spørge, som måske ikke får det gjort, men… men du får faktisk taget handsken op og siger: nu, det er jeg nødt til, nu gør jeg det?

D13: Ja. Det er jeg ligesom vant til. Altså… det er bare sådan helt generelt, at hvis der er noget man ikke kan finde ud af, så man jo bare sige, at sådan er det, og så man ligesom lære det… Ja.

### *1G. MEANS FOR DEALING WITH MATHEMATICAL CHALLENGES*[179]

Q16. How do you improve in Mathematics, if you face challenges? I mean, what can you do yourself? You did mention that asking is helpful, and that you have your brother. But if you are on a task and deal with it, and you think: "*Okay, I have to find a way of doing something?*"

D16: Well, number one: I have a lot of notes from class and such, because...well otherwise I forget it. And then I kind of get it written down in my own manner, so that I understand it myself. And then, well I do not know exactly, I suppose I just work on it really hard. Sometimes just in my head, because, well, I do not know why, but it just helps me imagining that I have to explain it to someone else, to imagine someone else sitting (next to me) and then kind of say: "O*kay, as you can see, then there is a triangle in this...something*". But actually I am just explaining it to myself.

179

Q16: (...) hvordan bliver man bedre til matematik når man har udfordringer til det? Altså, hvad kan man selv stille op? Og der har du i hvert fald nævnt, at det i hvert fald er godt at få spurgt, og du har din bror. Men hvis du selv sidder med det og sidder med en eller anden opgave hvor du tænker "*okay, nu skal jeg bare prøve at få gjort noget ved denne her*"?

D16: Altså, ét: jeg har en helt masse noter fra timerne og sådan nogle ting, fordi… ja ellers så glemmer jeg det. Og så får jeg det ligesom også skrevet ned på min egen måde, sådan så jeg ligesom selv kan forstå det. Og ja, jeg ved det ikke helt, jeg knokler vel bare på. Nogle gange sådan inde i mit eget lille hoved, fordi ja, jeg ved ikke hvorfor, men det hjælper mig bare ligesom at tænke at jeg skal prøve at forklare det til en anden person, at der sidder en anden person, og så ligesom at sige: "*okay, som du kan de her, så er der en trekant, i denne her… et eller andet*". Men ligesom at man forklarer det til en anden. Men egentlig, så forklarer jeg det bare til mig selv.

### *1H. REASONS FOR CHOOSING SCIENCE*[180]

Q17: Anyway, what you are considering after upper secondary school, well, it may be some chemistry or pharmacology, but why the natural sciences at all? What has given you a preference for science?

D17: It is just...I have always been interested in Chemistry and Science and such. And then, firstly: You know there will be a future in Denmark in science. And I like doing something which everybody does not just do. Most people choose (unintelligible) but I would like to do something other people do not do. Then, of course, you will get more insight into subjects which not many people do. Then I like standing out. And I think you do that in the Natural Sciences. And then I just love everything about standing in a laboratory coat wearing safe glasses and then just...yes. I have also done work placement twice. One time in [Biotechnology Company A] and another time in [Biotechnology Company B]. That was amazing. Just standing there and creating your own cells [?]... That was nice! (Donna laughs)

Q18: When did you do these work placements, was it in lower secondary school?

D18: Yes.

180

Q17: Ja. (…) Men altså, det du tænker efter gymnasiet med, jamen altså det må godt være noget kemi eller farmakologi, altså, hvorfor overhovedet naturvidenskab? Hvad er det der giver dig lyst til at det skal være noget naturvidenskabeligt?

D17: Det er bare… Jeg har altid interesseret mig for sådan noget som kemi og noget naturvidenskabeligt. Og så, ja ét: man ved ligesom, at der altid vil være en fremtid i Danmark omkring naturvidenskabelige ting… Og så kan jeg godt lide at gøre noget som alle ikke bare gør. Sådan, det er de fleste vælger jo (utydeligt), så jeg kunne ligesom godt lide at gøre noget som andre ikke gør. Så bliver man sådan, selvfølgelig klogere på nogle andre emner, men man kan ligesom sige, at det er ligesom nogle emner som der ikke er særligt mange der berører. Så jeg kan ligesom godt lide at ligesom at stå ud. Det synes jeg man gør i det naturvidenskabelige. Og så elsker jeg bare hele det der med at man kan stå og… kittel og briller og så bare… ja. Jeg har også været i praktik to gange. En gang på [Biotek-firma A] og en gang på [Biotek-firma B]. Det var jo for fedt. Og stå og lave sine egne celler og… Det var nice (Donna ler).

Q18: Hvornår, var det i folkeskolen du var i praktik der?

D18: Ja.

### *1I. 'THE SMARTER PERSON KNOWS MATHEMATICS'*[181]

Q19: I would like to return to something we talked about earlier, about how one is perceived when studying mathematics, and you talked about your sports team, that you liked it, you appreciated being the 'nerd'.

D19: Yes, definitely. You know, I love being the smart person. Well, the others know a lot about other things, that I do not know too much about, but in the long run, it is often perceived like the smarter person is the one who knows about mathematics and science and such things. And then, well, every time people have problems with something they do not know how to solve, then they come to me and say like: "*Oh, maybe you could help?*" And such. I like that a lot.

### *1J. COMPETENCE IN MATHEMATICS AS PERCEIVED BY CLASSMATES*[182]

Q20: But in class, would people ever come to you and then...

D20: It depends on the subjects... Mathematics? No. Most people know that I am lagging behind in that. English - a lot. I actually considered taking the language study programme [in upper secondary school], because I have always been really good at languages. Like German, fluent, English, fluent and so on, you know. And a little bit

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Q19: (…) jeg kunne godt tænke mig at spørge til noget af det samme igen som vi faktisk har snakket om, altså, med hvordan bliver man opfattet når man har matematik, og hvor du snakkede om dit "*sportshold*", men at du godt kunne lide det, du kunne godt lide at være nørden?

D19: Ja helt sikkert. Altså, jeg elsker at være den kloge person. Altså, de andre de ved sådan rimelig meget om så mange andre ting, som jeg ikke kender noget som helst til, men det er ligesom altid, det bliver altid set sådan, at dem der er klogest, det er dem som ved sådan noget med matematik og det naturvidenskabelige og sådan nogle ting. Og så, ja.. Altså så hver gang at folk de har nogle problemer eller et eller andet som de ikke rigtigt kan finde ud af, så kommer de altid til mig og er sådan "ej, kan du ikke lige hjælpe" og sådan noget. Det synes jeg er fedt.

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Q20: Men er det også en gang i mellem sådan i klassen, at der er nogen der kommer til dig og så…

D20: Det kommer helt sikkert an på emnerne… Matematik – nej. Det ved de fleste godt, at jeg halter rimelig meget på. Engelsk – rigtigt meget. Jeg overvejede også at tage noget sproglig linje, fordi jeg har altid været rigtig rigtig god til sprog. Sådan flydende tysk, flydende engelsk og så videre, ikke? Og en smule spansk og sådan. Det har altid kommet nemt til mig, men det har bare ikke været det der har været interesseret, interessant. Så det er sådan… Så vil jeg hellere tage det, som jeg egentlig interesserer mig for. Så ja… Men altså sproglige ting, kommer folk i hvert fald rimelig ofte hen til mig med. (Interviewer taler utydeligt ind over) Og så en engang i mellem hvis der er noget omkring kemilaboratoriet, nogle bindinger, hvad det er for nogle ting. Så… Men ikke i matematik.

of Spanish and such. It has always come to me easily, but it just has not been what I have been interested in. Then I would rather choose what I am actually interested in. But people often talk to me about language related issues. And sometimes, if there is something about the chemistry laboratory, or some chemical bonds, what it is. But not in Mathematics.

## THEMES IN DONNA'S 1ST YEAR

*The questionnaire answers and the interview transcripts form the basis of my interpretation and analysis of Donna's interpretations of her experiences with mathematics in the first year of upper secondary school. The interpretations are organised in themes concerning issues or situations.*

### *NOT FUN*

Donna experienced a great contrast in the transition from lower secondary school to upper secondary school; She used to like mathematics in lower secondary school, but now it is not fun any more, since she has to work much harder and it is hard for her to keep up (1A). She used to be the best in mathematics, but that is not the case any more (1A). Apparently, the experience of not being the best seems to somehow discourage her. Nevertheless, she seems to interpret the challenges as a positive feature as well.

In some sense, Donna does not find any distinctions between mathematics in upper secondary school compared to it in lower secondary school. "*It is just harder*" she says. Nevertheless, when explaining what she means by "harder" it turns out that the demands are quite different: "*You are not told what means to use*", she says, "*You have to figure out yourself where everything should be and where the angles are placed*". In this sense, it turns out that there is quite a contrast which is in contradiction to her first account that it is more or less the same (1B).

### *THE USE OF MATHEMATICS*

According to Donna it is important that everybody should learn mathematics, since otherwise "*people can cheat you*". This indicates that she sees mathematical abilities as a form of intellectual self defence which could be interpreted in favour of

education for citizenship.

At this point in time, Donna does not seem to recognise the use of mathematics in her other subjects, which she exemplifies by suggesting that *"Chemistry and Biology does not relate much to Mathematics"*. However, this is not because she does not see any use of mathematics outside its teaching in school; she recognises that mathematics is useful in business life which she exemplifies by payment of wages.

### *MATHEMATICS AS A DISCIPLINE*

Her idea of the activities of a professional mathematician seems to suggest a view of mathematics which is not purely static. Her reply suggests that mathematicians "*Tr[y] to find new formulae and ways to calculate*" and she indicates to be in favour of the idea of mathematics as *invented* by human beings rather than discovered by human beings.

### *CHALLENGES AND THE ROAD TO UNDERSTANDING*

In order to calculate, Donna needs to understand the mathematics she uses, and to understand, she needs to ask several times during a lesson, she explains (D12 in 1F). However, asking repeatedly is annoying for her, and she notes that her classmates, 'the others', do not ask, even though she thinks they do not understand either. The basis for this approach seems to get some explanation in this quote: "*it is just fundamental, if there is something you do not get, then you have to say '*that is just how it is,' *and then you have to, well, learn it*" (D13 in 1F). This is how she justifies her approach of asking until she grasps it and for overcoming the issue that at the same time as providing her with the means for acquiring relational understanding of the mathematics she does not immediately understand, asking questions is quite annoying for her.

It is clear to Donna, that mathematical abilities do not come automatically; she

knows that she has to work actively to achieve her goal of understanding. Donna has been diligent in taking notes, which she writes down in her own manner such that she can understand. These notes provide a reference when she needs to look up something (1G). Also, she has a strategy of imagining she is explaining what she does not yet understand to someone else. Even though she is just explaining things to herself, it is a help to her. In contrast to asking in class or getting help from her brother or even reading her notes, the strategy of imagining she is explaining to someone else is something she can do herself, and which comes from herself. Moreover, as a means for improving in mathematics, she mentions getting a task to explain to the class how to solve.

In general, Donna prefers working on her own, but at home she can get help from her brother, who studies (some?) mathematics in university (1D), but not from her parents, who did not take the Upper Secondary School Leaving Examination themselves (Q1-E).

#### *SELF-IMAGE AS BEING BRIGHT*

Being perceived as bright amongst her peers from the sports team seems to work as a driving force for Donna to keep up in mathematics. To her, the nickname 'the nerd' is a welcome compliment which accentuates her mathematical skills. Somehow her self-image seems to include that she *is* or *wants to become* successful. In this sense she seems to be rather competition oriented, and her competitors include herself (1D). Her brother plays a vital role for her to keep up in mathematics since he has also taken A-level mathematics at the same school (1D). She refers to her interpretation of the status mathematical and scientific knowledge compared to other fields:

> *"Well, the others know a lot about other things, which I do not know too much about, but in the long run, it is often perceived like the brighter person is the one who knows*

*about mathematics and science and such things"*(1.I.)

If it is a primary belief of hers that *the brighter person is the one who knows mathematics* and she likes being the bright person, then it makes sense to work hard to keep up in class in order to maintain the status of being the bright person amongst her peers. Amongst her peers in her sports team, she told, she has the role of being one people consulted with questions they somehow did not manage themselves. However, the same phenomenon is not occurring in class (1.J.), at least not if it concerns mathematics, but in other subjects she is frequently asked, especially in languages but also in chemistry.

At some point she makes a strange comment; she is asked how she deals with being stuck on a task at school, to which she replies "*I solve the task*". It may be that it is incompatible with her self-image to get stuck on a task; on the other hand she finds the greatest obstacle for her to improve in mathematics to be that she "*blank(s) out*" when she "*cannot work it out*". For Donna there seems to be a conflict between her self-image and her actual abilities in mathematics, which actually seems to work as a driving force.

### *RELATIONAL VERSUS INSTRUMENTAL ASPECTS IN RATIONALES*

Donna has an interest towards biotechnology which has had an influence on choosing the study programme with A-level mathematics. Nevertheless, she could have chosen to study something rather similar (a study programme consisting of A-level chemistry and biology and B-level mathematics), but since A-level mathematics is required for university studies in most study programmes within the natural sciences, she chose the study-programme matching both her interest in biotechnology and the level of mathematics required for her preference for further studies after upper secondary school (1.C.). In this sense, the choice of A-level mathematics is based on an *instrumental rationale* (Mellin-Olsen, 1981, 1987, see also

chapter III)[183], rather than on a genuine interest in the subject.

This means that for now, mathematics has the function of opening doors to the further education of her dreams, but she rather hopes not to have to deal with it in university. Her brother is studying it, and she thinks it looks rather difficult. Anyhow, she is willing to accept some mathematics (1E). She somehow concludes that as soon as you have learned mathematics, it is quite simple, but getting it is the hardest part. Apparently, Donna's support from her brother is beneficial, not only in terms of support for undertaking mathematical activities, but also as a source of ideas for what could be in front of her.

She chose her study programme (biotechnology) because she needs A-level mathematics for studying in university after graduation. The rationale behind choosing A level mathematics as a means for obtaining something else rather than a decision in favour of *Mathematics as a Discipline*, seems to be an instrumental rather than a relational rationale for choosing mathematics. At the point in time when she answers the questionnaire, she does not recognise the use of mathematics in her other subjects, biology and chemistry. From this point of view, too, studying mathematics is a means for accomplishing something else, rather than something obviously meaningful in her study programme.

#### *DONNA'S RESOURCES FOR OVERCOMING CHALLENGES*

From the first year interview with Donna just referred to, I will first emphasise the interplay between some aspects of beliefs; One aspect is *Mathematics at School*, since Donna discusses keeping up in Mathematics at School, and the other thing is the joy and the benefit of being good at mathematics when she is among her peers in

183 I interpret the definition of Mellin-Olsen's concepts of *social* versus *instrumental* rationales for learning mathematics to relate to whether it makes sense for passing examinations and getting the grades only or whether mathematics is valued in its own right because of its intrinsic values, such as the applicability or the beauty of it.

her sport. This would relate to the aspect *Mathematics and Me*. The expert role she can get from being good at mathematics in her peer group has a positive influence or her willingness to invest in understanding mathematics. A project for which the help and support from her brother has been crucial. She would not have been able to succeed without his help.

This can be seen as an interplay between the two aspects of beliefs concerning *Mathematics at School* and *Mathematics and Me*. These two aspects seems to support each other in the excerpt mentioned above since keeping up at school helps her maintain the expert role among her sports peers, and the role as an expert or 'nerd' among her peers makes it meaningful to work hard to keep up.

#### *DONNA'S IDEAS OF TERTIARY EDUCATION*

Donna seems interested in studying chemistry or pharmacology in university in her future, but she is not keen on the fact that these studies involve mathematics. Her impression of mathematics in university is that it is quite difficult. It would be tolerable, if there were not too much mathematics involved. Also she suggests that when you understand it, it is not too hard any more, but getting it is the critical part.

I will emphasise two aspects of Donna's beliefs about mathematics from this excerpt: The role of mathematics in other disciplines within the natural sciences, which relates to *Mathematics as a Discipline*, and *Mathematics and Me*, since we are talking about "getting it" and the role of mathematics in her future life. It appears as if she has an idea of the mathematics involved in other disciplines as being less complicated rather than the mathematics study in itself. Also she says something about acquiring mathematics, since talks about getting it as the hard part, and once you get it it is quite simple.

Donna's challenges getting mathematics has an influence on the kind and

amount of mathematics she is willing to accept in her future study. But also the other way around, since the kind of mathematics she will be dealing with is seen as something which is hard to get.

# DONNA'S BELIEFS IN 1ST YEAR

### *MATHEMATICS AT SCHOOL*

Mathematics at upper secondary school is extremely hard. And now you have to figure out much more yourself than in lower secondary school.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is invented and mathematicians in university try to find new formulae.

### *MATHEMATICS IN SOCIETY*

Mathematics is useful in a variety of contexts and should be learned by everybody

### *MATHEMATICS & ME*

Mathematics is not fun. Donna fights against lagging behind. She wants to be successful, but is only experiencing struggle. Nevertheless, mathematics is necessary for her future plans.

## DONNA'S 3RD YEAR QUESTIONNAIRE

| Q3-A | TRANSITION | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [7] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [184] | [Working on your own] [Project Work] |
| **3** | Did you like mathematics when you went to lower secondary school? [185] | [Yes, it was one of my favourite Subjects] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | It has decreased a little[186] |
| **New 4d** | In which ways has mathematics changed from first year to third year of upper secondary school? | It has improved[187] |

*Table 11: Donna's 3rd year questionnaire, part A – TRANSITION*

184 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

185 Options: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

186 Det er faldet lidt

187 Det er blevet bedre

| Q3-B | FOR SCHOOL | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | Because it is teaching us a lot about economy and other things that one should be responsible about[188] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | We use it in relation to calculations[189] |

*Table 12: DONNA'S 3rd year questionnaire, part B - MATHEMATICS AND SCHOOL*

| Q3-C | BEYOND SCHOOL | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | Design, economy, construction work[190] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? [191] | [Mathematics is discovered] |
| **10** | What do you think a professional mathematician at a university is doing?[192] | Tries to find new ways to describe our world, possibly by means of formulae[193] |
| **11** | Would you have to be a genius in order to study mathematics in university?[194] | [No] |

*Table 13: DONNA'S 3rd year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

188 Fordi det lærer os meget om økonomi og andet som man skal være ansvarlig omkring
189 Vi bruger det i sammenhæng med beregninger
190 Design økonomi byggearbejde
191 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]
192 Hvad tror du en professionel matematiker på et universitet laver?
193 Prøver at finde nye måder at beskrive vores verden på evt. ved hjælp af formler
194 [Yes]; [No]; [I do not know]

| Q3-D | IMRPOVING | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | To learn fast enough[195] |
| **12b** | What do you think is the best means for improving in mathematics? | Repetition |
| **12c** | What do you do to improve in mathematics? | (I) Pay attention and do my stuff [196] |
| **13a** | What do you do if you get stuck on a task at school? | (I) Try again or ask for help[197] |
| **13b** | What do you do if you get stuck on your homework? | (I) Try and (I) try and (I) try[198] |
| **13c** | What do you do if you get stuck on your written assignments? | (I) Try and (I) try and (I) try[199] |

*Table 14: DONNA'S 3rd year Questionnaire, part D – STRATEGIES FOR IMPROVING*

195 At lære hurtigt nok
196 Hører efter og laver mine ting
197 Prøver igen eller spørger om hjælp
198 Prøver og prøver og prøver
199 Prøver og prøver og prøver

| Q3-E | CHALLENGES & SUPPORT | | DONNA |
|---|---|---|---|
| # | *Question* | | *Answer* [200] |
| **14** | What issues involve more challenges to you? | a) Remembering | [4] Few challenges |
| | | b) Computing | [4] Few challenges |
| | | c) Figuring out the purpose of a task | [2] Several challenges |
| | | d) Finding a way to solve a task | [4] Few challenges |
| | | e) Reading and understanding the textbook | [2] Several challenges |
| **15** | Where can you find support for mathematical activities? | | [From classmates] [Other]: "Homework Café","Teacher" |
| **New 16a** | Do you find that you meet some special challenges related to succeeding in mathematics compared to your classmates? | | Not any more, no[201] |
| **New 16b** | Do you find that you have some some special strengths related to succeeding in mathematics compared to your classmates? | | No, I am average[202] |

*Table 15: DONNA'S 3rd year Questionnaire, part E – CHALLENGE & SUPPORT*

200 Options: [1]= The most; [2]= Several; [3]= Moderate; [4]= Few; [5]=The fewest

201 Ikke længere, nej.

202 Nej, er middel

| Q3-F | IN CLASS | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? [203] | [More than eight times] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [Yes] |
| **17c** | Are you content with that? | It is a shame because I am sure there are others who do not understand.<br>But besides that, yes[204] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? [205] | [More than eight times] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [Yes] |
| **18c** | Are you content with that? | Yes |
| **19a** | In your class, is it okay to be good at mathematics? | Yes. |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Yes. |

*Table 16: DONNA'S 3rd year Questionnaire, part F - MATHEMATICS IN CLASS*

203 Options: [0]; [1-3]; [4-8]; [More than 8 times]

204 Det er ærgerligt, for der er sikkert andre som heller ikke forstår. Men ellers, ja.

205 Options: [0]; [1-3]; [4-8]; [More than 8 times]

| Q3-XA | UNDERSTANDING | DONNA |
|---|---|---|
| # | *Question* | *Answer*[206] |
| **X.1.** | Have you recently experienced to *understand* what you worked with in mathematics? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.3.** | Have you, during upper secondary school experienced that you *understood* what you worked with and then s*ubsequently learned it by heart*? | [Yes] |
| | If yes, on which occasion? | Max/ min (but I still understand it)[207] |
| **X.5.** | Have you during upper secondary school experienced *understanding something* but *never learning it by heart?* | [Yes] |
| | If yes, on which occasion? | Proofs |

| Q3-XB | LEARNING BY HEART | DONNA |
|---|---|---|
| # | *Question* | *Answer*[208] |
| **X.2.** | Have you recently experiences having to *learn something by heart*? | [No] |
| | If yes, on which occasion? | - |
| **X.4.** | Have you, during upper secondary school experienced that you had to *learn something by heart*, and then, *subsequently understood it*? | [Yes] |
| | If yes, on which occasion? | Vectors |
| **X.6.** | Have you during upper secondary school experienced *learning something by heart without ever understanding it*? | [No] |
| | If yes, on which occasion? | - |
| **X.7.** | Additional comments on understanding or rote learning in mathematics | - |

206 Options: [Yes], [No] or [I do not know]

207 Maks/min (men jeg forstår det stadig)

208 Options: [Yes], [No] or [I do not know]

| Q3-XC | A-LEVEL EXAMINATION | DONNA |
|---|---|---|
| **X.8.** | *Topic* | *Answer*[209] |
| **(a)** | Parabola | [Rather not] |
| **(b)** | Exponential | [Rather not] |
| **(c)** | Pythagoras | [Readily] |
| **(d)** | Sine and cosine relations | [Readily] |
| **(e)** | Definition of differentiability | [Readily] |
| **(f)** | Sum and product of differential functions | [Readily] |
| **(g)** | Indefinite integral | [Readily] |
| **(h)** | Volume of solid of revolution | [Readily] |
| **(i)** | Differential Equations and their solutions | "Has not learned it"[210] |
| **(j)** | Vectors in the plane, including scalar product | [Readily] |
| **(k)** | Lines and planes | "Has not learned it" |
| **X.9.a.** | Which topic is your favourite?<br>- and why? | The appearance of a function.<br>- [I] Did it for the end of term examination[211] |
| **X.9.b.** | Which topic would you rather avoid?<br>- and why? | Exponential function<br>- [I] Find it hard to understand[212] |

*Table 17: Topics for oral and written examinations in A-level Mathematics for Upper Secondary School Programmes in Denmark 2013*[213]

209 Options: [Readily], [Okay], [Rather not], [I do not know]

210 Har ikke lært det

211 Funktionens udseende. - Var oppe i det til terminsprøven

212 Eksponentiel funktion. - Synes det er svært at forstå.

213 Topics common to both the Technical (htx) and the General (stx) upper secondary programme examinations are suggested.

| Q3-G | PLANS | DONNA |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Copenhagen University, molecular medicine, or Danish Technical University, technical biomedicine/ biotechnology[214] |
| **20b** | What kind of influence would your final marks in mathematics have on your plans (or hope for plans) after upper secondary school? | Great influence[215] |
| **20c** | What kind of influence would your experiences with mathematics in upper secondary school have on your plans after upper secondary school? | None[216] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [No] |
| **21b** | Comments: | My abilities are not that good[217] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [No] |
| **22b** | Comments | If I have to, I will toil for it[218] |

*Table 18: Donna's 3rd year Questionnaire, part G – PLANS*

214 Københavns Universitet – molekylær biomedicin eller DTU teknisk biomedicin / bioteknologi

215 Stor indflydelse

216 Ingen

217 Mine evner er ikke så gode

218 Hvis jeg skal så vil jeg knokle for det

# DONNA'S 3RD YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 13 March 2013 | 25:50 |

### *3A. FUN AND UNDERSTANDING*[219]

Q1: How do you feel about mathematics now?

D1: Much better. I actually think that mathematics has started to become fun again, so much better.

Q2: Why is it about to become fun?

D2: Because I have started to understand the topics. And when I understand the topics, then I see the logic of it and then everything begins to become much more fun than it were in year 1 and year 2, when I did not understand anything of anything, then it just was a pain to have mathematics. It is not like that any more.

### *3B. INFLUENCES FOR UNDERSTANDING*[220]

Q3: How did you reach this point?

D3: I do not really know. It was like, as soon as we started on some topics that I kind of understood, well, then I understood things, obviously, and then I just began to believe more deeply that I could do it, and then it worked

219

Q1: Hvordan har du det med matematik nu?

D1: Meget meget bedre. Jeg synes faktisk matematik er begyndt at blive sjovt igen, så meget bedre.

Q2: Hvorfor er det ved at blive sjovt?

D2: Fordi jeg er begyndt at kunne forstå emnerne. Og når jeg kan forstå emnerne, så kan jeg også se det logiske i det, og så bliver det hele en del sjovere end det for eksempel var i 1. og 2.g, hvor jeg ikke forstod noget som helst af noget som helst, så blev det bare en pine at skulle til matematik. Det er det ikke mere.

220

Q3: Hvordan er du nået her hen?

D3: Jeg ved det faktisk ikke helt. Det var som om, at så snart vi begyndte at få nogle emner som jeg ligesom kunne forstå, ja, så forstod jeg det jo sjovt nok, og så begyndte jeg bare at tro mere på at jeg godt kunne, og så fungerede det lidt bedre. Og det er også med alle de emner jeg ikke forstod før. Som om at nu ved jeg at jeg godt kan, så nu gør jeg det bare.

Q4: Hvad for nogle emner hører til blandt dem hvor du tænkte, nu har jeg fået et emne som jeg forstår?

D4: Omkring differentialligningerne synes jeg det begyndte at blive nemmere.

a little better. And that also counts for all the topics that I did not understand before. As if, I now know that I can, so, now I just do it.

*3C. TOPICS EVOLVING UNDERSTANDING*[221]

Q4: Which topics belong to those that made you think, now I have a topic that I understand?

D4: Around differential equations I thought it began to be easier.

Q5: Was it something about the teaching or something about the books, or something about the contexts that you think was significant? How come it was exactly differential equations?

D5: I believe it was because you found another way of calculating which was completely different from what we had done before, and suddenly new relations came up and you could see things in a larger perspective, and then maybe it also becomes more interesting to compute.

*3D. USEFULNESS OF MATHEMATICS*[222]

Q6: What are differential equations used for?

D6: They are useful for many things, mainly slopes and such, yes, to compute the distance to the other side of a river or something, and... we had a task in which we should build a bridge over a river and then find the position from

221

Q5: Var der noget ved undervisningen eller noget med bøgerne, eller noget med den sammenhæng som det indgik i, som du mener har betydning dér? Hvordan kan det være det lige var differentialligninger?

D5: Jeg vil tro det er fordi at man fandt en helt anden måde at regne på end hvad vi havde gjort før, og pludselig var der nogle andre sammenhænge, og man kunne se tingene i et større perspektiv, og så bliver det måske også lidt mere interessant at regne på.

222

Q6: Hvad kan man bruge de differentialligninger til?

D6: Dem kan man bruge til mange ting, mest sådan noget som hældninger, og sådan, ja, at beregne sig frem til hvor langt der er over til et andet sted i en å eller noget, i forhold til alt muligt mærkeligt, og… Vi havde på et tidspunkt en opgave hvor vi skulle bygge en bro hen over en å, og så finde det sted hvor det var mest økonomisk at bygge den her bro, og hvordan man bygger æsker sådan så der kan være mest i uden at… Sådan, forskellige ting, hvor det bliver lidt mere realistisk på en eller anden måde.

Q7: Kommer du til at bruge din viden om differentialligninger, når du slutter her efter gymnasiet?

D7: Det kan jeg godt forestille mig, ja. Nu skal jeg så på universitetet, hvor jeg også kommer til at have matematik. Så højst sandsynligt kommer der til at være et eller andet.

which it would be more economical to build it[223]; and how to build boxes such that they could contain the most without... you know, different stuff in that is more realistic in some way or the other.

Q7: Will your knowledge of Differential Equations be useful to you after graduation?

D7: I could imagine, yes. Since I am going to university, there will probably be something.

*3E. PLANS FOR FURTHER EDUCATION*[224]

Q8: What are you going to study at university?

D8: Something like molecular- and biomedicine.

Q9: Oh yes, you wrote that [in the Questionnaire]. And it would be at University of Copenhagen. So you have a complete plan?

D9: Yes, if I get the required average [of grades]. It is quite high, last year it was 11,3 [out of 12] to get admission. But if not, I know what the alternative would be.

223 *It seems that her concept image of* 'Differential Equations' *deviates from the concept definition. She may refer to* the *use of differentials in general*.

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Q8: Hvad vil du gerne læse på universitetet?

D8: Sådan noget molekylær- biomedicin.

Q9: Det skrev du faktisk også. Og det var på Københavns Universitet. Så du har simpelthen en færdig plan.

D9: Ja, hvis gennemsnittet bliver til det. Det er jo rimelig højt, 11,3 var det for at komme ind sidste år. Men hvis ikke, så ved jeg også hvad alternativet er.

### *3F. COOPERATION IN CLASS*[225]

Q13: Is there any difference in how much you need each other in class for solving tasks? Do you feel an increased need for cooperation or do you feel you can work more independently? Has it changed during upper secondary school?

D13: Generally I work better on my own, because I feel that I benefit more from that myself and I understand all of it on my own. But in first the year I felt more like cooperating with others, because I did not really understand things on my own. But generally I work better on my own, but sometimes, also now in the third year, we are forced to work in groups. But it is not always positive. Not for everybody, at least.

Q14: Do you decide the composition of groups yourselves, or are they composed on beforehand?

D14: We get to choose ourselves, but people are so much grouped together in advance, that it is not straightforward to find a new group. So you kind of have to work with the same people every time, even though you are not on par with each other. Then it is not optimal.

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Q13: Er der forskel på hvor meget I har brug for hinanden til at løse opgaver? Føler du at have mere brug for at arbejde sammen med andre, eller oplever du at kunne arbejde mere på egen hånd? Har det ændret sig for dig igennem gymnasiet?

D13: Jeg arbejder generelt altid bedst alene, fordi så føler jeg at jeg får mest ud af det selv og forstår det hele selv. Men i 1.g havde jeg mere lyst til at være sammen med nogen, fordi jeg forstod det ikke rigtig selv. Men generelt arbejder jeg personligt bedst alene, men der er da nogen gange, også her i 3.g, hvor vi bliver tvunget i grupper. Men det er så ikke altid lige positivt. For alle i hvert fald.

Q14: Er det så nogle grupper I selv vælger, eller er det nogle grupper hvor de er sat sammen på forhånd?

D14: Vi får selv lov til at vælge dem, men folk er så gruppeinddelt i forvejen, at det er ikke lige til at finde en ny gruppe. Så man er lidt tvunget til at arbejde sammen med de samme hver gang, til trods for at niveauet ikke er ens. Så det er ikke så optimalt.

### *3G. APPLICATION OF MATHEMATICS*[226]

Q16: Well, we have discussed that you aim at becoming a Molecular Biologist, and that university study will probably involve some mathematics. How about outside an educational setting? Could you employ the Mathematics you learn now outside an educational setting?

D16: Well, I think it would be difficult, because the topics we study are not just something you employ in everyday life. We do not do, like percentages or probability like that. Those topics we learned in lower secondary school already. You need to work in an educational setting to be able to employ it in your job; for it to be useful. Because, it is too special, for being employed in everyday life.

Q17: But now, you do employ it in your other subjects too?

D17: Yes, easily. Also the software and some efficient ways of computing stuff. So, yes. Especially in subjects like Biotechnology and Chemistry. So, yes.

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Q16: Ja, vi har snakket om, at du gerne vil være molekylærbiolog, og så det dér med: der kommer nok til at indgå noget matematik i din uddannelse senere hen ad vejen. Hvad med udenfor uddannelsessystemet? Kan du bruge den matematik du lærer nu udenfor uddannelsessammenhæng?

D16: Ja … Det tror jeg faktisk bliver svært, fordi vores matematikemner er ikke rigtig noget man bare sådan lige kan bruge i hverdagen. Vi har jo ikke noget, sådan, sandsynlighedsregning og procentregning på dén måde, det lærte man allerede fra folkeskolen af. Det skal man sidde og arbejde med uddannelse til at bruge det på sin arbejdsplads, for at man rigtig kan bruge det til noget. Fordi, det er alt for specifikt til, sådan, at kunne bruge det i almindelige sammenhænge.

Q17: Ja. Men du bruger matematik i dine andre fag nu også?

D17: Ja. Sagtens. Også matematikprogrammerne og nogle hurtigere måder at regne ting ud på og sådan noget, så ja… Specielt i sådan noget som Biotek og Kemi. Så ja.

### *3H. COMPUTING AND UNDERSTANDING WHY*[227]

Q18: In the questionnaire I ask: "*What is the greatest challenge in becoming better at mathematics?*" And: "*What is the best means for this*?" And you mention "*Learning fast enough*" as the greatest challenge

D18: Yes, especially...maybe it is because I have been there myself at the beginning [of upper secondary school], that I was not fast enough to understand what I was supposed to do, so I managed to solve the tasks, partly at least, by just computing things. But to acquire the fundamental understanding of why, well there was not much of that. And if you are lagging behind on a topic, for example, then you have to be quick to catch up, and you are not being waited for that much any more. So you have to invest 110% to catch up.

Q19: But how can you do that, yourself? You write "*Repetition*"?

D19: Yes, we used to have that a while ago, just like, conventional mathematics lessons and then in a subsequent lesson every Tuesday, I believe, we had something called "*Math Help*"; that was one lesson in which you could ask questions in some of the topics we had learned about already and you had a chance to go over it one more time, if necessary. And then it was optional if you wanted to come or not, but it enabled one to catch up a little.

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Q18: Jeg spørger i spørgeskemaet til, hvad er den største udfordring i forhold til at blive god til matematik, og hvad er det bedste middel, og det med at lære hurtigt nok nævner du som, at det er faktisk dér der kan ligge en udfordring.

D18: Ja. Specielt… Men det er måske fordi at jeg selv har prøvet det, dér i starten, at der var jeg ikke hurtig nok til at kunne forstå præcis hvad det var jeg skulle, så jeg kunne godt løse opgaverne, sådan til dels, ved at bare regne tingene ud. Men at få den grundlæggende forståelse til at hvorfor, jamen det var der ikke så meget af. Og hvis man kommer bagud i et emne for eksempel, så skal man altså være hurtig til at komme med, og der bliver ikke ventet så meget på én mere. Så man skal, altså give 110% før man kan følge med.

Q19: Og hvad kan man så gøre ved det selv? Og der skriver du "repetition".

D19: Ja, vi havde på et tidspunkt, bare sådan, vi havde almindelige matematiktimer, og så en time efter, hver tirsdag tror jeg, der havde vi noget som hed matematikhjælp, hvor der var én time, hvor man kunne stille spørgsmål til noget af det som vi tidligere havde lært, hvor man lige havde tid til at få det gennemgået én gang til hvis det var. Og så var det selvfølgelig frivilligt om man havde lyst til at komme eller ej, men det gav i hvert fald én en mulighed for at man, ja, kunne være lidt mere med.

Q20: Var det noget du brugte i starten?

D20: Ja, jeg benyttede mig rigtig rigtig meget af det. Det gjorde jeg. Bestemt.

Q20: Did you avail yourself of it (after school Math Help) in the beginning?

D20: Yes, I frequented it a lot. I did. Definitely.

*3I. FURTHER EDUCATION - PERCEIVED COMPETENCE*[228]

Q22: Now you write something that really puzzles me

D22: Oh dear!

Q23: Here, for one of the questions, I have to ask about this... "*Could you imagine aiming at pursuing an education involving a substantial amount of mathematics?*" Then you tick off [No] and your comment is "*My abilities are not that good*". Is this really your questionnaire? It does not sound in line with what you say now?

D23: eh..it does not look like my handwriting, at least ... yes, that is a little odd

Q24: Strange, is it not?

D24: "*My abilities are not that good*"? That sounds a little funny.

Q25: I do not get it.

D25: Well, I kind of understand what I meant, but eh..., well, it is because I do not think I am sufficiently good at mathematics for daring to take an education involving a substantial amount of mathematics. Even though I find mathematics incredibly interesting I am not sure I would be able to keep up.

Q26: But right now it is going quite well?

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Q22: Nu skriver du noget der undrer mig rigtig meget -

D22: Nå for søren.

Q23: Her til en af spørgsmålene, det bliver jeg nødt til at spørge til. (…) Her: "*Kunne du forstille dig at gå efter en uddannelse der indeholder en del matematik*?" Så sætter du kryds ved [Nej], og din kommentar er: "*Mine evner er ikke så gode*". Er det dit spørgeskema? Det lyder ikke som sådan du snakker nu.

D23: Øh… Det ligner bestemt ikke min håndskrift, i hvert fald. (…) Ja, det er s'gu lidt underligt.

Q24: Det er mærkeligt, ikke?

D25: "*Mine evner er ikke så gode*." Det lyder lidt sjovt.

Q25: Det forstår jeg ikke.

D25: Jamen, jeg forstår godt hvad jeg mener med det, men øh… Jamen, det er fordi at jeg ikke synes selv at jeg er god nok til matematik til at kunne turde tage en uddannelse der indeholder for meget matematik. Selvom jeg synes matematik er super, super spændende, så er jeg ikke sikker på at jeg ville kunne følge med i det.

Q26: Men det går ret godt nu?

D26: Ja, det går meget bedre nu.

D26: Yes, it is going quite well right now.

### *3J. FURTHER EDUCATION - PERCEIVED COMPETENCE VERSUS INTEREST*[229]

Q27: And now you are actually going to take an education involving a good deal of mathematics?

D27: Yes, But...I do not know, I just think I have a propensity for choosing what I find exciting without really considering what I think I am good at. For example, here in upper secondary school, I am much better at languages that at anything else, but then again I chose science. That is because I find it more exciting. My abilities are not as remarkable in mathematics as they are in the history of ideas, for example, but even so I chose mathematics as the major subject for my Study Programme Project[230]. So I choose what I find more exciting, and then I have to work a little harder for it. So yes, that is probably also one of the reasons that I dare choosing one involving mathematics, but I will make it if I have set my mind on it, you know.

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Q27: Og du skal jo sådan set ind på en uddannelse der indeholder en del matematik.

D27: Ja. Jamen… Det ved jeg ikke, jeg tror bare at jeg har en tendens til at vælge hvad jeg synes er spændende uden rigtig at overveje hvad jeg synes, hvad jeg egentlig er god til. For eksempel her på gymnasiet, jeg er langt bedre sproglig end jeg er alt muligt andet, og alligevel vælger jeg naturvidenskab. Det er fordi det er det jeg synes er mest spændende. Mine evner er ikke så fantastiske i matematik som de er i idéhistorie, for eksempel, men alligevel vælger jeg at skrive i matematik til SRP. Så jeg vælger altså det jeg synes er mest spændende, og så må jeg knokle lidt hårdere for det. Så ja, det er nok også lidt af grundene til at jeg tør tage en med matematik, men jeg kan nok godt klare det hvis jeg sætter hovedet efter det, ikke.

230 *An individually conducted interdisciplinary project involving one of the main subjects in the study programme, for Dana either Mathematics or Biotechnology. The project report will be subject to external evaluation.*

### *3L. MORE CHALLENGING ISSUES*[231]

Q33: (Referring to the questionnaire) What issues are more challenging? Most of them are placed in the middle (...) But it is more challenging to find out what a task is all about and to read and understand the textbook?

D33: Yes. When I get a task, I might spend some time on figuring out what it is that they want me to do. For example, we recently had one in which I just wondered, how on earth would I know? It said: "*Determine such and such*", and then I determine it in one way, but we were supposed to determine it by means of an equation. But if I arrive at exactly the same conclusion, then I cannot see why it should be done by means of an equation. To understand: What is it they want me to do in this task, that is sometimes the most difficult part. And then I think the textbook is really really poorly written. In the beginning, for example, I understood nothing, just to begin with, because they wrote: "*Given a triangle*". It was when we had trigonometry; it said: "G*iven a triangle*". Then I was lost already. Now it is a matter of course, yes, given a triangle, yes, of course. Then it is like, a little condescending. "*Then you obviously see that…*" I cannot see that! So it is quite intricate, and sometimes it seems a little condescending. It becomes a joke in class that it is like that, but it is a little.. are you supposed to be able to see that? Then you feel a little, well...

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Q33: Hvilke ting har du flest udfordringer med. De fleste har du lagt i midten her (...). Der er flere udfordringer med at finde ud af hvad en opgave går ud på og med at læse og forstå matematikbogen end med de andre ting.

D33: Ja. Når jeg får sat en opgave, så kan jeg godt bruge lang tid på at finde ud af, hvad er det de vil have mig til. Vi har for eksempel lige haft en hvor jeg bare sad og tænkte, jamen, hvor i alverden skulle jeg vide det fra? Hvor der står, at vi skal bestemme et eller andet, og så bestemmer jeg den på én måde, men vi skulle bestemme den ved hjælp af en ligning. Men hvis jeg når frem til præcis det samme resultat, så ser jeg ikke rigtig nogen grund til at gøre det ved hjælp af en ligning. At forstå: hvad er det de gerne vil have mig til i den her opgave, det synes jeg nogen gange er det sværeste. Og så synes jeg matematikbogen er virkelig, virkelig dårligt skrevet. Sådan til at starte med, for eksempel, der forstod jeg ingenting, bare til at starte med, fordi de skrev "givet er en trekant". Det var dengang vi havde om trigonometri; der stod "givet er en trekant". Allerede dér, der var jeg lost. Nu er det selvfølgelig, ja, givet er en trekant, ja, selvfølgelig er den det. Og så er det sådan, lidt nedladende, "Så kan du selvfølgelig se at…". Jeg kan ikke se det dér. Så det er meget kringlet, og nogen gange virker det sådan lidt nedladende. Det bliver så en joke i klassen at den er sådan, men det bliver da sådan lidt… Er det meningen man skal kunne se det dér? Så føler man sig sådan lidt… Ja.

***3M. CHANGE OF COMPETENCE - CHANGE OF ROLE IN CLASS***[232]

Q37: ...I understand that you expect more of yourself now than you did earlier?

D37: Yes, 100%.

Q38: Do you feel that in class? Do you feel that your role has changed?

D38: Yes, a lot. One could say, in the beginning, people waited for me a lot, and I felt that. It is not like people were negative or anything, but when I asked, maybe for the third or the fourth time if we could go over it again, then people might become a little: "*Come on, could you take that later or something, because the rest of us want to move on.*" I was always the one who had to ask the others for help, if I did not understand a task or did not know how to solve it. Then they gave me little hints, like: "*Try to look at it from this perspective,*" or: "*Try to to look it up in this chapter,*" or something like that. Now it is the other way around, and people can come to me to ask for help. And, yes, of course it is rather nice to be able to solve it yourself, you know.

Q39: I remember in first year, people were asking you for Biology questions and you were asking the them about mathematics...

D39: Exactly. And now it is just that people do ask, also in mathematics.

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Q37: … jeg kan forstå på dig at du faktisk forventer mere af dig selv nu end du har kunnet tidligere.

D37: Ja. 100%.

Q38: Kan du mærke det i klassen? Kan du mærke, at din rolle har ændret sig?

D38: Ja. Meget. Man kan sige, til at starte med, der var det jo meget, at folk ventede på mig, og det kunne jeg selvfølgelig også godt mærke. Det var ikke sådan så folk blev negative over det eller noget som helst, men når jeg spurgte, måske 3. eller 4. gang om at få det gennemgået igen, så kunne det godt være at folk blev sådan lidt, "*Arh, men så må du s'gu lige… Det må du tage senere eller et eller andet, for nu skal vi andre altså også se at komme videre*.". Det var altid mig der skulle spørge om hjælp fra de andre hvis der var en opgave jeg ikke rigtig kunne forstå eller ikke rigtig kunne løse. Så var det dem der gav mig nogle små tips, til at "*prøv at se det fra den hér vinkel*", eller "*prøv at se i det hér kapitel*", eller sådan noget, ikke… Nu er det så omvendt, hvor folk godt kan komme til mig for at spørge om hjælp. Så ja, det er selvfølgelig meget rart også at kunne løse det selv, ikke.

Q39: Ja, fordi jeg kan huske, at i 1.g, der snakkede vi om, at i biologi, der kom folk og spurgte dig, og i matematik, der var det dig der spurgte de andre.

D39: Ja, lige præcis. Så ja, så nu er det bare sådan så folk godt kan spørge, også i matematik. Så det, det er meget godt.

*3N. MATHEMATICS AS A DISCIPLINE AND THE NATURE OF MATHEMATICS*[233]

Q48: In the questionnaire I ask what you think a Mathematician in a university is doing? And you write: "*Tries to come up with new ways of describing the world, possibly by use of formulae*."

D48: Yes, but...eh..it is because I actually do not really know what they are doing. I could imagine that one would try to explain how things are related. It may not be the only thing they do, but eh... That is at least what I could imagine. It would be a nice hobby, one might say.

Q49: I also ask whether Mathematics is discovered or invented. You may have become used to that question now. You write: "*Mathematics is discovered*." Can you say something more about why you think that?

D49: Why it is discovered?

Q50: Yes.

D50: It is because, Mathematics has always been there. It is not like, we are just sitting and then, all of a sudden, then Mathematics is here. Mathematics has always been used in some way or another. We may not have known it, but Mathematics is also the relations between things, and relations between things have always been there. So...

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Q48: Jeg spørger i spørgeskemaet til, hvad tror du en professionel matematiker på et universitet laver? Og så skriver du: Prøver at finde nye måder at beskrive verden på, eventuelt ved hjælp af formler.

D48: Ja. Ja, men øh… Det er fordi, jeg ved egentlig ikke rigtig hvad de ellers skulle lave. Jeg kan forestille mig at man vil prøve at forklare hvordan tingene hænger sammen. Det er nok ikke det eneste de laver, men øh… Det kan jeg i hvert fald forestille mig. Det ville være en ret fed hobby, kan man sige.

Q49: Jeg spørger også, om matematik er opdaget eller opfundet. Nu har I snart vænnet jer til det spørgsmål. Du skriver: Matematik er opdaget. Kan du sige noget mere om hvorfor du tænker det?

D49: Hvorfor det er opdaget?

Q50: Ja.

D50: Det er fordi, matematik har jo altid været her. Det er jo ikke rigtig noget hvor vi sidder og tænker, lige pludselig, så er der matematik her, matematik er altid blevet brugt på en eller anden måde. Det kan godt være at vi ikke rigtig har vidst det, men matematik er jo også… Matematik er jo sammenhængen mellem ting, og der har jo altid været sammenhæng mellem forskellige ting. Så det er jo...

## THEMES IN DONNA'S 3RD YEAR

*These interpretations are based on both the questionnaire answers and the interview transcripts from Donna's 3rd Year.*

In her answers to the questionnaire for the 3rd year in upper secondary school, Donna now place mathematics at [7] on the favourite subject scale, while it was given a [2] the 1st year. On a question regarding how mathematics has changed from the 1st year one to the 3rd year in Upper secondary school, she writes "*It has improved*".

In the interview we now get a chance to hear Donna elaborate on some of the reasons behind this answer; When asked how she feels about mathematics now she responds that she not only feels much better about it (compared to the 1st year), but even that it has begun to become fun. This is a major change from the 1st year, when she thought the subject had become less fun, because she felt she was lagging behind.

### *ON UNDERSTANDING*

She explains this difference with the fact that she now feels she is beginning to understand the topics. In her response, she clearly links understanding and fun, and also the opposite, not fun, and not understanding, not to say a pain not understanding. When asked how she reached the point of beginning to understand, Donna starts explaining how understanding one topic led her to believe more in herself which again led her to understand better. And this appeared to her to influence the topics that she did not understand earlier (3.A.). She expresses that now she knows that she can, she just does it. This positive development seems to appear in relation to certain topics, which she understood initially. Donna

remembers 'differential equations' as the topic around which she began to understand. Around this topic new relations came up which revealed a larger perspective, a feature which led her to find it more interesting to compute. The change seems to appear quite suddenly, when she sees things in a new light (3.B.). Donna emphasises what she refers to as 'differential equations' for their applicability; "*different stuff that is more realistic in some way*" and she gives examples in which the use of the derivative might be of help for solving a problem of optimisation, which suggests that she might be thinking of differential calculus rather that differential equations. Nevertheless, these features seem to relate to the new perspectives (mentioned in 3.C.). She seems to be confusing the concepts, but the influence of the experiences with this area of mathematics seems clear. This way of calculating, its usefulness and the new perspectives it provides seem to lead her to better understanding.

### *UNDERSTANDING OR LEARNING BY HEART*

In her recent experiences with mathematics she indicates that in relation to the topic of vectors, she experienced that she initially learned some of it by heart and then subsequently understood it, whereas for maximum and minimum problems she understood it initially, and subsequently learned it by heart. She also ticks off that she has never in upper secondary school mathematics learned something by heart that she has not subsequently understood. But for some proofs, she understood them without ever learning them by heart.

### *SUPPORT*

It is noteworthy, that in the third year of upper secondary school, Donna does not mention "Siblings" as a source of support for mathematical activities, neither is her brother mentioned when she was asked how she deals with getting stuck with mathematical tasks. Instead, now, in the third year, she asks classmates, make use of

the school's "Homework Café" or get help from the teacher. Donna mentions 'after school mathematics help' as a means for overcoming the challenge of not understanding 'why' in mathematics. It appears as if she is aware that an instrumental understanding is not sufficient for her - that she is motivated to pursue relational understanding and the 'after school mathematics help', which was running in the beginning, was a means for that (3.H.).

*IN CLASS*

Working on her own is still a favourite approach of hers, but now she also ticks of project work as a favourite. The way she indicates to meet challenges with mathematics in different contexts seems a little different from the 1st year. When stuck on a task at school, she now says that she tries again or ask for help, whereas in the 1st year she would conclude that she solved the task, which did not really make any sense. Donna does not indicate to meet any special challenges compared to her classmates, nor does she find herself to possess any special strengths. On the contrary, she indicates to be 'average'. In year three Donna still does not hesitate to pose questions in class; she indicates that she would typically ask a question more than eight times during a lesson, which she finds to be more frequent than with her classmates. However, she is content with her style, only regretting that not more students follow her example, since she is sure she is not the only one who does not understand. Compared to her answers to the same questions in the 1st year questionnaire, she still asks questions frequently, but back then, it was followed by the comment "*it is not fun*", whereas she now seems to be more than fine with it (Q3-F). An important detail could be that compared to 1st year she now also answers questions more frequently than her classmates, which was not the case in the 1st year. Her response to the closed item concerning the number of times she typically would raise her hand to answer a question is [More than 8 times] (Q3-F). She believes this to be more often than most other students in class, a fact with which she seems to be

content.

Now in the 3rd year, she still does not indicate any difference in the acceptance of competence and the acceptance of difficulties in mathematics.

### *TEXTBOOK AND CONVENTIONS*

Some of the conventions in mathematics which the students meet in upper secondary school can form a barrier to learning. Donna mentions both figuring out what is meant in the formulation of tasks by the teacher, for example the term 'determine', but also the textbook is formulated in a way which is new to her and which seems condescending - not only to her, but apparently to the whole class (Donna 3.L. ).

Once again, we are reminded that Donna has experienced a development in her competence in mathematics from the 1st to the 3rd year in upper secondary school. In the 1st year, she was the one who had to ask the others for help in mathematics, while in other subjects, they would ask her. Now her classmates can ask her for help in mathematics too (Donna 1.J., 3.M.).

### *DONNA AND THE DISCIPLINE OF MATHEMATICS*

Some of the questions do not really relate to experiences from upper secondary school, but rather to the students' idea of some more general characteristics of mathematics. These issues are approached by asking about the nature of mathematics and about the activities of a research mathematician.

Apparently, the question of whether we are creating something new in mathematics does not make any sense to Donna (3.K.). Nevertheless, in her reply to whether mathematics is invented or discovered in the 3rd year questionnaire, she suggests it to be invented, whereas in the 1st year questionnaire, she suggested it to

be discovered. Indicating Mathematics to be invented, should from a theoretical point of view correspond to creating something new. From Donna's 3rd year interview (reply D48), we learn that she does not really have an idea of what a mathematician in a university would be doing, but she suggests that the person would be engaged in explaining how things are related (3.N.). She explains that she sees mathematics as the relation between things. And since relations between things have always been there, then mathematics must always have been there as well.

Since these beliefs do not relate to each other in what could be characterised as a logical sense, this might be an example either of the clustering of beliefs, preventing beliefs from different contexts to cross-fertilise each other, but it might also relate to the fact that these beliefs are not central to Donna; they are not held very strongly.

### *APPLICATIONS OF MATHEMATICS*

In her third year answer to the question of whether mathematics relates to some of her other subjects, she now writes that "*we use it in relation to calculations*". This represents a difference from her first year answer to the same question, in which she replied that she did not find any relationship between chemistry and biology and mathematics. In Donna's view, the mathematical topics she studies in upper secondary school are 'too specific' to be useful outside a school setting, only if you work with education, it would be useful (3G). However, what she learns in A-level mathematics can 'easily' be employed in other subjects, like chemistry or Biotechnology, both as efficient ways of computing stuff and the software she uses in mathematics. Outside school, she now exemplifies the use of mathematics by "*Design, economy, construction work*". This is a wider range of applications compared to the 1st year, when her example regarded wages in companies. But she fails to see the use of these aspects of mathematics outside school.

In her reply to the question of why it has been decided that everybody in Denmark should learn mathematics, Donna suggests the following reason: "*Because it teaches us a lot about economy and other stuff you should be responsible for*". I see this answer as referring to a rationale of *education for citizenship.* She still supports that mathematics is something that everybody should learn.

### *PLANS*

Donna's plans for study after upper secondary school involves going to university to study "Molecular Biomedicine[234]" (3.E.). It depends on her final grades from examinations and evaluations whether it will work. Last year the required average for admission to this education was 11,3 [12 corresponds to the mark A, and 10 corresponds to the mark B]. The plan seems to have changed a little since the 1st year, when she intended to study Chemistry. However, it is not out of sync with her initial dream of working in a Biotechnology company in her professional life in the future. It might even be a better or at least as good an option for that endeavour.

### *MATHEMATICS SELF CONCEPT AND CAREER DECISIONS*

Both from most of the answers to the third year questionnaire and from the dialogue in the third year interview, Donna seems to have acquired a good understanding in mathematics. It has become more fun and she is asked for help from her classmates (3.M.). It seems to be a successful experience, but for some reason, she has indicated a lack of confidence in her own abilities in her answer to one of the questions concerning whether she would be comfortable with choosing an education involving a good deal of mathematics. She actually does not remember having answered that, when I ask for the background to that answer (3.I.).

234 "Molekylær Biomedicin" or Master of Science in Molecular Biomedicine – a study programme at Copenhagen University.

# DONNA'S 3RD YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics has now become tractable, because it is now possible to understand what is going on. Now classmates come to her with questions in mathematics.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is discovered, and mathematicians in university try to find new ways to describe the world. This view seems to place mathematics as a research discipline in itself in a static mode, but the applicability of mathematics outside itself is dynamic.

### *MATHEMATICS IN SOCIETY*

Mathematics is useful everywhere, also for being a citizen in society and everybody should learn mathematics.

### *MATHEMATICS & ME*

Mathematics has become fun to Donna now in the third year of upper secondary school, since she is now a successful mathematics learner understanding both new topics and earlier ones. Donna has worked hard to reach this point, and she is now able to achieve her goals for tertiary education in a STEM-university study.

## DONNA'S BELIEFS TRANSPOSITION

Donna's belief in herself as a successful person has encouraged her to work hard – by means of imitative reasoning, if necessary. Nevertheless, her approach has paid of in terms of relational understanding meaning she is now on top of the situation in mathematics.

***STABLE:***

Donna's persistence for overcoming challenges. But, strangely enough; she does not perceive herself as something special as a mathematics learner; she sees herself as average.

***CHANGE:***

Donna's image of herself as a mathematics doer; she now knows she understands and that she can solve the tasks and get good grades in mathematics.

Donna has changes her view of the Nature of Mathematics; in the 1st year se saw mathematics as invented and now in the third year she sees it as discovered. However, the usefulness of mathematics for describing relations in the world has become visible to Donna due to her experiences in A-level mathematics in upper secondary school.

# *THE CASE OF DYLAN*

*Dylan is a male student from DELTA Technical Upper Secondary School, in a Biotechnology study programme which involves studying high-level mathematics. In the 1st year he rated mathematics [8] on the favourite subject scale, but in the 3rd year he dropped the rating by two steps to [6]. In the 1st year questionnaires, his preliminary plans for tertiary education was to study medicine in order to become a doctor, whereas in the 3rd year he would consider either medicine, if possible, or a bachelor's degree as a laboratory technician.*

| Dylan | Questionnaires | Interviews |
|---|---|---|
| 1st Year | 24 November 2010<br>(+ supplement<br>16 December 2010) | 31 March 2011 |
| 3rd Year | 17 December 2012 | 13 March 2013 |

*Table 19: Dylan: Dates for Questionnaires and Interviews*

## DYLAN'S 1ST YEAR QUESTIONNAIRE

| Q1-A | TRANSITION | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [8] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [235] | [Working on your own]; [Working in pairs] |
| **3** | Did you like mathematics when you went to lower secondary school? | [Yes, it was fine] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | Things I used to find difficult I now learned relatively fast. I learn new things more easily, generally better than in lower secondary school.[236] |
| **4b** | Is there anything you liked better before?' | Only the teacher, but not something worth mentioning.[237] |
| **4c** | Is there anything you like better now? | Everything, the way the work is organised, the tasks, the teacher, the way of learning.[238] |

*Table 20: DYLAN'S 1st year questionnaire, part A – TRANSITION*

235 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

236 Ting jeg havde svært ved lærte jeg i løbet af ret kort tid. Jeg lærer nye ting nemmere, generelt bedre end i folkeskolen.

237 Kun selve læreren, men ikke noget nævneværdigt.

238 Alt, arbejdsformen, opgaverne, afleveringerne, læreren, læringsformen.

| Q1-B | FOR SCHOOL | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | To get mathematical skills and for everybody to have the POSSIBILITY of taking an education with mathematics.[239] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **6** | What made you choose a study programme involving A-level mathematics? | Because I want to learn it on A-level[240] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | Physics builds a little on mathematics. Social Science also.[241] |

*Table 21: DYLAN'S 1st year questionnaire, part B - MATHEMATICS AND SCHOOL*

239 For at have matematiske færdigheder, og for at alle har MULIGHED for at få en uddannelse med matematik.

240 Fordi jeg gerne vil lære det på A niveau.

241 Fysik bygger lidt på matematik, det samme kan samfundsfag.

| Q1-C | BEYOND SCHOOL | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | With engineers, architects, retail, finance, everywhere one could find some mathematics.[242] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? | [Mathematics is discovered] |
| **10** | What do you think a professional mathematician at a university is doing? | He/she conducts research on new formulae, helps students, teach and pursues a career.[243] |
| **11** | Would you have to be a genius in order to study mathematics in university? [244] | [Yes] |

*Table 22: DYLAN'S 1st year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

242 Hos ingeniører, arkitekter, hos detailhandlen, finans, alle steder vil man kunne finde lidt matematik.

243 Han/hun forsker i nye formler, hjælper de studerende, underviser og skaber karriere.

244 Options:[Yes]; [No]; [I do not know]

| Q1-D | IMRPOVING | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | I do not really see any hindrances. It is a matter of volition/inclination.[245] |
| **12b** | What do you think is the best means for improving in mathematics? | Volition, inclination and that you YOURSELF want to learn.[246] |
| **12c** | What do you do to improve in mathematics? | Pay attention to the teaching I receive and prepare for the lessons of the day.[247] |
| **13a** | What do you do if you get stuck on a task at school? | First, I try to solve it myself, then I look up formulae and finally I ask for help, if possible.[248] |
| **13b** | What do you do if you get stuck on your homework? | I proceed to the next task; otherwise I ask a classmate.[249] |

*Table 23: DYLAN'S 1st year Questionnaire, part D – STRATEGIES FOR IMPROVING*

245 Kan ikke se nogen store hindringer i det. Det er et spørgsmål om vilje/ lyst…

246 Vilje, lyst, og det at man SELV vil lære.

247 Følger med i undervisningen jeg modtager, og forbereder mig til dagens timer.

248 Jeg prøver først selv at løse den, der efter kigger jeg på mulige formler og til sidst spørger jeg om hjælp hvis muligt.

249 Jeg går videre til næste opgave, og ellers spørger jeg en fra klassen.

| Q1-E | CHALLENGES & SUPPORT | | DYLAN |
|---|---|---|---|
| # | *Question* | | *Answer*[250][251] |
| **14** | What issues involve more challenges to you? | a) Remembering | [4] Few challenges |
| | | b) Computing | [3] Moderate challenges |
| | | c) Figuring out the purpose of a task | [5] The fewest challenges |
| | | d) Finding a way to solve a task | [4] Few challenges |
| | | e) Reading and understanding the textbook | [4] Few challenges |
| **15** | Where can you find support for mathematical activities? [252] | | [From parents]; [From siblings]; [From classmates] |
| **16** | Did you parents take the Upper Secondary School Leaving Certificate? [253] | | [Both] |

*Table 24: DYLAN'S 1st year Questionnaire, part E – CHALLENGE AND SUPPORT*

250 Svar: [1]=Flest; [2]=Flere; [3]=Midt imellem [4]=Få; [5]=Færrest

251 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

252 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From classmates]; [Other places] If other places, from where or from whom?

253 [Yes, my mother did]; [Yes, my father did]; [Both my parents did]; [None of them did]

| Q1-F | IN CLASS | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? | [1-3] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [No] |
| **17c** | Are you content with that? | Yes, entirely content. Independence is fine, since you do not always have the possibility of getting help.[254] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? | [1-3] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | No. It could be improved.[255] |
| **19a** | In your class, is it okay to be good at mathematics? | Yes, it commands a certain respect[256] |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Yes, everybody has a hard time[257] |

*Table 25: DYLAN'S 1st year Questionnaire, part F - MATHEMATICS IN CLASS*

254 Ja fuldt tilfreds. Selvstændighed er godt, da man ikke altid har mulighed for hjælp.

255 Nej, det kan forbedres.

256 Ja, det giver en vis respekt.

257 Ja, alle har det en smule svært.

| Q1-G | PLANS | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Pathology at Copenhagen University[258] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [No] |
| **21b** | Comments: | Pathologist[259] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [I do not know] |
| **22b** | Comments | - |

*Table 26: DYLAN'S 1st year Questionnaire, part G – PLANS*

258 Patologuddannelse på Københavns Universitet.

259 Patolog

# DYLAN'S 1ST YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 31 March 2011 | 26: |

***1A. TRANSITION & UNDERSTANDING***[260]

Q1: How has mathematics changed, from when you went to lower secondary school, till now, when you are in technical upper secondary school?

DY1: It has changed a lot. It was like, what you found hard earlier, it has become much easier now. That is also due to other teachers

Q2: What, for example, has become easier now?

DY2: Things like understanding equations and other mathematical tasks. When, earlier, then they did not … well compared to the explanation you get here, it makes a huge difference.

Q3: What was it that was lacking?

DY3: It was more that the explanation of equations and so on, that has become much easier to understand here.

Q4: May I ask more precisely, what it is about equations that you have got explicated?

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Q1: (...) hvordan matematik har ændret sig, fra du gik i folkeskolen og så til nu, hvor du går på HTX?

DY1: Det har ændret sig meget. Det var sådan at, det man før i tiden havde svært ved, det er sådan set blevet meget lettere nu. Det er så også på grund af, at man har nogle andre lærere.

Q2: Hvad er det for eksempel der er blevet lettere nu?

DY2: Altså sådan noget som at forstå ligninger og andre matematiske opgaver. Hvor, før i tiden, der var det jo, at de kom ikke… altså i forhold til den forklaring man får her, så er det jo en kæmpe forskel.

Q3: Hvad var det, du manglede (…)?

DY3: Det var mere at forklaringen på ligninger og så videre, som er blevet meget lettere at forstå her.

Q4: (…) må jeg prøve at spørge sådan helt præcist, hvad det er ved ligninger at det er du har fået forklaret nu?

DY4: Jamen (...) i folkeskolen, der startede de ud med at give en eller anden forklaring, der var sådan lidt.. den sad lidt løst i det. Der var ikke nogen egentlig regel for, hvordan man skulle gøre tingene og så videre. Det kom sådan lidt løst hen ad vejen, (…) hvor at her, der tager det måske en dag, så har jeg lært den ligning der. Fordi det ligesom bliver forklaret på en hel anden måde. En lettere måde.

DY4: Well, in lower secondary school, then they started out by giving some kind of explanation which was a little... it was kind of loose. There was not really any rule for how to do things and so on. It came along loosely little by little, whereas here, then it maybe takes a day, and then I learned this equation. Because it was explained in a completely different way. An easier way.

### *1.B. MATHEMATICS FOR ALL*[261]

Q5: Could you elaborate why everybody should learn mathematics?

DY5: Because... because maybe you choose a language oriented education; then everybody should have the chance to bring up mathematics, and they should also, one can say that they may not need it in their everyday life, but if they are in a situation, in which they needed some equations or formulae and so on, then they would need mathematics, if they did not know it already.

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Q5: (…) Kan du uddybe hvorfor alle skal lære matematik?

DY5: Fordi at… fordi du måske vælger en sproglig uddannelse (…) så skal alle have mulighed for at kunne komme ind på matematikken og de skulle også, man kan sige at de ikke kommer til at bruge det i deres hverdag, men hvis de kommer ud for en situation, hvor de skal bruge nogle ligninger eller nogle formler og så videre, så får de jo brug for matematikken, hvis ikke de har kunnet det i forvejen (...)

*1.C. A-LEVEL & FUTURE PLANS*[262]

Q6: And you have chosen A-level mathematics here?

DY6: Yes.

Q7: Is it something you need in your further education?

DY7: Yes, I need it if I continue my studies at university. For the education I want, A-level mathematics is required...

Q8: And it was medicine, was it not?

DY8: Yes.

Q9: Yes, pathology.

DY9: Yes.

Q10: Is it then tough luck that you have to take A-level mathematics, or could you, even if it was not exactly that particular education you had in mind, could you imagine taking A-level anyway?

DY10: I could, actually. But, for example, some people choose an education because they dislike some subjects, but they do not take this study programme [Biotechnology, author] But I take it rather for the sake of education, because then I

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Q6: Og du har så selv valgt matematik på A-niveau her?

DY6: Ja.

Q7: Er det noget, du skal bruge i din videre uddannelse (…)?

DY7: Ja, jeg skal bruge det, hvis jeg skal videre på uni. Den uddannelse jeg gerne vil have, der er det et krav at jeg har det på A, så…

Q8: Var det medicin du skrev, eller hvordan var det?

DY8: Ja.

Q9: Ja, patologuddannelsen.

DY9: Ja.

Q10: Er det så ærgerligt, at du har matematik på A, eller kunne du godt, selvom det ikke lige præcis var den uddannelse du havde forestillet dig, kunne du så forestille dig, at tage matematik A alligevel? (...)

DY10: Det kunne jeg faktisk godt. Men altså, (…) for eksempel, så er der nogen der bare tager en uddannelse fordi at de ikke kan lide den slags fag, jamen så tager de ikke den linje [Bioteknologi, red.]. Hvor at jeg tager det mere for uddannelsens skyld, fordi at så har jeg det. Så, ja jeg kunne godt have taget det på A alligevel, selvom jeg ikke havde taget den uddannelse.

Q11: Ved du hvad det er for noget matematik man skal bruge på den uddannelse du gerne vil have?

DY11: Nej, ikke præcist. Altså, jeg skal jo læse medicin, så det, jeg ved ikke hvor meget matematik der egentlig kommer ind over, men altså, jeg går ud fra at siden de kræver det på et A-niveau, så går jeg ud fra at man kommer til at bruge det til noget.

have that. Yes, I could have taken it on A-level, even if I had not planned to take that education.

Q11: Do you know what kind of mathematics you use in the university study you want?

DY11: No, not exactly. Well, I am going to study medicine, so, I do not know how much mathematics there is actually involved, but I presume, that since they require it on A-level, then I assume it will be applied for something.

***1.D. DEALING WITH CHALLENGES***[263]

Q12: What do you draw on, if you think that now you have got a task to solve and you do not know what to do about it what do you draw on, in a situation like that?

DY12: Well, to begin with, I need to understand the task. When I have done that, then... and if I do not know what to do, then I would at first grab the collection of formulae and then take a look if something reminiscent of the task appears in examples or such like. Then see if I can compare it and use it. If not, then ask for help, if that option is available. Typically, when I have a task, then it is something about combining several formulae, for kind of solving it. But otherwise, find some intermediate calculations to indicate to the teacher what I am attempting.

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Q12: Hvad trækker du på, hvis du synes at, nu har jeg fået en opgave her, som jeg skal løse, og jeg ved faktisk ikke, hvad jeg skal stille op med den. Hvad griber du og gør i, i sådan en situation?

DY12: Jamen altså, til at starte med, så skal jeg jo forstå opgaven, hvad det går ud på. Når jeg har gjort det, så er det jo, og jeg ikke kan finde ud af det så vil jeg jo først gribe til formelsamlingen og så se om der er noget der minder om opgaven i eksempler og lignende. Så prøve at se om jeg kan sammenligne det og bruge det. Hvis ikke, så spørge om hjælp, hvis jeg har mulighed for det. (...) Typisk når jeg har en eller anden opgave, så bliver det noget med at sammensætte flere formler, for ligesom at løse det. Men altså, ellers (utydeligt), så finde nogle mellemregninger, så læreren måske har en anelse om, (...) hvad jeg prøver på.

*1.E. MORE UNDERSTANDING*[264]

Q13: You have actually improved in solving equations after you got a solid explanation of what it is about?

DY13: Yes.

Q14: Is there anything else you think enables you to improve in mathematics? Both in terms of yourself and the teacher?

DY14: I would say that to some extent it is your own mathematical knowledge, and (…) I come from a family in which everything is measured by means of a vernier caliper and micrometer and such. (…) It is said, that it changes people to come to upper secondary school from lower secondary school. That everything becomes more serious and that you reach a new level, even if your grades tell the opposite. But, I would say, just going to upper secondary school improves everything.

Q15: (…) Do all of you make a greater effort, compared to earlier, or...?

DY15: (…) probably, there would be a difference from when they started (…) but for me, it has been rather significant.

Q16: In which way?

DY16: That you … it brightens, well, you understand things faster. I do not know (…) if (…) you have become more

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Q13: (…) du er faktisk blevet bedre til at løse ligninger efter du har fået en… en ordentlig forklaring på, hvad det handler om.

DY13: Ja.

Q14: Er der andet, du synes, der gør, at du bliver bedre til matematik (...) både med hvad du selv kan gøre og hvad læreren kan gøre, for at du bliver bedre til at lære matematik? Endnu bedre, selvfølgelig.

DY14: (…) jeg vil sige, at det jo nok skyldes en lille bitte smule ens egen matematiske viden, og (…) nu kommer jeg jo selv fra en familie hvor at det er meget teknisk det hele, hvor at det hele bliver målt med skydelære og mikrometer og så videre. (...) Det siges jo også, at det ændrer folk at komme på gymnasiet fra folkeskolen. At alting bliver mere seriøst og man kommer op på et andet niveau, selvom ens karakter måske siger det modsatte. Men altså, jeg vil sige i det hele, bare det, at man kommer på gymnasiet, det trækker det hele op.

Q15: (…) gør I mere ud af det, alle sammen, end I gjorde før, eller? (...)

DY15: (…) der vil nok være sket en forskel fra de begyndte (…) men altså, for mig selv der har den været rimelig stor.

Q16: På hvilken måde?

DY16: At du ... det lysner, altså det, du forstår hurtigere tingene. Jeg ved ikke, (…) om (...) man er blevet mere moden, eller hvad det er, men af en eller anden grund, så forstår man tingene hurtigere og man er hurtigere til at løse tingene. Hvor man før i tiden sad og terpede de her opgaver for at komme igennem dem, og alligevel så ikke forstå dem ret godt.

mature, or what it is, but for some reason, you understand things faster, and you are faster in solving things. Whereas earlier you would cram these tasks to get through them, and yet not understand them very well.

#### *1.F. THE TEXTBOOK*[265]

Q17: What about the textbook? Is it, is it easy to understand and use when you work? If you compare with lower secondary school, for example?

DY17: Well, I must say that it is much easier to find formulae for solving tasks. It is the way the tasks are organised in the book I have now, it is organised in a way that is more understandable, but it also allows for several different kinds of calculations and computations of the tasks. And, in the old book, then you had to search for formulae, or get an extra book with collections of formulae only. But there are also things in this book for which you need a collection of formulae, and then it becomes by means of notes from the blackboard that you solve the task.

#### *1.G. TECHNOLOGIES FOR LEARNING*[266]

Q18: What about computers and calculators, do you use them differently now in upper secondary school, compared to lower secondary school?

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Q17: Hvad med matematikbogen? Er det, er den god at forstå, er den god at bruge, når man skal arbejde? Hvis du skal sammenligne med (…) hvad med i folkeskolen for eksempel?

DY17: Altså, jeg vil sige at, så er det langt nemmere at finde formler frem til at løse opgaver. Det er den måde opgaverne er stillet op på i den bog jeg har nu. Der er det stillet op på en måde som er mere forståelig og, men det giver også mulighed for flere slags beregninger og udregninger af opgaverne. Og, altså den gamle bog, der var det sådan at du skulle lede efter formler i den, ellers fik du givet en ekstra bog kun med formelsamlinger. Men der er også ting i denne her bog, hvor at du har mangel på formelsamlinger og der bliver det jo via noter fra tavlen, at du løser opgaven.

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Q18: Hvad med computere og lommeregnere, bruger I det på en anden måde nu i gymnasiet end I gjorde i folkeskolen?

DY18: Ja. Der vil jeg så sige, at det kommer også lidt an på læreren. Den lærer jeg havde i folkeskolen, han var generelt ikke så meget for computere, så der var ikke så meget. Men altså før i tiden, (...) så blev det hele skrevet på papir og du var tvunget til at kigge i dine egne noter og i dem du har i bogen (...). Hvor her, der har du mulighed for at du kommer ind i det hele, da teknologien den giver dig langt højere mulighed for at lave opgaver, for at stille opgaver op og så videre.

Q19: Hvordan er det, bruger I TI-Nspire (...), eller er det nogle andre programmer I bruger?

DY19: (…) Vi bruger det der er med i Office-pakken, det der Math Type og så et eller andet, og også bare vores lommeregnere. (...)

DY18: Yes. Or I would say; it depends on the teacher. The teacher I had in lower secondary school, he was not too fond of computers, so that was not much. (…) but earlier, everything was written by hand, and you had to look up in your own notes and in those in the book (…) but here, you can access everything, since technology, to a much further extent, enables you to do tasks, to do lay-out on tasks and so on.

Q19: How is it, do you use TI-Nspire, or is it some other software that you use?

DY19: (…) we make use of what is in the office-package, this Math Type and also just our calculators.

***1.H. CHALLENGES***[267]

Q20: About mathematics, do you feel that it is actually something that you think: "*I can actually manage, and when challenges come along, then I can actually come up with means for dealing with them*." Would that be how you feel?

DY20: Well, more or less (…) It depends to some extent on what kind of task it is. For some tasks, it is more or less just to plunge into them and then start, whereas other tasks, they are more like: "*What is this actually*?" So...

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Q20: Har du det sådan at matematik, det er sådan set noget du synes, det kan jeg faktisk godt finde ud af og når der kommer nogle udfordringer så kan jeg sådan set, jeg kan godt finde på noget at gøre (...). Er det sådan, du har det? Eller…

DY20: Ja nogenlunde (…) Det kommer lidt an på hvad det er for nogle opgaver. Altså nogen opgaver, der er det jo bare at kaste sig ud i det og så begynde, hvor der er andre opgaver, der er sådan lidt… hvad er det egentlig for noget. Så…

*1.I. PLANS AND POSITION*[268]

Q21: Is there any specific reason why you want to become a medical doctor?

DY21: Well, firstly, it is a high education. You place yourself highly in society with an education in medicine. And something else, which is also attractive, is that gives a good income.

Q22: Yes, but there might be other sort of education programmes that enable you to get a high education, a good position in society and a good income?

DY22: Yes. Well, as it is said, you work with people, you work with living things (…) but really, dealing with stocks or being a construction entrepeneur, that does not really have my interest. I cannot really see anything in it. And also, those sectors suffer a lot from unemployment right now. So I cannot really see they will need new people there.

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Q21: (…) Er der en bestemt grund til, at du gerne vil være læge?

DY21: Altså, for det første så er der jo, ligger der jo det i det at det er en høj uddannelse. Man ligger forholdsvist højt i samfundet, når man står med sådan en uddannelse inden for medicin. Og så, noget andet der trækker, det er også lidt, at det giver gode penge (...)

Q22: Ja. Altså man kan sige, det kan jo være at der er nogle andre uddannelser, hvor du også tænker, nå her kan jeg også godt få en høj uddannelse, hvor at jeg får en god position i samfundet og hvor jeg får en løn, som jeg synes er god.

DY22:Ja… Altså du, som det siges, du arbejder med mennesker, du arbejder med levende ting. (...) Men altså, det med at sidde og handle med aktier, eller en eller anden anden ting, som entreprenør eller noget, et eller andet andet, det står ikke rigtigt i min interesse for det. Jeg kan ikke rigtigt se det helt store i det. Og så der jo det med, at (i?, red.) den sektor der er der jo (…) også mange arbejdsløse lige nu, så jeg kan ikke rigtigt se at man vil komme til at mangle nogen der.

### *1.J. MATHEMATICS FOR CAREER?*[269]

Q23: Could you imagine studying Mathematics?

DY23: As a study programme?

Q24: As applying for it at university: "*Now I opt for the discipline of Mathematics*"?

DY24: No.

Q25: Try to say some more about why not?

DY25: I think it would become too dry, too boring. It has never really caught my interest to study mathematics. I tried it through out lower secondary school. Through out upper secondary school, and then again at university, I think it would be too much to deal with mathematics again. Because it is like Danish. I am not really interested in it, because you know the language already, you speak it every day, you have learned it over many years, it is not something … I cannot see anything challenging or exciting about it.

### *1.K. PRESTIGE*[270]

Q26: Did it matter, when you chose this Biotechnology study programme, that it is about Biology; do you think that it is particularly fun or are the other aspects more important,

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Q23: Kunne du forestille dig at vælge at læse matematik?

DY23: Som linje eller?

Q24: Ja, som, nu søger jeg ind på universitetet, nu vælger jeg faget matematik.

DY24: Nej.

Q25: Nej. Prøv at fortæl bedre om hvordan det kan være.

DY25: Jeg tror, det ville blive for tørt, altså for kedeligt. Det falder ikke rigtigt i min interesse at stå med, at løse matematik. Altså, jeg har prøvet det før (…) gennem folkeskolen. Gennem gymnasiet også nu og så stå igen på universitetet, jeg tror det ville blive for meget at stå med matematik igen. Fordi det er lidt ligesom dansk. Det interesserer jeg mig som sådan heller ikke for, fordi du kan sproget i forvejen, du taler det til dagligt, du har lært det i flere år, der er ikke noget... jeg kan ikke rigtigt se nogen udfordringer eller noget spændende ved det.

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Q26: Betyder det noget, (…) når du har valgt en bioteknologi-linje her, altså, er det inden for biologien, du synes, at det især er sjovt eller, eller er det, vejer de andre ting højere med, okay jeg vil godt have en høj uddannelse med en ordentlig løn, hvor man bliver respekteret i samfundet?

DY26: Det ved jeg sådan set ikke rigtigt. Altså, det er sådan lidt… Jeg tror det ligger det der med, at der er en lidt prestige i samfundet over at man er læge i forhold til hvis man er matematiker eller lignende. Der er jo en vis forskel. Også i det hvor at, matematiker, jamen det er der jo i høj grad mange af, så jeg ser det ikke rigtigt som noget særligt.

about getting a high education, a good income and being respected in society?

DY26: I do not know, really. It is kind of like...I think it relates to the prestige in society when you are a doctor compared to if you are a mathematician or something. There is a certain difference. And also since, mathematicians, well there are so many of them, so I do not perceive it as something special.

*1.L. CHOICE OF UPPER SECONDARY EDUCATION*[271]

Q27: But (..) now you chose Biotechnology. How come you chose this study programme?

DY27: It was actually due to prospects for further education. I could have chosen mathematics-physics, which was also a possibility, if I were to take a machine-related education, but anyway, I chose this study programme, biotechnology due to further education. So, I choose to look at; what about the rest of my life? What about your education there? Then I would rather take one from which I can move on, where I have something to move on from.

271

Q27: Men (…) nu har du valgt den med bioteknologi. Hvad, hvordan kan det være, du valgte lige den linje?

DY27: Det er faktisk på grund af (...) videre uddannelse. Altså jeg kunne måske have valgt matematik-fysik, som også var et bud på det, hvis jeg skulle have en eller anden maskinel uddannelse i stedet, men altså, jeg tager den linje, bioteknologi på grund af videre uddannelse. Så… (...) jeg vælger så at se på (…) hvad med resten af livet. Hvad med din uddannelse der. Så vil jeg hellere tage en hvor jeg ligesom kan komme videre, hvor at jeg har noget at gå fra.

## THEMES IN DYLAN'S 1ST YEAR

*Dylan's questionnaire answers and the interview transcripts form the basis for my analysis of Dylan's account of his experiences with mathematics in the 1st year of upper secondary school. Before presenting my analysis, I will provide a summary of Dylan's 1st year questionnaire and interview. The analysis is organised in themes concerning issues or situations. # refers to the enumeration of the questionnaire questions.*

### MATHEMATICS AT SCHOOL

Dylan does not see any hindrances for learning mathematics, but perceives it as a matter of volition (DYLAN: Q1-D). Dealing with mathematical challenges at school involves finding and combining formulae (DYLAN: 1.D.). The textbook in upper secondary school is valued for exposing different ways of computing and calculating and for its usefulness for providing formulae for solving tasks (DYLAN: 1.F.). In this sense, formulae and their application seems to dominate Dylen's beiefs of *Mathematics at School*.

Dylan's way of interpreting what Mathematics at School, has many references to tasks and the application of formulae. These feature could be characterised as belonging to what Mellin-Olsen would call a "task discourse" (Mellin-Olsen, 1990), for example his computational beliefs about *Mathematics at School* (Dylan: 1.F.). Lithner suggest different categories of imitative reasoning to refer to incidents of "grab a formulae and try it out"-related strategies (DYLAN: 1.D.) for reasoning (Lithner, 2008).

### TRANSITION

Dylan is learning better now, and he likes the way the work is organised, the tasks and the way of learning. Among other things, it seems easier to Dylan to

understand equations now, and they even seem to cost him less effort (DYLAN: 1.A.) and in general he experiences that he understands new things faster, compared to earlier, when he used to cram for the exercises in lower secondary school, without ever understanding them too well (DYLAN: 1.E.). "The teacher" is mentioned both in terms of something Dylan liked better before, and something he likes better now (DYLAN: Q1-A #4b, #4c). What he likes better relates to the way the teacher organises the teaching, but he may have had a better personal relation to his teacher in an earlier setting.

### MATHEMATICS AS A DISCIPLINE

Dylan' believes that Mathematics is discovered, and mathematicians conduct research on new formulae (#10). To study mathematics in university, one has to be a genius (DYLAN: Q1-C #11) and Dylan is actually one of the few students who suggests this. Nevertheless, Dylan compares studying Mathematics to the studying the subject Danish, which is somehow just a familiar tool, rather than exciting in its own right (DYLAN: 1.J.). In this way Dylan indicates to see the discipline of mathematics as a box of tools; useful for other subjects, but not interesting on its own terms.

### MATHEMATICS IN SOCIETY

As Dylan sees it, Mathematics is everywhere and everybody should have the possibility of taking an education with mathematics (DYLAN: Q1-B). Also, one might need mathematics later in life, either in everyday life situations or in change of career situations (DYLAN: 1.B.)

### MATHEMATICS, PROFESSIONS & PLANS

Mathematicians are much less respected in society compared to doctors, Dylan says, and he also finds that there are so many of them (mathematicians). So he does

not perceive it to be anything special (DYLAN: 1.K.).

Dylan's plans for tertiary education - studying Medicine – are influenced by his interest in getting a good position in society; being well-respected and getting an attractive salary (DYLAN: 1.G.). In contrast to this, Dylan associates the mathematics-physics study programme in upper secondary school with machine-oriented tertiary education and professions, which does not have his interest. Instead, he preferred the biotechnology programme; not because of the biology in it, in itself, but rather because of its usefulness for admittance to medical school (DYLAN: 1.L.). Also, he finds it more exciting than working in finance or business, which is also plagued by unemployment due to the current crisis (DYLAN: 1.I.).

### CHALLENGES & SUPPORT

None of the suggested types of mathematical challenges appears to challenge Dylan too much (DYLAN: Q1-E). He finds support for mathematical activities in his family – he describes both his parents as having an upper secondary school leaving certificate, and both parents and siblings are mentioned for possible support (DYLAN: Q1-E).

### PLANS

As current plans for tertiary education, Dylan mentions pathology (#20), at Copenhagen University (DYLAN: Q1-G). However, it is not clear if he knows what it means; "*You work with people*", he says, whereas pathologists actually deal with dead bodies. Dylan chose a study programme with A-level mathematics due to his plans for his future. Nevertheless, mathematics may be used somehow in studying medicine, he supposes, since it is a mandatory prerequisite (DYLAN: 1.C.).

## DYLAN'S 1ST YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics in technical upper secondary school seem both on a higher level, but also easier to learn due to the teaching. Dylan often focuses on the solution of tasks by the application of formulae when talking about *Mathematics at School*.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is discovered and one needs to be a genius to study mathematics in university. Mathematics is a discipline of formula-based computations.

### *MATHEMATICS IN SOCIETY*

Everybody should learn mathematics, since everybody could end in a situation demanding mathematical skills. There is an abundance of mathematicians in society, but they do not enjoy any particular esteem.

### *MATHEMATICS & ME*

Dylan is quite focused on his future plans in his choice of his A-level mathematics study programme, and his future plans orient themselves towards a high position in society. Since Mathematics is a necessary prerequisite for studying medicine it is relevant to Dylan to engage in studying it at school.

# DYLAN'S 3RD YEAR QUESTIONNAIRE

| Q3-A | TRANSITION | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [6] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [272] | [Working in pairs]; [Project Work] |
| **3** | Did you like mathematics when you went to lower secondary school? [273] | [Yes, it was fine] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | More challenging[274] |
| **New 4d** | In which ways has mathematics changed from first year to third year of upper secondary school? | Has become considerably harder[275] |

*Table 27: DYLAN'S 3rd year questionnaire, part A – TRANSITION*

272 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

273 Options: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

274 Mere udfordrende

275 Blevet væsentlig sværere

| Q3-B | FOR SCHOOL | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | It is a matter of course in order to manage oneself[276] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | Physics, biotechnology – calculations, curves, functions.[277] |

*Table 28: DYLAN'S 3rd year questionnaire, part B - MATHEMATICS AND SCHOOL*

276 Det er en selvfølge for at kunne begå sig

277 Fysik bioteknologi – beregninger kurver funktioner

| Q3-C | BEYOND SCHOOL | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | Architecture, statistics, a lot about business.[278] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? [279] | [Mathematics is invented] |
| **10** | What do you think a professional mathematician at a university is doing? | Immerses in the function of formulae, works on projects[280] |
| **11** | Would you have to be a genius in order to study mathematics in university? [281] | [No] |

*Table 29: DYLAN'S 3rd year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

278 Arkitektur statistik meget med erhverv

279 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]

280 Fordyber sig i formlers funktioner arbejder på projekter

281 [Yes]; [No]; [I do not know]

| Q3-D | IMRPOVING | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | The pace[282] |
| **12b** | What do you think is the best means for improving in mathematics? | Theory → practice – in tasks[283] |
| **12c** | What do you do to improve in mathematics? | Read up on things and solve tasks[284] |
| **13a** | What do you do if you get stuck on a task at school? | Read theory – move on to the next task.[285] |
| **13b** | What do you do if you get stuck on your homework? | I skip them or read up on theory[286] |
| **13c** | What do you do if you get stuck on your written assignments? | Same as previous |

*Table 30: DYLAN'S 3rd year Questionnaire, part D – STRATEGIES FOR IMPROVING*

282 Hastigheden
283 Teori - > praksis – i opgaver
284 Læser op på ting og løser opgaver
285 Læser teori – går videre til næste opgave
286 Springer dem over eller læser op på teori

| Q3-E | CHALLENGES & SUPPORT | | DYLAN |
|---|---|---|---|
| # | *Question* | | *Answer* [287] |
| **14** | What issues involve more challenges to you? | a) Remembering | [3] Moderate challenges |
| | | b) Computing | [2] Several challenges |
| | | c) Figuring out the purpose of a task | [3] Moderate challenges |
| | | d) Finding a way to solve a task | [3] Moderate challenges |
| | | e) Reading and understanding the textbook | [3] Moderate challenges |
| **15** | Where can you find support for mathematical activities? [288] | | [From classmates] |
| **New 16a** | Do you find that you meet some special challenges related to succeeding in mathematics compared to your classmates? | | Yes, pace[289] |
| **New 16b** | Do you find that you have some some special strengths related to succeeding in mathematics compared to your classmates? | | Logic[290] |

*Table 31: DYLAN'S 3rd year Questionnaire, part E – CHALLENGE & SUPPORT*

287 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

288 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From classmates]; [Other places] If other places, from where or from whom?

289 Ja – hastighed

290 Logik

| Q3-F | IN CLASS | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson?[291] | [1-3] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [No] |
| **17c** | Are you content with that? | No |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? [292] | [1-3] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | No |
| **19a** | In your class, is it okay to be good at mathematics? | "Yes."(sic.) |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Yes and no[293] |

*Table 32: DYLAN'S 3rd year Questionnaire, part F - MATHEMATICS IN CLASS*

291 Options: [0]; [1-3]; [4-8]; [More than 8 times]

292 Options: [0]; [1-3]; [4-8]; [More than 8 times]

293 Både og

| Q3-XA | UNDERSTANDING | DYLAN |
|---|---|---|
| # | *Question* | *Answer*[294] |
| **X.1.** | Have you recently experienced to *understand* what you worked with in mathematics? | [Yes] |
| | If yes, on which occasion? | Project[295] |
| **X.3.** | Have you, during upper secondary school experienced that you *understood* what you worked with and then s*ubsequently learned it by heart*? | [Yes] |
| | If yes, on which occasion? | Project |
| **X.5.** | Have you during upper secondary school experienced *understanding something* but *never learning it by heart?* | [Yes] |
| | If yes, on which occasion? | Certain formulae[296] |

| Q3-XB | LEARNING BY HEART | DYLAN |
|---|---|---|
| # | *Question* | *Answer*[297] |
| **X.2.** | Have you recently experiences having to *learn something by heart*? | [Yes] |
| | If yes, on which occasion? | Project |
| **X.4.** | Have you, during upper secondary school experienced that you had to *learn something by heart*, and then, *subsequently understood it*? | [Yes] |
| | If yes, on which occasion? | Tasks in general[298] |
| **X.6.** | Have you during upper secondary school experienced *learning something by heart without ever understanding it*? | [Yes] |
| | If yes, on which occasion? | Tasks in general |
| **X.7.** | Additional comments on understanding or rote learning in mathematics | - |

294 Options: [Yes], [No] or [I do not know]
295 Projektopgave
296 Visse formler
297 Options: [Yes], [No] or [I do not know]
298 Alm opgaver

| Q3-XC | A-LEVEL EXAMINATION | DYLAN |
|---|---|---|
| **X.8.** | *Topic* | *Answer*[299] |
| **(a)** | Parabola | [Okay] |
| **(b)** | Exponential | [Rather not] |
| **(c)** | Pythagoras | [Readily] |
| **(d)** | Sine and cosine relations | [Readily] |
| **(e)** | Definition of differentiability | [I do not know] |
| **(f)** | Sum and product of differential functions | [I do not know] |
| **(g)** | Indefinite integral | [Rather not] |
| **(h)** | Volume of solid of revolution | [Okay] |
| **(i)** | Differential Equations and their solutions | [I do not know] |
| **(j)** | Vectors in the plane, including scalar product | [Okay] |
| **(k)** | Lines and planes | [Rather not] |
| **X.9.a.** | Which topic is your favourite? - and why? | Cosine, sine and tangent. I understand it 100%[300] |
| **X.9.b.** | Which topic would you rather avoid? - and why? | Integrals/differentials, I do not understand them[301] |

*Table 33: DYLAN: Topics for oral and written examinations in A-level Mathematics for Upper Secondary School Programmes in Denmark 2013*[302]

299 Options: [Readily], [Okay], [Rather not], [I do not know]

300 Cos sin tan jeg forstår det 100%

301 Integral/differential forstår det ikke

302 Topics common to both the Technical (htx) and the General (stx) upper secondary programme examinations are suggested.

| Q3-G | PLANS | DYLAN |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Laboratory technician education or pathology[303] |
| **20b** | What kind of influence would your final marks in mathematics have on your plans (or hope for plans) after upper secondary school? | Great importance – it influences my decision [304] |
| **20c** | What kind of influence would your experiences with mathematics in upper secondary school have on your plans after upper secondary school? | Interest, function, work, education[305] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [I do not know] |
| **21b** | Comments: | Laboratory technician or medicine[306] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [I do not know] |
| **22b** | Comments | Physicist/mathematics[307]<br>[in terms of avoiding, author.]<br>*Other comments:*<br>Not as enthusiastic as previously (1st year – 2nd year) (sic.)[308] |

*Table 34: DYLAN's 3rd year Questionnaire, part G – PLANS*

303 Laborantuddannelse eller patologi
304 Stor betydning – det er med til at vælge
305 Interesse funktion arbejde uddannelse
306 Laborant / medicin
307 Fysiker / matematik
308 Ikke så begejstret som førhen – (1g 2g) (sic.)

# DYLAN'S 3RD YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 13 March 2013 | 45:30 |

### *3.A. VARIATIONS ON FAVOURITE SUBJECT SCALE*[309]

Q1: How do you feel about Mathematics right now?

DY1: Now, today, I am fine, but lately (…) February – March I think it has been quite (…) I am being challenged more now, than I was during the last two years.

Q2: Has it something to do with the topics, you deal with? Or is it rather something related to the amount of time for understanding each thing?

DY2: It is kind of a combination, because it is both due to our use of this "MathCalc" software, where we have to learn how to enter things, and that takes a certain amount of

309

Q1: Hvordan har du det med matematik lige nu?

DY1: … Lige i dag har jeg det faktisk meget godt med det, men inde for den seneste tid, (…) februar-marts, der synes jeg det har været ret så, […] Jeg bliver udfordret mere nu end jeg har været gennem de sidste to år.

Q2: Har det noget med den type emner I har at gøre? Eller har det noget at gøre med hvor lang tid der er til at forstå hver ting?

DY2: Det er lidt en kombination af det, fordi at det har både været med at vi så skulle bruge det hér MatCalc program, hvor vi så skal lære at sætte tingene ind, og det tager jo en vis tid, hvis man ikke er sat ordentligt ind i det fra starten. Og så er det også de emner vi så trækker ind over det, som vi så skal sætte ind i det. Der bliver det også sådan med, at man skal ligesom også selv få sin hjerne med til at kunne få det til at køre.

Q3: Er det rumgeometri I har haft her januar-februar, eller hvad har der ligget dér for nogle emner?

DY3: Der er noget analytisk plangeometri, det er… Ja. Eller… Vektorer i rummet, også.

Q4: (…) Det her spørgsmål med, på en skala fra 1 til 10, hvor 10 er dit yndlingsfag, hvor ligger matematik? Der sagde du: "Nu ligger det på 6", det vil sige en lille smule over middel. Fik den et ekstra dyk her efter jul, eller har den ligget dér?

DY4: Både og, det svinger jo lidt hvor man kommer til nogle dele hvor at man godt kunne tænke, at det går ned til 4 eller sådan noget, men så er der nogle andre dele, hvor man ligesom har sine bedre perioder og sine bedre ... tingene til at klare sig i det, så omkring dér, ja, omkring middel, vil jeg sige.

time, if you are not into it already. And then it is also those topics we combine with it. So, it is something about getting your brain ready for dealing with it.

Q3: Is it geometry in Euclidean space you have dealt with this January-February, or what are the topics?

DY3: There was some analytic plane geometry … and vectors in Euclidean space as well.

Q4: This question; "*On a scale from 1 to 10, on which 10 is your favourite subject, where would you place mathematics*?" You answered [6], which is a little above the middle score. Did it get a sudden dip, here after Christmas, or has it stayed on this level?

DY4: Yes and no, it varies as you get to some parts where it goes down to 4 or something, but other parts when you have a better period and better … things to handle it, so about that, yes, around middle, I would say.

***3.B. LESS APPLICABILITY***[310]

Q5: Can you still apply what you learn in mathematics to your other subjects?

DY5: I think it has diverged, mathematics in relation to other subjects, and those things I have learned lately, then I think it has got dispersed a lot, and Mathematics is at one end and the other subjects completely at the other end, so it is hard to connect the subjects.

Q6: So, those parts of Mathematics that relate to the other subjects are not part of the topics you work with now?

DY6: No, not right now. We dealt with them earlier and it is actually fine that you then know these things and can integrate them in the subject you work with now, but no – we do not apply something we have just learned recently to the other subjects.

310

Q5: Kan du stadigvæk bruge det du lærer i matematik i de andre fag du har?

DY5: Jeg synes (...) det er gået mere i hver sin retning, matematikken i forhold til de andre fag, og de seneste ting jeg har lært, der synes jeg det har taget en meget stor spredning, og matematikken ligger i den ene side, og så, de andre fag ligger helt ovre i den anden side, og det er svært lige at koble de fag sammen.

Q6: Så de dele af matematik der indgår i de andre fag, det er ikke de emner I arbejder med nu?

DY6: Nej, ikke lige nu. Vi har jo haft dem før, og der er det jo sådan set fint nok at man så kan tingene og kan integrere det i det fag man nu arbejder i, men ikke - Vi bruger ikke noget vi lige har lært, og så kører ind over de andre fag.

### *3.c. Nature of Mathematics*[311]

Q7: Concerning the discipline of Mathematics, I ask: "*Is mathematics discovered or invented*?" Then you ticked off "*invented*".

DY7: Yes. It is kind of whether you should define it in this or the other way... It is a hard one. Because … yes. It is kind of complex, I think.

Q8: Does mathematics exist already, and then we discover it? Or is it something we human beings invent?

DY8: It is created by human beings. Like the wheel. And if one person had not done it, I think someone else would.

### *3.d. Mathematics as a Science*[312]

Q9: What is a mathematician in a university doing? Then we have here: "*Immerses oneself into the function of formulae and works on projects*". Is there anything new to invent in mathematics?

DY9: Definitely (…) for example the many long formulae we have today, they can probably be reduced, likewise they can be extended and include more factors and so on. And you could probably put more different things into Mathematics.

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Q7: (…) Fagområdet matematik, jeg spørger: "Er matematik opdaget eller opfundet"? Så skriver du, du har sat kryds ved "opfundet".

DY7: Ja. (…) Det er jo sådan lidt, hvorvidt man skal definere det som det ene eller det andet. (...) Den er lidt svær. Fordi at… Ja. Det er lidt komplekst, synes jeg.

Q8: Findes matematik (…) allerede, og så opdager vi den? Eller er det noget vi mennesker opfinder?

DY8: Den er menneskeskabt. Lige så vel som hjulet. Og hvis det ene menneske ikke havde gjort det, så tror jeg nok at der havde været et andet.

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Q9: Og hvad laver en matematiker på et universitet? Så er vi her: "Fordyber sig i formlers funktion og arbejder på projekter". Er der mere nyt at opfinde indenfor matematik?

DY9: Det er der helt sikkert (...) for eksempel de mange lange formler vi har i dag, de kan sikkert også forkortes, ligeså vel som de kan forlænges og indeholde flere faktorer og så videre. Og man vil sikkert kunne sætte flere forskellige ting ind i matematikken.

### *3.E. LEARNING MATHEMATICS*[313]

Q10: This thing about improving in mathematics, you answer: "*What is the greatest hindrance for improving in mathematics*?": "*The pace*".

DY10: Yes. It is my own pace. It has been like this, that for certain things, I can easily apply them, but I do not necessarily understand them too well. So if it requires going the opposite way, that you understand it before you apply it, then it may take me some more time; and if you are supposed to be acquainted with things and end up being able to say, "*I know these things*".

### *3.F. IMPROVING*[314]

Q11: I also ask what are the best means for improving in mathematics, and then you write: "*Theory*" and "*practical tasks*", and then there is an arrow between them. Is that to say something about the sequencing, that eh..., theory first, and then practice?

DY11: It should be understood in the sense that, instead of getting theory now, and then do some tasks, then I find that it is preferable to try out the theory on some tasks, to kind of find out if the theory you just learned, if it can be applied in these tasks. And like, if the theory which should be applied in these tasks, if you understood it. Yes. Instead of getting the theory in an uninterrupted sequence and then deal with the tasks in another one.

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Q10: Det dér med at lære matematikken, der siger du, (til, red.) "Hvad er den største hindring for at kunne blive bedre til matematik?", der siger du: "Hastigheden".

DY10: Ja. Det er jo min egen hastighed. Det har været sådan at til visse ting, der kan jeg sagtens anvende det, men jeg forstår dem nødvendigvis ikke så godt. Så hvis det så kræver, at det går den anden vej, at man forstår det før man kan bruge det, så er det, det godt for mig kan tage lidt længere tid, og hvis man skal være inde i tingene, og ligesom skal frem til at kunne sige, at "det her, det kan jeg godt".

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Q11: Ja. Jeg spørger også til, hvad er det bedste middel til at blive bedre til matematik, og der siger du: "Teori og praktiske opgaver". Og så er der en pil imellem dem, er det for at sige noget om rækkefølgen, at øhm, først noget teori, og så nogle praktiske opgaver?

DY11: Den måde det skal forstås på, det er mere, at i stedet for at du har teori nu, og så bagefter bliver sat ud i nogle opgaver, så mener jeg, at det er godt at kunne afprøve den nye teori på nogle opgaver, for ligesom at finde ud af, jamen den teori du har fundet, kan du bruge den i de her opgaver? Og ligesom om at, teorien der skal bruges til de her opgaver, om du har forstået den. Ja. I stedet for at du får teorien i én køre og så opgaverne i en anden.

### *3.G. STRENGTHS*[315]

Q13: I also ask if you find that you have some special strengths in terms of doing well in mathematics compared to others. Then you write: "*Logic*".

DY13: Yes, it relates to if you see the logic in it, that of course things should be done in this manner, then you arrive a little faster at understanding them and being able to solve tasks with it. So, things that do not appear logical to me, as well as more complex and long equations, they are a little harder for me.

### *3.H. TEXTBOOK*[316]

Q14: Are you satisfied with your textbook? Is it a good help when you are doing Mathematics?

DY14: Yes and no. Sometimes, if you want to use it as a reference book, then it is not that useful, because it makes references and cross-references that do not make any sense, when, for example, an index in which it is said that it will describe a formula on this or that page, whereupon it refers, on that page, to something which is not really there. And then there are also some other items for which you might say that it, well, is fine in explaining things, and it is thorough, and you get some ready made formulae you can put values into and use. But the way it is organised and its lay-out is rather confusing.

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Q13: Jeg spørger også til, om du synes du har nogle særlige styrker i forhold til at klare dig godt i matematik sammenlignet med andre. Der skriver du: "Logik".

DY13: Øh, ja. Og det er det med, at hvis man ligesom kan se logikken i det her, at selvfølgelig skal det gøres på dén måde, så kommer man jo lidt hurtigere til en forståelse, og til at kunne løse opgaver med det. Så, ting der ikke ligger mig sådan, lige logiske for, såvel som mere komplekse og lange ligninger, det er lidt sværere for mig.

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Q14: Er du tilfreds med jeres matematikbog? Er den en god hjælp når man skal lave matematik?

DY14: (…) både og (…) nogle gange, hvor man gerne vil bruge den som opslagsværk, så er den ikke så brugbar at bruge, fordi at den lave nogle henvisninger og krydshenvisninger, som ikke rigtig giver mening. Hvor at, for eksempel et stikordsregister, hvor at der står, at den vil beskrive en formel på dén og dén side, hvorefter den så henviser, på dén side, så henvises til noget andet, som egentlig ikke er der. Og så er der også et par andre punkter, hvor man kan sige at, jamen, der er den god til at forklare ting, og den går i dybden med det, og man <u>får nogle færdige formler</u> man kan sætte ind og bruge. Men selve ordenen og den måde det er sat op på, det er lidt forvirrende.

### *3.I. COOPERATION*[317]

Q15: How much does one cooperate in your class, now when you are at the end of the 3rd year? Is everybody minding their own business? Or is there still time to talk to one another?

DY15: Well, we are all under a pressure of time, and this and that in our spare time, so it is not always possible to talk to each other about things. But I think there are certain things that still demand that you cooperate and get more of the project work done in groups.

### *3.J. MATHEMATICS AND PLANS FOR FUTURE*[318]

Q16: The Mathematics, you learn, is it something that will become useful to you later on?

DY16: As it looks right now, my tertiary education only requires C-level Mathematics. And this relates mainly to the fact that my average of grades in general does not permit admission to a further education programmes which demand it on B-level or A-level. But I do hope that the mathematics I have learned will prove useful to me in my tertiary education, so it is not just a wasted effort. Yes.

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Q15: (…) Hvor meget arbejder man sammen, her, nu hvor I er i slutningen af 3.g? Kører alle deres eget ræs? Eller har man stadig tid til at snakke med hinanden?

DY15: Altså, vi er jo alle sammen under tidspres, om at, noget af det ene og det andet i vores fritid og så videre, så det er ikke altid muligt at man kan snakke sammen om tingene. Men jeg synes der er visse ting der stadig kræver at man arbejder sammen om tingene og får mere af projektundervisningen i gruppevis.

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Q16: Det matematik du lærer, er det noget du skal bruge senere? Efter gymnasiet?

DY16: Som det ser ud nu, så kræver min videregående uddannelse, det kræver kun matematik på C-niveau. Og det handler jo sådan set mest om at mit snit ikke giver adgang til en højere uddannelse hvor jeg kunne få brug for det på B- eller A-niveau. Men jeg håber da i hvert fald på at kunne bruge det matematik jeg har, at kunne anvende det i videregående uddannelser, sådan så det ikke bare er spildt. Ja.

### *3.K. NEW SIDES OF MATHEMATICS?*[319]

Q17: Have you become acquainted with new facets of Mathematics in the technical upper secondary school programme?

DY17: I would say that we have got a good deal of new knowledge, and learned a lot about Mathematics, and it is actually just an extension to the Mathematics we learned in lower secondary school, so...

Q18: So you do not think you have got to know new facets of Mathematics?

DY18: Well, I think it is more natural now. I do not think there is that many more facets to it, but... It is still just Mathematics, even though one really can immerse oneself into it.

### *3.L. APPLICABILITY IN SOCIETY*[320]

Q19: Well, what you learn here, is it something you can apply in society? Do there exist people in society that have jobs in which that Mathematics you learn is applied?

DY19: ...Yes. Engineers, as a good example. They calculate this and that, on strength, carrying capacity and... yes, actually, just think of a bridge that is to be built. One might think that it does not relate much to Mathematics, but that is the foundation for calculating certain things.

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Q17: Har du lært nogle nye sider af faget at kende her på HTX, som du ikke kendte til før?

DY17: Jeg vil da sige, at jeg har da fået en hel del ny viden og har lært en hel masse omkring matematikken, og det er jo sådan set bare en viderebyggelse fra folkeskolens matematik, så…

Q18: Så det er ikke sådan så du synes du har fået nogen nye sider af matematikken (…)?

DY18: Jamen, jeg synes der er mere naturligt. Jeg synes ikke ligesom at der er så mange nye sider i det, men… Det er jo stadig bare matematik, selvom man virkelig kan gå i dybden med det.

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Q19: Altså, det I lærer her, er det noget man kan gå ud og bruge i samfundet? Altså, findes der folk ude i samfundet som har jobs hvor man bruger den matematik I lærer her?

DY19: … Ja. Ingeniører som et godt eksempel. De skal regne på det ene og det andet med, omkring styrke, og ren bæreevne og… Ja, faktisk, tænk på bare for eksempel en bro, at den skal bygges. Man kan måske godt tænke, at det ikke hænger så meget sammen med matematikken, men det er det der ligger til grund for at man kan udregne visse ting.

### *3.M. MATHEMATICS AND PLANS FOR EDUCATION*[321]

Q20: Now I have a peculiar note here … it must relate to: "*What is your experience with Mathematics?*", "*What influence does it have on your plans after upper secondary school?*" You write: "*Interest*". Has it had an impact on your interests in general, or an impact on your interest in Mathematics, your experiences in upper secondary school?

DY20: Obviously, it has meant that I would not chose a tertiary education which required A-level mathematics, because it is not within Mathematics I would unfold myself

Q21: Yes, so what you are saying is that it influences on which kind of job, you would go for, and which kind of education you would opt for? (…) And the prospects you indicate in the questionnaire are: "*Laboratory Technician or Pathology*".

DY21: Yes. For Pathology, I do not have the required average of grades, for studying medicine. So right now, I steer towards applying for the education programme as a laboratory technician, and to take a bachelor's degree.

Q22: ...yes, and you write that you would prefer to avoid being a Physicist or study Mathematics for the reason that you mentioned.

DY22: Yes.

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Q20: Nu har jeg en rigtig sjov note her (...) jeg tror det må være til "hvad er din oplevelse med matematik?" Hvilken betydning det har for dine planer efter gymnasiet. Du skriver: "Interesse". Har det betydet noget for hvad du interesserer dig for, eller har det betydet noget for din interesse for matematik? Dine oplevelser her i gymnasiet?

DY20: … Det har selvfølgelig betydet, så, at jeg ikke ville tage en videregående uddannelse, hvor jeg ville have Matematik A som et krav for at kunne arbejde videre på den, fordi at det er ikke indenfor matematikken at jeg vil gøre mig i. Ja.

Q21: Ja. Så du siger, det har også betydning for hvilket arbejde du går efter og hvilken uddannelse du går efter? (…) Og de ideer du har på banen her i spørgeskemaet: "Laborantuddannelsen eller patologi".

DY21: Ja. Patologi det har jeg så ikke snittet til, at kunne læse medicin. Så lige nu går jeg efter at søge ind på laborantuddannelsen, og så tage en bachelor i det.

Q22: … Ja, du skriver, du vil gerne undgå at være fysiker eller at læse matematik, af samme grund som du har nævnt.

DY22: Ja.

### *3.N. MORE THEORY – LESS APPLICATION*[322]

Q23: And finally, you write that you are not as enthusiastic now as you were in the first and second year.

DY23: Yes, it relates to, that, yes, now we got these formulae, and then we just apply them in the task and so on, where here, in the third year, then it is something related to getting a lot of theory which fundamentally (sic.!) - as such - does not relate to what you should be able to apply. And, well, we have been assigned some tasks that I think have been, well, somewhat complex, and rather far from what I thought the subject was about.

Q24: So, this side of mathematics, involving theory and being far from applications, that will be left behind, when you are done here?

DY24: Yes, but. Everything has actually – it has been good that you have learned all this, and it has demanded this and that, and it could all just be like playing a game. SO... But anyhow, it is not something I will move on with...yes.

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Q23: Og til sidst, der skriver du, at du er ikke så begejstret som du var i 1.g og 2.g.

DY23: Ja. Det handler om, at man ligesom kunne sige, at i 1.g og 2.g, der var det sådan, at man kunne… Det var nærmest mere en leg. Det handler om, at, jaja, nu har vi fået de her formler, og dem anvender vi bare i opgaven og så videre, hvor, her i 3.g, der er det mere noget med, at man får en hel masse teori som man i grunden - som sådan ikke ligger så tæt på det man egentlig skal kunne anvende. Og, altså, vi har fået stillet nogle opgaver, som jeg synes har været, ja, noget komplekse, og lidt langt fra hvad jeg troede var faget.

Q24: Så den side af matematik, med teori og langt fra anvendelse, den lægger du bag dig, når du er færdig herfra?

DY24: Ja, men altså. Det hele har sådan set - det har været godt, at man har fået lært alt det her, og det har jo krævet det ene og det andet, og det hele kan bare være en leg. SÅ… Men som sagt, så er det jo ikke noget jeg vil studere videre i… Ja.

***3.O. INSTABILITY OF RATING***[323]

Q25: Is there anything else you want to say?

DY25: Yes, in relation to this, I think it depends a little on when, when you get the questionnaire regarding which attitude you have towards it, because, if we had had the questionnaire one week from now, when we work in a different manner, then I might have had a different opinion to what I have answered now.

Q26: Is there anything you think has changed since you answered the questionnaire?

DY26: No, not radically.

Q27: Well, we talked about this scale from 1 to 10, we did talk about it having some oscillations.

DY27: Yes, but it is about the same, it relates to involving your ups and downs in your deliberations on everything, how it has been.

***3.P. RATING TODAY***[324]

Q28: Yes...to return to the beginning, when you came in, you said: "*Just today I am actually fine with mathematics*! *It is slightly more okay compared to how it has just been.*"

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Q25: Har du mere du gerne vil sige nu?

DY25: Ja, altså, i forhold til, til det her, så synes jeg, at det kommer lidt an på hvornår man … at man ligesom får spørgeskemaet hvilken holdning man har til det, fordi hvis vi havde fået spørgeskemaet om - en uge fra nu, hvor at vi arbejder anderledes, så havde jeg nok haft en lidt anden mening omkring det jeg har haft besvaret ved det.

Q26: Er der nogen ting du synes har ændret sig siden du svarede på spørgeskemaet?

DY26: Nej, ikke sådan… Ikke vildt.

Q27: Altså, vi snakkede om den dér med, på en skala fra 1 til 10, den har vi i hvert fald snakket om, den har ligget og svinget.

DY27: Ja. Men det ligger omkring det samme, og det handler jo om at man tager sine opture og nedture med i sine overvejelser om hvordan det har været.

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Q28: Ja. (...) For at vende tilbage helt til begyndelsen, dér da du kom ind, så sagde du: lige i dag har jeg det faktisk udmærket med matematik. Det I sidder med lige nu, Det er lidt mere okay end det lige har været.

DY28: Ja. Og det er fordi, at vi arbejder meget i projekter, hvilket gør, at visse projekter, det er der jo selvfølgelig forskellige meninger om, og der er visse projekter jeg ikke har syntes så godt om. Og der er visse projekter der har ligget i perioder hvor man ikke lige har haft motivationen til at have om lige det her emne. Det har selvfølgelig gjort at man ikke rigtig har gået i dybden med det eller har interesseret sig så meget for det.

DY28: Yes. And it is also because we work rather project oriented, which means that some projects, of course there are different views on this, and there are some projects which have run at times when you did not have the motivation for dealing with exactly this topic. This has implied, of course, that you have not worked with it all that thoroughly or have not been all that interested in it.

*3.Q. RATING NEXT WEEK*[325]

Q29: So, how do you think you will be doing in a week?

DY29: Well, in a week, then I think I would feel better about the project we work on now. Right now, just today, it is about preparing for the end of term exam, for which we have added some new things that we learn about, but also some old things, which we catch up on in a somewhat different way to what we did earlier with the theory (…) one can take some theory, and then one can work with it, and you can do some tasks and see how much one can manage, and then … work a little differently with the tasks, and thereby learn more.

Q30: Are you in control of how to mix things now, or has the agenda just been organised in a way that suits you better?

DY30: The agenda today has been organised really well, I think. Of course we have had a lot of thorough theory previously, when we did not have this interplay, that makes it a lot better.

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Q29: Så hvordan tror du du har det om en uge?

DY29: Jamen, om en uge, der tror jeg jeg har det bedre med det her projekt vi er i gang med nu. Lige nu, her i dag, der handler det om forberedelse til en terminsprøve, hvor vi både har fået trukket nogle nye ting indover, som vi lærer om, men også nogle gamle ting, som vi samler op på på en lidt anderledes måde end vi før har gjort med teorien. (…) Man kan tage noget teori, og så kan man prøve at arbejde med det, og så kan man tage nogle opgaver og ligesom se hvor meget man nu kan, og… Ja. Man kan sådan, arbejde lidt forskelligt på opgaverne, og deraf lære mere.

Q30: (…) Styrer I selv hvordan I blander tingene nu, eller er det bare programmet der er tilrettelagt, så det passer bedre til hvad du synes om?

DY30: Programmet synes jeg at, i dag, er blevet tilrettelagt ret godt. Vi har selvfølgelig fået en masse indgående teori førhen, hvor vi netop ikke har haft den her vekselvirkning. At vi så samler op på den her måde, med at man kan stadig trække teori ind over det, men også have den her vekselvirkning, det gør det ligesom en del bedre (...)

***3.R. TOPICS***[326]

Q31: Those topics for examination (…) there are some for which you write "*I do not know*"; that would for example be "*differential calculus*" and "d*ifferential equations*", and then there is something for which you say: "*Rather not*": "I*ntegral calculus*", "*Determination of indefinite integral*", "*lines and planes in Euclidean Space*", "*Equations of the Plane*", "*Distance between point and plane*" and "*the exponential function*". So, there are things you would rather not have for exam questions, and also some things you do not really know if you would be comfortable having for exam questions, at this point of time.

DY31: Yes, it relates to (…) there are certain things that it is good to be forced to have, such that you read up on them. If it results in something negative, and you get a poor grade or a bad impression of yourself at the exam, then you know, that this, I do not want to work with this. And those things that you already know are alright for you to have for exam questions, well then it is of course because you are able, or that you find the topic interesting...

Q32: And for "*readily*" goes: "r*ight-angled triangles*", "*Pythagoras*", "S*ine and cosine*", "*arbitrary triangles*", "*Law*

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Q31:: De her emner man kan trække til eksamen (…) Der er nogle hvor du skriver "Ved ikke", og det er for eksempel med differentialregning og med differentialligninger, og så er der nogle her hvor du siger "Helst ikke": Integralregning, Bestemmelse af Stamfunktion, Linjer og Planer i Rummet, Planens Ligning, Afstanden mellem Punkt og Plan, og Eksponentialfunktionen. Så du har nogle du helst ikke vil op i, og så har du også nogle du ikke rigtig ved om du vil op i, på det tidspunkt dér.

DY31: Ja. Det handler jo lidt om, (...) at der er jo selvfølgelig visse ting det kunne være rart at blive tvunget op i for at man ligesom læser op i det. Hvis der så kommer noget negativt ud af det, at man ikke får så god en karakter eller så godt indtryk til sig selv af det til eksamen, så ved man ligesom, at det her, det vil jeg ikke arbejde med. Og de ting man så i forvejen har lyst til at komme op i, jamen det er jo selvfølgelig fordi man kan det, eller man synes det er et interessant emne (...)

Q32: Og det som er "Meget gerne", det er Retvinklede Trekanter, Pythagoras, Sinus og Cosinus, Vilkårlige Trekanter, Sinusrelationerne og Cosinusrelationerne (…) dem vil du gerne trække til eksamen.

DY32: Ja. Man kan selvfølgelig sige, det er simpel matematik når man ser på det i lange - på de tre år. Det var de første vi lærte. Men derfor er det også det, vi har haft mulighed for at kunne bruge i længst tid, hvor at de nye ting vi har lært senere hen, der har vi ikke kunne anvende det i ligeså mange opgaver, eller vi har ikke haft mulighed for at kunne gøre det, fordi vi netop har skulle videre med det næste emne. Så der er selvfølgelig nogle, visse ting, for eksempel differentialregning, vektorer i rummet, og stamfunktioner og så videre, som ikke er så lette at trække indover alle mulige andre projekter… Ja... Ja.

*of Sines*", "*Law of Cosines*" (…) those you would readily have for the exam?

DY32: Of course one can say that it is simple mathematics when you look at it in a three years' perspective. It was the first things we learned. This is what we have been able to use for the longest time, whereas the new things we learned later on, these things we have not been able to apply for just as many tasks, or we have not had the chance to do so, because we had to move on to the next topic. So there are of course some things, for example differential calculus, vectors in Euclidean space, indefinite integral and so on, that are not as easy to apply on other projects.[327]

### *3.5. THE ROLE OF CREATIVITY*[328]

Q33: There are some people who are allowed to be creative, when they do mathematics, but is this also possible as part of common upper secondary school mathematics? Or does it feel more like it is about learning something by heart, when you deal with mathematics in upper secondary school?

DY33: I do not think you get much of a chance for working in that way, in terms of "*thinking out of the box*". And there are also parts where one thinks that "*well, could it be done in this way?*" When on the other hand, one also could think: "*Well, but that is not necessary.*" So why do it, if it is not necessary for achieving certain things?

327 *In the Technical programme, one part of the oral examination is that the student draws at random an examination topic relating to one of the projects they have done during the three years in upper secondary school. Dylan refers to the issue that the more advanced mathematical topics have not been part of that many projects, if they are introduced only later in the course, where the early projects with more fundamental topics are easier to relate to any topic.*

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Q33: Der er nogle der får lov til at være kreative når de laver matematik, men kan man også få det ind når man har almindelig gymnasieundervisning? Eller kommer det til at føles som om at det handler om at lære noget udenad når man har matematik i gymnasiet?

DY33: (…) Jeg synes ikke man har så meget mulighed for at arbejde på den måde, med ligesom at tænke mere ud af boksen. Og der er også en del hvor man tænker at, "Jamen det kunne du egentlig gøre på denne her måde", hvor man på andre måder tænker, "Jamen, det er ikke nødvendigt". Så hvorfor så gøre det hvis det ikke er nødvendigt for at, ligesom, opnå nogle ting?

***3.T. PLANS***[329]

Q34: What you want to do after upper secondary school? A plan that is on the table right now, is to start in the laboratory technician programme. Where does it take place?

DY34: It takes place at DOWNTOWN University College in Copenhagen.

Q35: And what is it that you like about that education? What males you think it is something for you?

DY35: It is the more practical tasks. It is something about being in the laboratory and deal with some practical tasks, manual work and more technical stuff. Of course, there is also a good deal of analysis to conduct on computers, and computations and so on, but that is not what I focus on when I have chosen this education. Yes.

Q36: Yes. But you do like laboratory work?

DY36: Yes.

Q37: Is it something that has been substantiated in your current study programme [Biotechnology]? Have you had a chance to...

DY37: Yes. Yes, we have, in Biotechnology, during the first and second years, we spent a considerable amount of time in the laboratory, and in the third year, then it has been replaced by "Technics and Processes" where we have been in the laboratory and done practical tasks. So...yes.

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Q34: Det du gerne vil efter gymnasiet, en plan der er oppe nu, det er i hvert fald at starte på laborantstudiet. Hvor foregår det henne?

DY34: Det foregår på "DOWNTOWN University College", inde i København.

Q35: Og hvad er det du godt kan lide ved det studie? Hvad er det ved det studie, der gør at det er noget for dig?

DY35: Det er de mere praktiske opgaver. Det er noget hvor man går i laboratoriet og arbejder med nogle praktiske opgaver, ligesom får brugt hænderne, og det bliver lidt mere teknisk. Selvfølgelig, så er der også en hel del analyse man skal lave på computer, og udregninger og så videre, men det er ikke det jeg sætter fokus på, når at det er jeg har valgt den hér uddannelse. Ja.

Q36: Ja. Men du kan godt lide laboratoriearbejdet?

DY36: Ja.

Q37: Er det noget du har fået bekræftet her på den linje du har valgt? Har I haft mulighed for at…

DY37: Ja. Ja, vi har, på Bioteknologi, der har vi, gennem første og andet år, der har vi haft en hel del tid i laboratoriet, og på tredje år, der er det så blevet overtaget af Teknik og Proces, hvor at man har gået i laboratoriet og lavet praktiske opgaver dér. Så… Ja.

## THEMES IN DYLAN'S 3RD YEAR

*These interpretations are based on both the questionnaire answers and the interview transcripts from Dylan's Third Year.*

### *DYLAN'S 3RD YEAR QUESTIONNAIRE*

#### MATHEMATICS AT SCHOOL

Mathematics has become considerably harder (DYLAN: Q3-A).

There seems to be less connection between mathematics and the other subjects for the time being (DYLAN: 3.B.).

Dylan experiences a more theoretical focus in the mathematics teaching in the 3rd year. It is far from what he believed was the subject mathematics, the tasks are quite complex and it is quite far from the more applicable side of mathematics he used to somehow enjoy (DYLAN: 3.L.).

The best feature of the textbook seems to be its usefulness for providing ready made formulae in which you can enter values and compute (DYLAN: 3.H.).

The teaching is organised around projects, and sometimes the motivation for the exact topic has not been too extensive which has led to less effort invested in learning. (DYLAN: 3.M.).

#### MATHEMATICS AS A DISCIPLINE

Mathematics is created by human beings, and if one person had not invented it somebody else would have (DYLAN: 3.C.). In university, the activities of mathematicians may make formulae longer or shorter or make them contain more factors (DYLAN: 3.D.). Dylan perceives mathematics as related to other scientific

subjects in terms of calculations and "curves" (DYLAN: Q3-B).

### MATHEMATICS IN SOCIETY

Mathematics is perceived useful for anybody (DYLAN: Q3-B).

### CHALLENGES & SUPPORT

Now in the 3rd year, computing apparently seems more challenging to Dylan than remembering, figuring out the purpose of a task, finding a strategy for solving it or for understanding the textbook (DYLAN: Q3-E). Also the *pace* is an obstacle for Dylan now (DYLAN: Q3-D). *Pace* is part of what Mellin-Olsen refers to as the task discourse (1990). Now in third year, parents and siblings are no longer among the resources for support for mathematical activities, but classmates are. He finds his sense of logic a strength compared to classmates (DYLAN: Q3-E).

### PLANS

Now his idea of studying medicine for future education has been supplemented by the thought of becoming a laboratory technician, which is an education at bachelor's level. The actual decision will be influenced by the average of the grades he obtains in general, and his experiences with mathematics in upper secondary school has influenced on the ideas for work and further education in a negative direction (DYLAN: Q3-G). However, it turns out that Dylan's average of grades will not allow for him to apply for medical school. Instead, he thinks of taking a bachelor of professions as a degree laboratory technician. If so, he will not need the high-level mathematics for admission, but he hopes it will prove useful anyway (DYLAN: 3.J., 3.K.). The education as laboratory technician is attractive because of its practical and technical elements, which he has experienced during the three years in technical upper secondary school (DYLAN: 3.S.).

### RATING

The rating of mathematics is lower, and mathematics has become considerably harder (DYLAN: Q3-A).

Favourite subject scale rating:"around the middle" - and he has felt quite challenged recently (DYLAN: 3.A.). "Today", the day of the interview, they work in a way that suits Dylan much better; they work in groups and they can have frequent interaction between theory and practice (DYLAN: 3.N.). Maybe "next week" Dylan would have rated mathematics at 6 or 7 (DYLAN: 3.O.)

### UNDERSTANDING

It is important for Dylan to understand the mathematics he is working with, and it plays a significant role in terms of his preferences for topics for the examination (DYLAN: 3.P., 3.Q.). Relational understanding takes time for him, and Dylan prefers frequent alternation between theory and practical tasks (DYLAN: 3.E.). Project work has functioned as a source for understanding for Dylan (DYLAN: Q3-XA). Understanding a topic works as an indicator of whether Dylan prefers one topic for the other for examination, and he seems somewhat comfortable only with half of the suggested topics (Pythagoras, Sine and Cosine, volume of solid revolution and vectors in the plane at this point of time (DYLAN: Q3-XC).

## DYLAN'S 3RD YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics has become much harder and the teaching involves sides of mathematics Dylan did not think was mathematics; too much theory and not just application of formulae for solving tasks. An increased emphasis on understanding.

### *MATHEMATICS AS A DISCIPLINE*

Now Dylan sees mathematics as invented, and one should not necessarily be a genius to study mathematics in university. Fundamentally, Mathematics is an applicatively discipline to Dylan and the theoretical aspects of it seems useless to him.

### *MATHEMATICS IN SOCIETY*

Mathematics should be learned by everybody as a matter of course in order to manage oneself.

### *MATHEMATICS AND ME*

Dylan's relation to mathematics is sensitive to variations in understanding relating to the different topics. Dylan now deselects an education demanding A-level mathematics due to an average of grades insufficient for admitting him to study medicine, but also because mathematics was never of interest to Dylan in its own right.

## DYLAN'S BELIEFS' TRANSPOSITION

*STABLE:*

Dylan's view of Mathematics in Society is unchanged. It is necessary and everybody should learn it. Dylan's beliefs about Mathematics at School are still showing many signs from the "task discourse" and the crucial role of formulae in this; maybe even more than in 1st year (#12b, c). Mathematics never had an intrinsic value to Dylan. He wanted to study it for instrumental purposes.

*CHANGED:*

Experiences in school lead Dylan to deselect an education demanding A-level mathematics for admittance. He has seen sides of mathematics that he never thought belonged to it and which he does not appreciate; its theoretical sides. Moreover, Dylan now believes mathematics is invented (#9) and that one should not necessarily be a genius in order to study mathematics in university (#11). All types of challenges has become even more challenging compared to 1st year; "*Computing*" still involving more than the others - from [*Moderate*] to [*More*] - and "*Figuring out the purpose of a task*" from involving [*The fewest*] to [*Moderate*] challenges (#14). In the 1st year he could get support from both [*Parents*], [*Siblings*] and [*Classmates*] – now in the 3rd year he depends on his peers only (#15).

# *THE CASE OF GRACE*

*Grace is a female student from Gamma Technical Upper Secondary School, in a mathematics-physics study programme which involves studying A-level mathematics. In 1st year, she considers chemical engineering for tertiary education, and in the 3rd year she is almost sure she wants to study food and nutrition engineering, which is a branch of chemical engineering. On the favourite subject scale she rates mathematics with an [8] in the 1st year and with a [4] in the 3rd year, decreasing by 4 steps, if the two questionnaires are compared.*

| Grace | Date for Questionnaire | Date for interview |
|---|---|---|
| 1st Year | 29 November 2010 | 6 April 2011 |
| 3rd Year | 11 December 2012/15 January | 14 March 2013 |

*Table 35: Grace: Dates for Questionnaires and Interviews*

In the next sections, Grace's answer to the first year questionnaire and interview transcripts from the 1st year interviews are displayed. Then an account of Grace's interpretation of mathematics in first year in upper secondary school will be given in a narrative form leading to an analysis in terms of the four aspects of beliefs; *Mathematics at School*, *Mathematics as a Discipline*, *Mathematics in Society*, and *Mathematics & Me*.

# GRACE'S 1ST YEAR QUESTIONNAIRE

| Q1-A | TRANSITION | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [8] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [330] | [Working on your own]; [Working in pairs]; [Group Work]; [Project Work] |
| **3** | Did you like mathematics when you went to lower secondary school? [331] | [Yes, it was one of my favourite subjects] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | I do not think it has changed that much[332] |
| **4b** | Is there anything you liked better before?' | I think it was better organised in lower secondary school! (Both, red.) In terms of peace to work, concentration (for which the initiative came from the whole class and the teacher!)[333] |
| **4c** | Is there anything you like better now? | No, except maybe the projects and the group work.[334] |

*Table 36: GRACE'S 1st year questionnaire, part A – TRANSITION*

330 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

331 [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

332 Jeg synes ikke det har ændret sig ret meget.

333 Jeg synes der var meget bedre styr på det i folkeskolen! Mht. afleveringer, arbejdsro, koncentration (med initiativtagning fra hele klassen og læreren!)

334 Næhhh, det skulle lige være projekterne og gruppearbejdet.

| Q1-B | FOR SCHOOL | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | In practice mathematics is important to master. Many places you meet mathematical challenges.[335] |
| **5b** | Is mathematics something you think everybody should learn? | [-] |
| **6** | What made you choose a study programme involving A-level mathematics? | Because I possibly will need mathematics in my (further, ed.) education.[336] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | Mathematics is often used in other subjects! Projects, tasks, etc.[337] |

*Table 37: GRACE'S 1st year questionnaire, part B - MATHEMATICS AND SCHOOL*

335 I praksis er matematik et vigtigt fag at kunne . Mange steder møder man matematiske udfordringer.

336 Fordi jeg muligvis skal bruge matematik i min uddannelse.

337 Matematik bruges ofte i andre fag! Projekter, opgaver etc.

| Q1-C | BEYOND SCHOOL | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | One applies mathematics for calculating graphs, statistics, equations etc. You make a lot of calculations.[338] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? | [Both] |
| **10** | What do you think a professional mathematician at a university is doing? | Renews mathematics, challenges it, examines it and deals with it in depth with sides of mathematics which has not been challenges that much previously. Besides that, they educate themselves, such that they can pass on their knowledge to others.[339] |
| **11** | Would you have to be a genius in order to study mathematics in university? [340] | [No] |

*Table 38: GRACE'S 1st year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

338 Man bruger matematik til at udregne grafer, statistikker, ligninger etc. Man laver mange beregninger.

339 Fornyer matematikken, udfordrer den, undersøger og går i dybden med sider af matematikken man ikke har udfordret så meget førhen. Udover det uddanner de sig, så de kan give deres viden videre til andre.

340 Options:[Yes]; [No]; [I do not know]

| Q1-D | IMRPOVING | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | That you maybe does not have the mathematics genius gene! (I, red.) need time for recalculation of tasks.[341] |
| **12b** | What do you think is the best means for improving in mathematics? | DETERMINATION, focus! The brain.[342] |
| **12c** | What do you do to improve in mathematics?[343] | [I] challenge myself! :-) [344] |
| **13a** | What do you do if you get stuck on a task at school? | I ask for help, after I have read through the task 117 times myself :) hehe[345] |
| **13b** | What do you do if you get stuck on your homework? | The same as above |

*Table 39: GRACE'S 1st year Questionnaire, part D – STRATEGIES FOR IMPROVING*

341 At man måske ikke har geni-matematikgenet! Skal bruge tid på at genregne opgaver.

342 VILJE, koncentration! Hjernen.

343 Hvad *gør* du for at blive endnu bedre til matematik?

344 Udfordrer mig selv! :)

345 Jeg spørger om hjælp, efter jeg selv har læst opgaven igennem 117 gange :) hehe

| Q1-E | CHALLENGES & SUPPORT | | GRACE |
|---|---|---|---|
| # | *Question* | | *Answer*[346] |
| **14** | What issues involve more challenges to you? | a) Remembering | [4] Few challenges |
| | | b) Computing | [4] Few challenges |
| | | c) Figuring out the purpose of a task | [4] Few challenges |
| | | d) Finding a way to solve a task | [3] Moderate |
| | | e) Reading and understanding the textbook | [4] Few challenges |
| **15** | Where can you find support for mathematical activities? [347] | | [From siblings]; [From classmates]; [Other places]: The teacher, the Internet, BOOKS![348] |
| **16** | Did you parents take the Upper Secondary School Leaving Certificate? [349] | | [Yes, my mother did] |

*Table 40: GRACE'S 1st year Questionnaire, part E – CHALLENGE AND SUPPORT*

346 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

347 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From classmates]; [Other places] If other places, from where or from whom?

348 Lærer, internettet, BØGER.

349 [Yes, my mother did]; [Yes, my father did]; [Both my parents did]; [None of them did]

| Q1-F | IN CLASS | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? | [More than 8 times] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [I do not know] |
| **17c** | Are you content with that? | - |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? | [More than 8 times] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [I do not know] |
| **18c** | Are you content with that? | - |
| **19a** | In your class, is it okay to be good at mathematics? | YES! A-level Mathematics :-) [350] |
| **19b** | In your class, is it okay to have difficulties in mathematics? | It is, I suppose, to a lesser extent.[351] |

*Table 41: GRACE'S 1st year Questionnaire, part F - MATHEMATICS IN CLASS*

350 JA! Matematik A :)

351 Det er det vel, i mindre grad.

| Q1-G | PLANS | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | I have no definite plans so far. Maybe chemistry engineering.[352] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [Yes] |
| **21b** | Comments: | I cannot say for sure, but yes, I probably will[353] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | - |
| **22b** | Comments | - |

*Table 42: GRACE'S 1st year Questionnaire, part G – PLANS*

352 Indtil videre har jeg ingen klare planer. Måske kemi-ingeniør.

353 Jeg kan ikke sige det med sikkerhed, men ja, det skal jeg højst sandsynligt.

## GRACE'S 1ST YEAR INTERVIEW

*Excerpts from Grace's first year interview are displayed. They each contain elaborations to the questionnaire answers and the issues relating the four aspects of Grace's beliefs about mathematics and to the formation of them during year 1 in upper secondary school.*

| Date | Duration (mm:ss) |
|---|---|
| 6 April 2011 | 28:23 |

### *1A. NOW EVERYBODY CAN HELP*[354]

Q1: The first thing I will ask is how you think mathematics has changed from lower secondary school till now, when you are in the technical upper secondary programme?

G1: Well, I went to [GREENFIELD Private School], at this school, the teaching is at a really high level, so I do not think it has changed that much. But you do feel that you have started to use the calculator a lot compared to lower secondary school, when everything was calculated by hand and on paper. But otherwise I do not think it has changed that much.

Q2: What about the difference between attending a mathematics class in lower secondary school for learning

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Q1: Det første, jeg vil spørge om, det er, hvordan du synes, matematik har ændret sig, fra du gik i folkeskolen, og så til nu, hvor du går på HTX.

G1: Altså jeg gik på [Privatskolen] før, og der foregår undervisningen også på et ret højt plan, så jeg synes ikke, den har udviklet sig særlig meget, ændret sig. Men man kan godt mærke, at man er gået over til at bruge lommeregner rigtig meget i forhold til i folkeskolen, hvor alt bare skulle regnes ud i hånden på papir. Men ellers synes jeg ikke, at det har ændret sig så meget. (...)

Q2: Hvad med (...) forskellen fra at gå i en folkeskoleklasse og så lære matematik og så til at gå (...) i en HTX-klasse og lære matematik.

G2: (...) fra folkeskolen af, (…) der havde jeg én veninde, som var nogenlunde på samme plan med mig, så vi lavede alting sammen. Men her, der er det sådan lidt alle, der er sat ind i matematikken. Så man føler, man kan spørge alle om hjælp. (...)

Q3: (...) gør det noget ved den måde, at det er at have matematikundervisning på, at I er 32 elever om én lærer?

G3: Det synes jeg faktisk ikke, fordi nu her, som jeg sagde, med at vi har det som A-fag, så er det ikke kun læreren, man kan spørge om hjælp nu, hvis man er i tvivl. Det er lige så meget eleverne, (…)

Mathematics compared to attending a mathematics class in technical upper secondary school for learning Mathematics?

G2: From lower secondary school, I had one friend who were more or less on the same level as me, so we did everything together. But here, it is more or less everybody who are acquainted with Mathematics. So you feel you can ask everybody for help.

Q3: Does it change anything that now you are 32 students to one teacher?

G3: Actually not, because, as I said, since we take it as an A-level subject, then it is not only the teacher one can ask for help now, but just as much the students.

***1B. MATHEMATICS FOR ALL?***[355]

Q4: Then I also ask: Who should learn mathematics?

G4: Well, I think that everybody should learn Mathematics. It is an important subject to know. And it is somehow general. I think everybody should learn the Mathematics. But if you are heading for a future in which Mathematics is a part, then, of course, it is important. But you cannot say, that because you are to stand in a kiosk selling ice cream, then you do not need to know Mathematics. Everybody should know Mathematics.

355

Q4: (...) Så spørger jeg også til, hvem der egentlig skal lære matematik.

G4: Altså jeg synes jo, alle bør lære matematik. Det er et vigtigt fag, det er godt at kunne. Og det er sådan lidt, det er lidt alment. Altså jeg synes, alle bør lære matematikken. Men hvis man vil have en fremtid inden for noget, hvor at det er ligesom, at matematik skal indgå, så er det selvfølgelig vigtigt. Men man kan jo ikke sige, at bare fordi man skal stå i en iskiosk og sælge is, så skal man ikke kunne matematik. Altså alle skal kunne matematik.

### *1.C. CHOOSING A-LEVEL MATHEMATICS*[356]

Q5: You chose a study programme encompassing A-level Mathematics. What made you choose this study programme?

G5: Well, in primary school, I did not like Mathematics. But then I moved to [GREENFIELD Private School], then I got a teacher who was incredible good, and Mathematics is fun, when you understand it. And then it became a little competitive, when you were one of those in class, who needed extra tasks, because you were ahead and came to the blackboard to explain. And then I started feeling like doing an extra page in the mathematics book to get ahead, so I just like mathematics, so I thought it could be fun to take A-level Mathematics at the technical programme. It sounds kind of cool to say.

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Q5: Og du har så valgt en studieretning, hvor matematik er på A-niveau. Hvad fik dig til at vælge denne her studieretning?

G5: Altså i folkeskolen, da jeg gik i de mindre klasser sådan op til 7.-8., der kunne jeg ikke lide matematik. Men så da jeg flyttede skole over til [Privatskolen], der fik jeg en lærer, som altså hun var virkelig dygtig, og matematik er sjovt, når man forstår det. (...) og så gik der lidt konkurrence i det, når man var sådan en af de der i klassen, der skulle have lidt, altså der havde brug for nogle ekstraopgaver, fordi at man var færdig med tingene før de andre og skulle op og forklare ved tavlen. (...) så fik jeg lyst til at sidde og lave sådan lige en side ekstra i matematikbogen for at komme foran, så jeg kan bare godt lide matematik, så tænkte jeg, det kunne være meget sjovt at have på A-niveau på HTX. Det lyder også lidt sejt at sige. (...)

### *1.D. CHOOSING THE TECHNICAL PROGRAMME*[357]

Q6: Is it more cool to to you to take it [Mathematics] in the technical programme, rather than in the general programme?

G6: Yes, I do. People may say that all the nerds are out here. But I do not care, because those who call us nerds, they will end up working for us anyway.

Q7: You are a handful of girls in a class with boys, what is it like?

G7: Earlier, I did not talk that much to the boys. I was also, in the beginning: This will be special, because, then you only have these four girl to cooperate with in Mathematics, but it is not like that at all. Because the boys here, you have to prove that you can, because they are like: "*Oh, but you cannot, just be quiet, right?*" and "*your result is not correct*". But I set them straight. I want to prove that just because you are a girl, I does not mean that you are dumb. But of course, there are not many girls. But it is not a problem. I do not work with the girls in Mathematics. Then I hang out with my little boy gang, and it is fun.

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Q6: Er det sejere, synes du, at have det på HTX end på STX?

G6: Ja, det synes jeg. Det kan godt være, folk siger, at alle nørderne går herude. Men det er jeg ligeglad med, fordi dem, der kalder os for nørder, det er jo dem, der ender med at arbejde for os alligevel. (...)

Q7: I er en håndfuld piger i en klasse med drenge, hvordan er det?

G7: Førhen, der snakkede jeg ikke så meget med drengene, (…) jeg var også meget i starten, sådan lidt: Det bliver lidt specielt, fordi så har man måske kun de her 4 piger, man kan arbejde sammen med i matematik, men sådan er det jo slet ikke (…) for drengene her, man skal ligesom bevise, at man godt kan, for de har den der: "Jamen, I kan jo ikke, I skal bare lige tie lidt stille, ikke også," og "Dit svar er ikke rigtigt." Men jeg får sat dem lidt på plads nu her, jeg vil gerne bevise over for dem, at bare fordi man er pige mellem alle de her drenge, så er det ikke, fordi man er snotdum. (...) men selvfølgelig der er jo ikke mange piger. Men det er ikke noget problem, jeg arbejder ikke sammen med pigerne i matematik. Der er jeg sammen med min lille drengebande, og det er sjovt.

Q8: Ja, ja, de brokkede sig, da du gik.

G8: Ja (…) de starter med at sidde sådan lidt:

(”Drengene”, red.): "Ej, det kan du ikke”, ”bare ti stille”, og ”det er jo rigtigt”, og ”hvor er du langsom."

(”Grace”, red.): "Og prøv at se, jeg har lavet lige en opgave ekstra end dig, ikke også?"

(”Drengene”, red.): "Jamen, må vi lige se, hvordan du gjorde?"

(”Grace”, red.): "Næh, så skal du bare lade være med at snakke sådan om mig."

Så man skal lige, man skal lige have dem i ørerne. De skal have respekt over for en. Det får de også hurtigt.

Q8: Yes, they were complaining when you left?

G8: Yes, they start by saying like: [Boys]:"*Oh, you cannot*", "*Just be quiet*", "*That is not correct*" and "*You are just so slow*". [Grace]: "*And look, I just made one more task than you!*" [Boys]: "*But could we just see how you did?*" [Grace]: "*No, then you should not talk about me like that!*". So, you have to, you have to keep them up mark. They should respect you. And they get that rather fast.

***1.E. MATHEMATICS RELATING TO OTHER SUBJECTS***[358]

Q9: Do you find what you learn in Mathematics to be useful in your other subjects?

G9: I do. Here at this school, they are really good at combining the subject with each other. This means, that we may have Technology and Chemistry and Physics and Mathematics, for which we do a project and write a report, all all these subjects should be part of it. And you learn to use Mathematics in different ways and in different contexts. So they are really good at combining.

***1.F. MATHEMATICS IN SOCIETY***[359]

Q10: What is Mathematics used for outside school?

G10: Yes, I should answer that?

Q11: Well, you did answer: You use Mathematics for calculating graphs, statistics, equations and so on, and you do a lot of computations. Can you say something about where you do that, if you are not at school?

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Q9: Synes du, du kan bruge det, du lærer i matematik, i dine andre fag?

G9: Det kan jeg. Her på skolen, der er de rigtig gode til at kombinere fagene med hinanden. Dvs., vi har måske teknologi og kemi og fysik og matematik, hvor vi laver et projekt og skriver en rapport, og der skal alle de her fag indgå i. Og så lærer man at bruge matematik på forskellige måder og i forskellige sammenhænge. Så det er de rigtig gode til at kombinere. (...)

359

Q10: Hvad bruger man matematik i Verden uden for skolen (…)?

G10: Ja, skal jeg svare på det?

Q11: Altså du har i hvert fald svaret: Man bruger matematik til at udregne grafer og statistikker og ligninger og videre, og man laver mange beregninger. Kan du sige noget om, hvor man gør det, når man ikke lige er i skole?

G11: Når man regner skat og løn og værdipapirer. Altså man bruger matematik mere end man egentlig lige går og tror. Og det, ja, hvis man lige er ude og shoppe og skal regne ud, hvor meget tøj man skal have og... Men der er jo mange steder, det ses også på større arbejdspladser, hvor de skal se, hvordan virksomhedens økonomi er, så bruger de også grafer til at afmåle, hvordan det går. (...)

G11: When you calculate tax and salary and stocks. So you use Mathematics more than you should think. And, yes, if you are out shopping and should calculate how much clothes you should buy, and...but there are many places, it is also seen at larger work places, where they need to check how the economy of the company is doing, then they use graphs to measure how is is going.

***1.G. NEW TEACHING - NEW ISSUES EMPHASISED?***[360]

Q12: Is there anything that is emphasised more now compared to earlier? To argue for it or is it also some of what you think is the same as when you went to [GREENFIELD Private School]?

G12: I think it is rather the same, but here you should for every...also when we use a calculator, then it is easy just to enter a function and get a result, but then it is important, and that is also emphasised by our teacher [GAMMAGAARD], that we remember to write down our intermediate results. Also how we get from this to that and what we have done and why and how. So yes, you have to argue for it a lot.

***1.H. SUPPORT FOR MATHEMATICAL ACTIVITIES***[361]

Q13: We talked about where you can seek support and help, where you say: Well, but you use your peers a lot.

G13: Yes, you do. Especially since all of us, we are brought together, us who are good and really like mathematics. Then it is not hard to find one who could help you. There are many who knows something and then there are others

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Q12: Er det noget, som du synes, der bliver lagt mere vægt på nu end før - at argumentere for det eller er det også noget af det, du synes, at det er det samme, som da du gik på [Privatskolen]?

G12: Jeg synes, det er meget det samme, men her der skal man altså for hvert, også når nu vi bruger lommeregner, der er det jo nemt bare lige at trykke en funktion ind og så få et resultat, men der er det jo vigtigt, det gør eller understreger vores lærer [GAMMAGAARD] også - at vi husker at skrive vores mellemregninger. Og hvordan vi kom fra dét til dét, og hvad vi har gjort og hvorfor og hvordan. (...) Så, jo, man skal argumentere meget for det.

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Q13: Vi har også været sådan lidt inde på, hvor man søger støtte og hjælp, (...) hvor du siger: Jamen, man bruger faktisk kammeraterne meget.

G13: Ja, det gør man også. (...) især her hvor vi alle sammen, vi er blevet samlet, os der er gode og virkelig godt kan lide matematik. Så det er ikke svært at finde nogen, der kan hjælpe en. Der er mange, der ved noget, og så er der nogle andre, der ved noget andet, og så er der noget, man er lidt usikker på, hvor andre er stærke. (…) Nu når ens forældre ikke kan følge med mere(...) så er det godt, at man også har sådan et godt forhold til sine klassekammerater (...) Det er bare: "Jamen, så hjælper du mig med den her, så hjælper jeg dig med den." (...)

who knows other things and then there are things one is insecure about, when others are strong. Now, when your parents cannot keep up any more, then it is good that you have such nice relations to your peers. It is just: "*Well, but if you help me with this, I may help you with that*"

***1.I. ACCEPT OF DIFFERENCE IN COMPETENCE IN MATHEMATICS***[362]

Q14: It is perceived positively that you are good at mathematics, in class?

G14: Yes, it is, definitely. And also, if you solved a task and other people have difficulties with it, then suddenly somebody need help, and it is also easier to understand your peers' ways of arriving at a result compared to the teachers. Because the teacher does not give an answer, the teacher rather gives some guidance. Whereas it is not because we come and ask "*Can you help me?*" "*Yes, here you have the result*", but rather like: "*Yes, you do like this, and do you remember what we did in the other task?*". They take the time compared to the teacher.

Q15: You also say: "*Well, it is okay to have difficulties in mathematics, but it is of course better to be good at it*"

G15: Well, of course some people have problems. There are many who do not see the logic of Mathematics, but there are also people who can look at a task and then: "*Well, it has to be like that you deal with it like this*". And it is also okay, but it is better to be good at mathematics.

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Q14: (…) Det bliver opfattet positivt, at man er god til matematik i klassen?

G14: Ja, det gør det, helt bestemt. Og det er også, hvis nu der er en opgave, man har løst, hvor andre folk har det svært ved den (...) så står der lige pludselig nogen og skal bruge hjælp, og det er også nemmere (...) at forstå ens klassekammeraters måde at komme frem til et svar på end det er med lærerens. Fordi læreren kommer jo ikke med et svar, læreren kommer med noget vejledende hjælp. Hvorimod altså det er ikke fordi vi kommer over og spørger: "*Kan du hjælpe mig med den*?" "*Jo, her har du facit*." Men det er lidt mere: "*Jo, men så gør du det, og kan du huske, hvad vi gjorde i forrige opgave?*" Giver sig bedre tid end læreren gør.

Q15: Du siger også: "*Jamen, det er okay at have svært ved matematik, (...) men det er selvfølgelig bedre at være god til det*."

G15: Jamen, altså selvfølgelig er der nogen, der har problemer. Der er mange, som ikke kan se det logiske i matematik, mens der også er en del, der kan se på et stykke, og så: "*Jamen ved du hvad, det må være sådan dér, at man gør det*." Men man bliver nødt til, altså man skal ville det, men alligevel så, der er en del, der har svært ved det, ikke? Og det er jo også okay, men det er bedst at være bedst til matematik.

### *1.J. PLANS AFTER GRADUATION*[363]

Q16: Then we have arrived at the plans after graduation, what you want to do then. "*No clear plans, but maybe Chemistry engineer*".

G16: I think it is hard to say what you want afterwards. Somehow, I would like to be a veterinarian, and otherwise I would like to be a TV host and Chemistry engineer. And that is three completely different things. I think Chemistry engineer would be exciting. But I have not really acquainted myself with what they do, besides that they are nerds.

Q17: And then you smile.

G17: Yes, it is fun. But yes, I also have an interest for animals and I have a good relation to them. So I would also like to be a veterinarian. But that is not why I chose this study programme. It is because I like the subjects, and I know it will be within this field.

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Q16: Så er vi nået til der med planerne efter gymnasiet, hvad du gerne vil der. "Ingen klare planer, men måske kemiingeniør."

G16: (...) jeg synes, det er svært at sige hvad man vil bagefter. (…) Et eller andet sted, så kunne jeg godt tænke mig at være dyrlæge, og et andet sted så kunne jeg godt tænke mig at være tv-vært og kemiingeniør og det jo tre vidt forskellige ting (…) Jeg synes, kemiingeniør ville være spændende. Men jeg har ikke sat mig så meget ind i, hvad de egentlig laver, ud over at det er nørder.

Q17: Og så smiler du.

G17: Ja, det er sjovt. Men jo, jeg interesserer mig også for dyr, og det der, ja, har et godt forhold til dem. Så jeg kunne også godt tænke mig at være dyrlæge (...) Men det er ikke derfor, jeg har valgt den her linje (...) Det er, fordi jeg godt kan lide fagene, og jeg ved det bliver noget inden for det. (...)

### *1.K. CHOOSING TECHNICAL OR GENERAL UPPER SECONDARY EDUCATION ?*[364]

Q18: But now you are here, at least. And you found out how to decide, when you had to chose?

G18: Well, it was actually difficult, because my best friend, who actually was on the same level in mathematics as me, and with whom I cooperated in mathematics, when I said technical upper secondary school, she just said: "You are not doing that!" And then I said: "Why not?" "But, do you realise that there are only boys, and no cool parties?""Cool parties? They should not determine what you want you should become in your future, whether there are cool parties or not!" But it was my mother, she heard in a TV show that girls were needed in this mathematics-science field. Then she came home and suggested it to me, and then I said: "*That would be exciting*". Now, my brother was also out here. Well, but it ended up with me going here – and I am very, very pleased about that.

Q19: So you do not feel that you sacrifice yourself for the cause?

G19: No, I feel I should choose what I think is better for me. Then you just have to listen to yourself. And now they are like: "*Okay, I do understand why you chose this programme*", because they can tell that I am really happy about it.

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Q18: Men nu er du i hvert fald her. Og du kunne finde ud af at beslutte dig for, hvad du skulle, da du skulle vælge...

G18: Altså det var faktisk svært, fordi min bedste veninde, som egentlig også var på samme matematikniveau som mig, som jeg arbejdede sammen med, hun sagde, da jeg sagde HTX, så sagde hun også bare: "*Det gør du bare ikke*!" Og så sagde jeg: "*Hvorfor ikke*?" "*Jamen, er du godt klar over, der er kun drenge derude, og der er ikke nogen fede fester*." Fede fester? Det er da ikke det, der skal afgøre, hvad du skal blive i fremtiden, om der er fede fester eller ej. (…)(…) men det var min mor, hun hørte et tv-program om, at de manglede nogle tøser i denne her matematik-naturvidenskabelige branche. Så kom hun hjem og foreslog det for mig, og så sagde jeg: "*Det kunne være meget spændende.*" (…) Nu var min storebror så også herude. (…) Så nå, men det endte med, at jeg kom herud - og det er jeg meget, meget glad for.

Q19: Så du føler ikke, at du ofrer dig for sagen (...)?

G19: Næ, jeg føler, at jeg skal vælge det, som, jeg synes, er bedst for mig. (…) Så må man bare lytte til sig selv. Og nu er de også sådan lidt: "(…) okay, jeg kan godt forstå, du valgte den linje," for de kan mærke, jeg er rigtig glad for det.

*1.L. DESELECTING SUBJECTS FROM GENERAL UPPER SECONDARY EDUCATION*[365]

Q20: You are not sorry to miss Classical Studies and...?

G20: Oh, no [swearword] definitely not. It is...and Geography and History, they are these kind of heavy subjects, I think – and I do not need them. So I am really happy and content about avoiding them.

*1.M. ON THE NATURE OF MATHEMATICS*[366]

Q21: Is Mathematics discovered or invented?

G21: I think both, actually.

Q22: Many answers exist, I mean, you may say the answer that makes sense to you.

G22: I think the human beings find out different ways to calculate things. So, somehow it is invented by us human beings, I suppose. But it is also, it is also. But they also discovered how to do these things. But, yes, we have found out how to calculate different things. So I think it must be inv... no, I think it must be a mix. It is a mixed answer.

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Q20: Du er ikke ked af de fag, du går glip af med oldtidskundskab og?

G20: Åh, nej. (Bandeord) Det er jeg godt nok ikke. Det er... Og geografi og historie, det er lidt de der tunge fag, synes jeg, kedelige fag - og jeg skal ikke bruge det til noget. Så det er jeg rigtig glad for, jeg slipper for.

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Q21: Er matematik opdaget eller opfundet?

G21: (...) Jeg tror egentlig både og. (...)

Q22: Der findes mange svar, altså så du må sige det svar, du synes, er meningsfuldt.

G22: Jeg tror, mennesker finder ud af forskellige måder, man kan regne ting på. Så et eller andet sted, så er det vel opfundet af os mennesker. Men det er jo også... Det er jo også. Men de har jo også opdaget, hvordan man gør de her ting. (...) Men jo, vi har vel fundet ud af, hvordan vi skal regne forskellige ting ud. Så jeg tror, det må være opf... Ej, jeg tror, det er en blanding. Det bliver et blandings-svar.

*1.N. ON MATHEMATICS AS A SCIENCE*[367]

Q23: What would a mathematician at a university be doing?

G23: Mathematicians conduct research on new ways to compute or calculate things. Find easier ways, maybe. I do not know, but conduct research in some new algorithms, I think.

Q24: How would one know, I mean, are you thinking: "*What would one be doing at a university*" and "*What would one be doing in mathematics*" and combine them, or do you know someone or what?

G24: Well, when one has an education, one stays at university with some mathematics, well, then it must be to understand it better. And to make other people wiser, so in that sense I think one conducts research in new ways to do the different things, to make everything easier. New operations. I do not know completely, but no, I do not know anyone, who...

*1.O. MATHEMATICS IN FUTURE LIFE*[368]

Q25: Mathematics is done other places than at university, so, well, you will be doing some mathematics, if you proceed in chemistry engineering?

G25: Yes, I will. But I have not made myself acquainted specifically with what they are doing, but mathematics is used for everything, I would say.

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Q23: Hvad laver en matematiker på et universitet?

G23: Matematikere forsker i nye måder at regne eller udregne ting på. Finder nemmere måder måske. Det ved jeg ikke, men forsker i nogle nye regnemetoder, vil jeg gætte på.

Q24: (...) hvor ved man det fra, altså sidder du og tænker: "Hvad laver man på et universitet og hvad laver man i matematik" og sætter det sammen eller kender du én eller hvordan?

G24: Altså når man har en uddannelse, (...) man sidder på universitetet med noget matematik, jamen så må det være for, at man skal blive klogere på det. Og for at gøre andre folk klogere, så på den måde tror jeg man forsker i at finde nogle nye måder at gøre de forskellige ting på, at gøre det hele nemmere. Nye regnearter. Det ved jeg faktisk ikke helt, men nej, jeg kender ikke nogen, der...

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Q25: Men man laver matematik andre steder end på universitetet, så altså du kommer vel også til at lave en del matematik, hvis du går i kemiingeniørretningen.

G25: Ja, det gør jeg. Men nu har jeg ikke rigtig sat mig ind i, hvad det egentlig, hvad de egentlig laver helt specifikt, men man bruger matematik til alt, vil jeg sige.

## THEMES IN GRACE'S 1ST YEAR

From examining the questionnaire answers and the interview transcripts forms, I get a basis for my interpretation and analysis of Grace's account of her experiences with mathematics in the 1st year of upper secondary school. The interpretations are organised in themes concerning issues or situations.

### *MATHEMATICS AT SCHOOL*

#### CLASSROOM NORMS

It is Grace's impression that success in mathematics is somewhat more accepted in class compared to failure (GRACE: Q1-F).

#### BELIEFS ABOUT MATHEMATICS LEARNING

Success in mathematics is ascribed to ones genes, which is something that is given, as well as factors such as determination and concentration (GRACE: Q1-D). While the former is something that you cannot change, the latter rests on your own effort.

#### COOPERATION AND SUPPORT

From cooperation with one friend—in a private school—to possible cooperation with anyone or everyone from class – everybody is on the same track (1.A), and then everybody can help you (1.I), which is nice, since your parents cannot any more.

It does not seem to bother her to be of very few female students in a class with mainly male students. Nevertheless, she explains how she need to win their respect, but also that she succeeds in it. You have to prove that you are not a complete idiot in mathematics just because you are a girl, she explains. And it seems that she

prefers to work with a group of male students in mathematics rather than the other female students (1.D).

### DIDACTICAL CONTRACT

The group and project work in technical upper secondary school is valued by Grace, but her teacher and her classmates do not match the standard she is used to from compulsory school. Somehow there seems to be a mismatch between what could be her idea about the didactical contract and what she actually encounters (GRACE: Q1-A).

In primary school, Grace did not fancy mathematics much, but in lower secondary school she got a highly proficient mathematics teacher, and started to understand mathematics. "*[M]athematics is fun, when you understand it*", Grace states (1.C). Another element Grace mentions from lower secondary school, is the competitive element; extra-curricular tasks, if you had completed the compulsory tasks already, getting to explain things at the blackboard and computing ahead in the textbook – then she liked mathematics and felt like choosing it on A-level in upper secondary school (1.C).

The didactical contract in the 1st year of upper secondary school is not 'raise your hand-mathematics'. The teacher gives an instruction for five minutes, and then they change to group work (1.J). The teacher does not tell you how to do it, but gives a more supervising kind of advice (1.K). Moreover, the teacher emphasises the demands for giving the reasoning behind the results, even though, or maybe because of, the widely used calculator (1.H).

### MATHEMATICS LEARNING

Some students have a hard time in mathematics, because they do not see the logic in it. And Grace says that is okay – but it is better to be best at mathematics

(1.K).

### *MATHEMATICS AS A DISCIPLINE*

Her view of mathematics seems to have dynamical aspects (GRACE: Q1-C).

### MATHEMATICS IN OTHER SUBJECTS

Some of the projects are integrated in corporation with other subjects – they are good at that in the technical upper secondary school programme (1.E).

### THE NATURE OF MATHEMATICS

People find different ways to calculate things, so in some way it is invented by human beings. But they also discovered how to do these things, so it must be a mix (1.N).

### ACTIVITIES OF UNIVERSITY MATHEMATICIANS

If you stay in a university with mathematics, it must be to become more insightful on it. And to provide insight to other people. So in that sense I think they do research in finding new ways for doing the different things, to make everything easier. New mathematical operations.

### *MATHEMATICS IN SOCIETY*

Grace sees mathematics in use 'everywhere' both in a school context where it is applied in projects and in her other subjects, but also in a wider societal context. She uses it in project work, she expects to be using it in her tertiary education (GRACE: Q1-B).

### MATHEMATICS IN SOCIETY

Grace find that mathematics is used more than you should think. In larger corporations they use graphs for evaluating the economy (1.F).

### *MATHEMATICS AND ME*

#### "ME" AS A MATHEMATICS LEARNER

Grace does not indicate to have any severe challenges; in fact she indicates to have no challenges at all. Finding a way to solve a task involves moderate challenges to her, which is slightly more than other potential issues (GRACE: Q1-E).

Many resources for support for mathematical activities are available to Grace (family, friends the internet, books and the teacher) (GRACE: Q1-E).

Grace seems to participate actively in class; both in terms of addressing issues she may be in doubt of and of contributing with things she knows (GRACE: Q1-F).

#### PLANS

One idea for educational plan after graduation is chemical engineering, which would involve some more mathematics. Grace seems to be fine with this (GRACE: Q1-G).

Grace has future possibilities as different as Veterinarian, TV host or chemistry engineer in her ideas (1.L). Grace chose this study programme, not because she knew what she wanted to study afterwards, but because she liked the subjects. She has not yet investigated the role of mathematics in chemistry engineering, but mathematics is everywhere, so there must be some (1.O).

## GRACE'S 1ST YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

The 'supervising' kind of mathematics teaching in the first year of upper secondary school differs from the more competitive and task oriented teaching she knew from lower secondary school. There is more collaboration in class now, but it seems less accepted to have difficulties with mathematics than to be good at it.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is both invented and discovered. I is a dynamic thing that involves developing new mathematics. You do not have to be a genius to study mathematics at university, but a strong genetic disposition helps.

### *MATHEMATICS AND SOCIETY*

Mathematics is everywhere and everybody should learn it. It is used in professions.

### *MATHEMATICS AND ME*

Understanding is important in mathematics, and mathematics is fun when you understand it. Grace sees no real challenges in dealing with mathematics. She expects to be needing it in her desired tertiary education as chemical engineer.

## GRACE'S 3RD YEAR QUESTIONNAIRE

| Q3-A | TRANSITION | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [4] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [369] | [Group Work] [Project Work]; |
| **3** | Did you like mathematics when you went to lower secondary school?[370] | [Yes, it was one of my favourite Subjects] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | In a negative way[371] |
| **New 4d** | In which ways has mathematics changed from first year to third year of upper secondary school? | Not much – a little harder[372] |

*Table 43: GRACE'S 3rd year questionnaire, part A – TRANSITION*

369 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

370 Options: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

371 I en negativ retning

372 Ikke meget - lidt sværere

| Q3-B | FOR SCHOOL | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | It is an important subject to master[373] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | In physics – chemistry etc. - you do calculations for more or less everything [374] |

*Table 44: GRACE'S 3rd year questionnaire, part B - MATHEMATICS AND SCHOOL*

373 Det er et vigtigt fag at kunne

374 I fysik - kemi osv - man laver beregninger i stort set alt

| Q3-C | BEYOND SCHOOL | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | More or less everywhere – in most work places[375] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? [376] | [Both] |
| **10** | What do you think a professional mathematician at a university is doing? | Conducts research in the subject[377] |
| **11** | Would you have to be a genius in order to study mathematics in university? [378] | [I do not know]<br>Both yes and no[379] |

*Table 45: GRACE'S 3rd year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

375 Stort set alle steder - på de fleste arbejdspladser

376 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]

377 Forsker i faget

378 [Yes]; [No]; [I do not know]

379 Både og

| Q3-D | IMRPOVING | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | The teaching should be improved[380] |
| **12b** | What do you think is the best means for improving in mathematics? | A good teacher[381] |
| **12c** | What do you do to improve in mathematics? | (I, red.) Do my homework[382] |
| **13a** | What do you do if you get stuck on a task at school? | (I, red.) Ask the person next to me[383] |
| **13b** | What do you do if you get stuck on your homework? | (I, red.) Contact a classmate[384] |
| **13c** | What do you do if you get stuck on your written assignments? | Same as previous |

*Table 46: GRACE'S 3rd year Questionnaire, part D – STRATEGIES FOR IMPROVING*

380 Undervisningen skal forbedres"
381 En god lærer
382 Læser lektier
383 Spørger personen ved siden af
384 Kontakter de andre fra klassen

| Q3-E | CHALLENGES & SUPPORT | | GRACE |
|---|---|---|---|
| # | *Question* | | *Answer* [385] |
| **14** | What issues involve more challenges to you? | a) Remembering | [3] Moderate challenges |
| | | b) Computing | [4] Few challenges |
| | | c) Figuring out the purpose of a task | [4] Few challenges |
| | | d) Finding a way to solve a task | [3] Moderate challenges |
| | | e) Reading and understanding the textbook | [4] Few challenges |
| **15** | Where can you find support for mathematical activities? | | [From classmates] [Other]: Professional teacher[386] |
| **New 16a** | Do you find that you meet some special challenges related to succeeding in mathematics compared to your classmates? | | No – or yes – students who have parents they can ask[387] |
| **New 16b** | Do you find that you have some some special strengths related to succeeding in mathematics compared to your classmates? | | It varies[388] |

*Table 47: GRACE'S 3rd year Questionnaire, part E – CHALLENGE & SUPPORT*

385 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

386 Prof. Underviser

387 Niks - eller jo - elever der har forældre de kan søge hjælp hos

388 Det svinger

| Q3-F | IN CLASS | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? [389] | [1-3] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [Yes] |
| **17c** | Are you content with that? | No – but we are so many and it is the same two people who are quick as a flash and get the task immediately – I need time[390] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson?[391] | [1-3] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | No – but we are so many and it is the same two people who are quick as a flash and get the task immediately – I need time[392] |
| **19a** | In your class, is it okay to be good at mathematics? | Yes |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Well, no.[393] |

*Table 48: GRACE'S 3rd year Questionnaire, part F - MATHEMATICS IN CLASS*

389 Options: [0]; [1-3]; [4-8]; [More than 8 times]

390 Nej - men vi er så mange og det er de samme to der er lynhurtige og fanger opgaven med det samme – jeg skal bruge tid

391 Options: [0]; [1-3]; [4-8]; [More than 8 times]

392 Nej - men vi er så mange og det er de samme to der er lynhurtige og fanger opgaven med det samme – jeg skal bruge tid

393 Tjaaa nej

| Q3-XA | UNDERSTANDING | GRACE |
|---|---|---|
| # | *Question* | *Answer*[394] |
| **X.1.** | Have you recently experienced to *understand* what you worked with in mathematics? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.3.** | Have you, during upper secondary school experienced that you *understood* what you worked with and then s*ubsequently learned it by heart*? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.5.** | Have you during upper secondary school experienced *understanding something* but *never learning it by heart?* | [I do not know] |
| | If yes, on which occasion? | - |

| Q3-XB | LEARNING BY HEART | GRACE |
|---|---|---|
| # | *Question* | *Answer*[395] |
| **X.2.** | Have you recently experiences having to *learn something by heart*? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.4.** | Have you, during upper secondary school experienced that you had to *learn something by heart*, and then, *subsequently understood it*? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.6.** | Have you during upper secondary school experienced *learning something by heart without ever understanding it*? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.7.** | Additional comments on understanding or rote learning in mathematics | - |

394 Options: [Yes], [No] or [I do not know]

395 Options: [Yes], [No] or [I do not know]

| Q3-XC | A-LEVEL EXAMINATION | GRACE |
|---|---|---|
| **X.8.** | *Topic* | *Answer*[396] |
| **(a)** | The Parabola | [Okay] |
| **(b)** | Exponential | [Readily] |
| **(c)** | Pythagoras | [Readily] |
| **(d)** | Sine and cosine relations | [Okay] |
| **(e)** | Definition of differentiability | [Okay] |
| **(f)** | Sum and product of differential functions | [Okay] |
| **(g)** | Indefinite integral | [Okay] |
| **(h)** | Volume of solid of revolution | [Readily] |
| **(i)** | Differential Equations and their solutions | [Rather not] |
| **(j)** | Vectors in the plane, including scalar product | [Rather not] |
| **(k)** | Lines and planes | [Okay] |
| **X.9.a.** | Which topic is your favourite? - and why? | It depends[397] |
| **X.9.b.** | Which topic would you rather avoid? - and why? | It depends |

*Table 49: Topics for oral and written examinations in A-level Mathematics for Upper Secondary School Programmes in Denmark 2013*[398]

396 Options: [Readily], [Okay], [Rather not], [I do not know]

397 Det er meget forskelligt

398 Topics common to both the Technical (htx) and the General (stx) upper secondary programme examinations are suggested.

| Q3-G | PLANS | GRACE |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Veterinarian, preferably[399] |
| **20b** | What kind of influence would your final marks in mathematics have on your plans (or hope for plans) after upper secondary school? | Great influence[400] |
| **20c** | What kind of influence would your experiences with mathematics in upper secondary school have on your plans after upper secondary school? | I do not know[401] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | - |
| **21b** | Comments: | - |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | - |
| **22b** | Comments | - |

*Table 50: Donna's 3rd year Questionnaire, part G – PLANS*

399 Dyrlæge helst
400 Stor betydning
401 Ved jeg ikke

# GRACE'S 3RD YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 14 March 2013 | 16:27 |

### *3A. STARTING TO MAKE MORE SENSE*[402]

S1: How do you feel about mathematics now?

G1: I feel good. I think I am better than I were in first and second year. So that must be positive.

Q2: When you answered the questionnaire, asking: "*On a scale from 1 to 10, on which 10 is your favourite subject, where is it now?*" It is not amongst the greatest favourite subjects.

G2: No, but it is definitely – now I do not even remember what I answered back then.

Q3: No, but the most important would be how you feel now?

G3: How I fell now? I think it is on "8" right now.

Q4: What has been important for you to reach, where you are now, about it?

402

Q1: Hvordan har du det med matematik?

G1: Jeg har det godt. Jeg synes jeg har fået det bedre end jeg havde det første og andet år. Så det må være positivt.

Q2: Da du svarede på det spørgeskema, der spurgte til, på en skala fra 1 til 10, hvor matematik er dit yndlingsfag, hvor ligger det nu? Det ligger ikke sådan oppe blandt de største yndlingsfag.

G2: Nej, men det er i hvert fald - Nu kan jeg ikke engang huske hvad jeg svarede dengang.

Q3: Nej, men det vigtigste, det er jo sådan set hvordan du har det nu.

G3: Hvordan jeg har det nu? Jamen, så tror jeg det ligger på en 8'er nu.

Q4: Hvad har haft betydning for at nå dertil hvor du er nu med det?

G4: Jeg synes bare at det begynder at give mere og mere mening. Og, ja, hvis man virkelig hænger fast, for jeg synes lidt i starten, at hvis der var noget der var rigtig svært, så gjorde man sig måske ikke ligeså umage for at skulle lære det, fordi man tænkte "*Det skal nok komme hen ad vejen*", og det gjorde det bare rigtig svært, i stedet for at man bare hænger ved og bliver ved med at spørge læreren hvis der er noget man ikke forstår, bliver ved med at spørge sine kammerater. Til sidst har man bare så godt styr på det at det ender med at være en der hjælper de andre, ikke.

G4: I just think it makes more and more sense. And yes, if you really hang on, because I thought in the beginning, that something was really hard, then maybe you did not make that much of an effort for learning it, because you thought: "*It will come, eventually*". And that just made it really hard, in stead of hanging on and keep asking the teacher, if there is something you do not understand, keep asking your peers. Eventually, one is just so much in control of it, that one ends up helping the others.

***3.B. BETTER TEACHING***[403]

Q5: What is the best means for improving in mathematics, and what is the greatest hindrance? Your answer relates a lot to the teaching.

G5: Yes. I think the teaching has improved a little. I have been discontented with my teacher, because I do not think he is that good in teaching. I think, that when he is about to teach us something, he tells about it, but he does not get into it, well, I do not think it makes too much sense. And if you ask him, you do not get an answer that is of any use. What you have to do is to read, read ourself. Make sure to read at home. A lot yourself, so you more or less get a good grip of what he is talking about, and then keep asking. And then it will come, eventually..

403

Q5: Hvad er det bedste middel for at blive bedre til matematik, og hvad er den største hindring for at blive bedre til matematik? Og det handler meget om undervisningen, det svar du giver.

G5: Ja. Jeg synes undervisningen er blevet lidt bedre. Jeg har været lidt utilfreds med min lærer, fordi jeg synes ikke at han er særlig god til at undervise. Jeg synes, at når han skal lære os noget, så fortæller han ligesom om det, men han kommer ikke rigtig ind i det, altså, jeg synes ikke rigtig det giver så meget mening. Hvis man så spørger ham, så får man ikke rigtig noget svar man kan bruge til noget. Det man så skal gøre, det er jo så at læse, at læse selv. Sørge for at læse derhjemme. En masse selv, sådan så man (...) får ordentligt forhold på det han snakker om, og så ellers blive ved med at spørge. Og så kommer det til sidst.

### *3.C. GETTING ANSWERS* [404]

Q6: Where do you get the better answers to your questions?

G6: Well, from my classmates. Or in the book [Carstensen & Frandsen, 1998]

Q7: Can you always manage yourself? You are thirty people in the class, so you could ask the teacher, but probably you would not be the only one?

G7: What I experience more frequently is that one ask each other in class, because there are some students who just knows everything before they learned it, almost. It has just become routine because our teacher has been teacher in so many years that...I just do not think you get that many answers that are useful, so I think it is in class one should ask.

Q8: What about the textbook? You are fine understanding it, it appears?

G8: Yes. It explains and set up some boxes with: "*These formulae, can you see this and this?*" and it gives examples, so it is quite easy to understand. So it is a fine book.

404

Q6: Hvor får du de bedste svar på dine spørgsmål?

G6: Jamen, det gør jeg hos mine klassekammerater. Eller i bogen (Carstensen & Frandsen, 1998/1999, red.).

Q7: (…) Kan I altid klare den selv? (...) I er jo tredive i klassen, så man kan sige, I kan godt spørge læreren, men det er der vel også andre der vil.

G7: (...) Det jeg oplever mest, det er at man spørger hinanden i klassen, for der er nogle som bare virkelig virkelig bare kan det hele, inden de har lært det, nærmest, ikke. Så det er nemmere at spørge dem, fordi de kan forklare det på en måde som, altså… Det er blevet lidt rutine for vores lærer fordi han har været lærer i så mange år at… Jeg synes bare ikke man får så mange svar man kan bruge til noget, så jeg synes det er i klassen man skal spørge.

Q8: Hvad med matematikbogen (Carstensen & Frandsen, 1998/1999, red.)? Den har du det fint med at forstå, ser det ud til.

G8: Ja. Jaja, men den forklarer det og stiller det op i nogle kasser med "De her formler, kan du se dét og dét" og kommer med nogle eksempler, så det er ret nemt at forstå. Så det er en fin bog.

### *3.D. LEARNING BY HEART/ ROTE LEANING?*[405]

Q9: Have you ever needed to learn something by heart, because you did not have time to understand, for example?

G9: No. I am not sure I understand the question.

Q10: No, but it may not be relevant in this context.

G10: I do not think so, because then you look it up in your notes and such, we take notes on everything said. Then you find it there. But there are some things, like sine and cosine and this and the other, some fundamental formulae that one knows because they are used that often. Then you might as well learn them by heart.

### *3.E. PROVING*[406]

Q11: This thing about proving in Mathematics, how much emphasis does it have in the manner you are taught?

G11: But what do you mean about proving? From an experiment, maybe? Set up something and prove it?

Q12: I am considering, well, if you are to prove theoretically that some Mathematics is true, that a Mathematical relation is valid?

G12: That is very rare. It is. I wish we had had some more of it, because those questions, when we get to the examination, then none of us knows what to write. So it is kind of silly that we do not do more of it.

---

405

Q9: Ja. Har du nogensinde brug for at lære noget udenad, fordi der ikke er tid til at forstå det, for eksempel?

G9: Nej. Jeg er ikke sikker på jeg lige forstår spørgsmålet.

Q10: Nej, men det er heller ikke sikkert det overhovedet er relevant i den her sammenhæng.

G10: Det tror jeg ikke, fordi så slår man det op i sine noter og sådan nogle ting, vi noterer jo alting der bliver sagt. Så finder man det der. Men der er nogle ting, alt så noget sinus-cosinus og det ene og det andet, nogle bestemte grundlæggende formler som man ligesom kan nu fordi man skal bruge det så tit. Så kan man ligeså godt lære det udenad.

406

Q11: Det dér med at bevise ting i matematik (...) hvor meget fylder det i den måde I har faget på?

G11: (...) Jamen hvordan med "bevise"? Altså, ud fra et forsøg, måske? Opstille noget og vise det på den måde?

Q12: Jeg tænker på, jamen, at hvis man skal, sådan, bevise teoretisk at noget matematik er sandt, at en eller anden matematisk sammenhæng, at den gælder.

G12: Det er meget sjældent. Det er det. Jeg ville ønske vi havde noget mere af det, fordi de spørgsmål, når de kommer i prøven, så er der ikke rigtig nogen af os der ved hvad vi skal skrive. Så det er lidt dumt vi ikke har mere om det.

### *3.F. MATHEMATICS AS SUCH*[407]

Q13: The Discipline of Mathematics, what is it? What kind of game are you playing, when you are dealing with Mathematics? What is it about?

G13: But, it is a big broad game, if one can say so. I think you use it for more and more, Mathematics. One of the most important subjects to learn. And I think you use it in many subjects, across

Q14: You use it in… Physics and...

G14: And Chemistry.

Q15: And Chemistry.

G15: But you use it a lot in Chemistry. But then you use it by means of Chemistry formulae. But that is also a kind of Mathematics, is it not. And likewise with Mathematics, I think it is everywhere. Maybe not in Philosophy, that much, but otherwise it is something you really use.

### *3.G. MATHEMATICS IN SOCIETY*[408]

Q16: But the Mathematics, you use, can it be used outside school as well?

G16: Yes, maybe it can. It is not too often that you experience that you need to find the dot product or vectors in space, but of course you can use it otherwise, outside school.

---

407

Q13: Fagområdet matematik, altså, hvad er det for en størrelse? (…) Hvad er det for en leg man leger når man har med matematik at gøre? Hvad går det ud på?

G13: Jamen, det er en stor bred leg, hvis man kan sige det sådan. Jeg synes man bruger det til mere og mere, matematikken(...) Noget af det vigtigste fag at lære. Og jeg synes man bruger det i mange fag, på kryds og tværs, (...)

Q14: Bruger du det i…fysik og ...

G14:: Og kemi.

Q15: Og kemi.

G15: Man bruger det i hvert fald meget i kemi. Men der bruger man det jo via kemi-formler. Men det er jo stadig en slags matematik, ikke. Og sådan, ligeledes er det også med matematik, så jeg synes det er der i alt. Måske ikke i filosofi ligeså meget, men ellers synes jeg virkelig det er noget man bruger.

408

Q16: Men den matematik I bruger (...) Den kan man bruge udenfor skolen også?

G16: Ja, det kan man måske. Det er ikke så tit man lige oplever at man skal finde skalarprodukter eller vektorer i rummet, men selvfølgelig kan man også bruge det ellers, udenfor skolen.

### *3.H. MATHEMATICS AS SUCH II*[409]

Q17: How emerges Mathematics?

G17: Yes, it is something we created. It is something we have arrived at, and kind of determined that it is like this and like that, and if we do like this, it must give that. I think some of it is nature, it is something that has always been logical, but it is something that we have created, human beings have created in the last instance.

Q18: I ask: "*What is a professional mathematician at a university doing*?" "*Doing research in the subject*". But what is it about, the doing research in the subject? What is there to do?

G18: I think, among other things, it is to conduct research in the subject, well, to come up with more, to elaborate on the Mathematics, to find, maybe, easier solutions, methods, if it is a subject, as I think it is, Mathematics, that we human beings have come up with, then there must be someone who keeps inventing, and finding easier ways to do things. But yes, a Mathematician at a university, they can also do other things. Now, I want to become a food technology engineer, and it is amongst other things also Mathematics, also a large part of that subject. And Chemistry. So you use it a lot.

409

Q17: Hvordan opstår matematik? Har den altid været der, eller er det noget vi har lavet?

G17: Ja, det er noget vi har skabt. Det er noget vi i hvert fald har fundet frem til, og så ligesom har fastlagt, at det er sådan og sådan, og hvis vi gør sådan, må det give sådan. Jeg tror noget af det er noget natur, det er noget som altid har været logisk, men det er noget vi har skabt, mennesket har skabt, i sidste ende.

Q18: (…) Jeg spørger: "Hvad laver en professionel matematiker på et universitet?" "Forsker i faget". Men hvad går det ud på, at forske i faget? Hvad er der at lave?

G18: Jeg tror blandt andet det er at forske i matematikken, altså, (...) finde på mere, uddybe det, finde måske nemmere løsninger, fremgangsmåder hvis det er et fag, som jeg mener at det er, matematik, som vi mennesker har fundet på, så må der ligesom være nogen der bliver ved med at finde på, og finde på nemmere måder at gøre det på, ikke. Men ja, en matematiker på et universitet, de kan også beskæftige sig med andre ting. Nu vil jeg gerne selv være noget fødevarevidenskabs-ingeniør, og det er blandt andet også matematik, også en stor del af det fag. Og kemi, ikke. Så du bruger det jo meget.

### *3.I. MATHEMATICS AND ME*[410]

Q19: Is that something new you have come up with [Food Technology Engineer]?

G19: Yes.

Q20: How new?

G20: It was Thursday, a week ago. I wanted to become a Chemistry engineer. And this was kind of a branch of Chemistry engineering. Some food/nutrition Engineering also, that I find really interesting.

Q21: What is it that you liked about that subject?

G21: I just think it is exciting. Also the thought of earning a lot of money at the end. It is of course nat what should determine it, but a civil engineer education, they are needed, one should think of whether you can get a job, when you are graduated, and that is hard to predict, when it is five years ahead. (…)

Now, I was watching the news with my mother yesterday, and all the time it was about, "*TOOTHPASTE, what is it doing to the body?*" So this is a field in constant development and it is not going to change, I think. One will always predict how you can make food more healthy. So I think there is a future in it. So I chose to focus on it.

410

Q19: Er det noget nyt du har fundet ud af? (at Grace gerne vil være fødevareingeniør)

G19: Ja.

Q20: Hvor nyt?

G20: Det var i torsdags for en uge siden. Jeg kunne godt tænke mig at være kemi-ingeniør, nemlig. Og så var det her lidt en gren af kemi-ingeniøren. Noget fødevare/ernæringsingeniør, også, som jeg synes lyder rigtig rigtig spændende. Så det er jeg ret opsat på at jeg skal. (...)

Q21: Hvad var det det du godt kunne lide ved det fag?

G21: Jeg synes bare det er spændende. Og så også tanken om (muntert) at man kommer til at tjene mange penge til sidst, ikke? Nej, det er selvfølgelig ikke det der skal afgøre det, men en civilingeniør-uddannelse, dem er der brug for, man skal tænke på om man kan få et job når man er færdig, det er svært at sige når der er fem år til. (…)

Nu sad jeg og så TV-avis med min mor i går, og hele tiden, så handlede det om, "Colgate, hvad går det ind og gør ved kroppen?" (...) så det her, det er et fag der hele tiden udvikler sig, det kommer aldrig til at gå den anden vej, tror jeg. Man vil altid forudsige, hvordan kan du gøre maden sundere (...). Så det synes jeg, der er fremtid i det. Så derfor vælger jeg at fokusere på det.

### *3.J. MATHEMATICS IN FUTURE LIFE*[411]

Q22: Will you use the mathematics you learn now, when you continue there?

G22: Yes. I will. Probably not on the same level. Well, I do not know, if it gets harder, but, the requirements are actually that you have studied A-level mathematics, so I will probably use it. But it is probably mainly chemistry I will use. But also mathematics.

Q23: Is it then just the exam you will need, or will you use the instruments you have now?

G23: I think I will be using the instruments. Probably not in the same way, but yes, some of it I will.

### *3.K. MATHEMATICS AT SCHOOL/DIDACTICAL CONTRACT*[412]

Q24: So you have not been scared away from having to do with mathematics after this?

G24: No. Not at all, I only think it has become better. I was probably a little down in the first and the second year, because I was not satisfied with the teaching, compared to what I was used to. But I think it has improved after we talked to our teacher and such. I think it has improved and it also makes more sense to me.

---

411

Q22: Kommer du til at bruge den matematik du lærer nu, når du skal videre dér?

G22: Ja. Det kommer jeg til. Nok ikke på samme plan. Jamen, det ved jeg selvfølgelig ikke, om det går hen og bliver sværere, men… Kravet er kun… Næh, kravet er faktisk at man skal have haft matematik på A-niveau, så det kommer jeg nok til at skulle bruge, ja. Men det er nok mest kemien jeg skal vægte på dér. Men også matematik.

Q23: Er det så bare det at have eksamen i det du skal bruge, eller kommer du til at bruge de redskaber du har nu?

G23: Jeg tror jeg kommer til at bruge redskaberne. Nok ikke på samme måde, men jo, noget af det gør jeg.

412

Q24: Så du er ikke blevet skræmt fra at have med matematik at gøre efter det her?

G24: Nej. Overhovedet ikke, jeg synes kun det er blevet bedre. Jeg var godt nok lidt nede ved 1.-2., for jeg syntes godt nok ikke at jeg var tilfreds med den måde undervisningen var på i forhold til hvad jeg var vant til. Men jeg synes det er blevet bedre, efter at vi har snakket med vores lærer om det og sådan nogle ting. Jeg synes det er blevet bedre, og det giver mere og mere mening for mig.

### *3.L. MATHEMATICS AT SCHOOL/SOCIOMATHEMATICAL NORMS*[413]

Q25: What did you gain from talking to the teacher?

G25: He had this thing about that if you handed in a task and found the right solution and everything, but that you had not solved it in the same way as him, then it was wrong. He is more willing to compromise, and earlier, if we were taught something on the blackboard, or theory, then he told us something, and then we were supposed to go home and read it afterwards. And I think that...I do not get that order, in stead of reading at home and then go over it at school, because you cannot ask to something you have not read. So that has been changed, and also...I just think the whole way of teaching has improved.

### *3.M. FAVOURITE SUBJECT SCALE*[414]

Q26: If I return to the time, when you answered the questionnaire. At this time, you rated Mathematics 4, on a scale from 1 to 10. Was there something specific at this point of time, that gave it a dip?

G26: I just think it was because I was so unhappy with the teaching. I just did not think it made any sense. The whole class just sat there and were speechless, besides the three with engineering parents, who can teach them at home. I was just sad about that. So it went from a favourite subject to be something for which you had to pull yourself together. And that is a little sad, when you have it as an A-level subject, which is taught many times a week.

---

413

Q25: Hvad fik I ud af at snakke sammen med læreren?

G25: (...) Han havde det meget med, hvis man afleverede opgaver og sådan nogle ting, og man fik det rigtige resultat og løsning og det ene og det andet, men at man ikke havde løst opgaven som han gjorde det, så var det forkert (…) han går mere på kompromis, og så, førhen, der skulle vi lære om noget på tavlen, eller teori, så fortalte han os om noget, og så skulle vi bagefter gå hjem og læse om det. Og det synes jeg, at… Jeg kan ikke forstå den rækkefølge, i stedet for at man læser om det derhjemme, og så gennemgår man det i skolen, fordi du kan ikke spørge ind til noget du ikke har læst om. Så det blev der også byttet om på, og så… Jeg synes bare at hele undervisningsmåden blev meget bedre.

414

Q26: Hvis jeg lige vender tilbage til tidspunktet på den hér spørgeskemabesvarelse. På det her tidspunkt var du nede og give matematik en 4 på en skala fra 1 til 10. Var der noget bestemt på det her tidspunkt, som gav den et dyk?

G26: Jeg tror bare det var det at jeg var så utilfreds med undervisningen. Jeg syntes bare ikke det gav nogen mening. Hele klassen sad bare og var helt paf, næsten, udover de tre der bare har ingeniørforældre, der kan lære dem derhjemme. Det var jeg bare ærgerlig over. Så der gik det fra at være yndlingsfag til virkelig at være noget hvor man bare skulle tage sig sammen. (...) Og det er lidt ærgerligt når det er et A-fag, som man har så mange gange om ugen.

## THEMES IN GRACE'S 3RD YEAR

*From investigating Grace's account of her 3rd year experiences with mathematics, as they are expressed in the questionnaire answers and in the interview, I will now comment on Grace's third year.*

### *MATHEMATICS AT SCHOOL*

Apparently the subject has not changed much since the 1st year, except that she finds it somewhat harder (GRACE: Q1-A).

There is a discrepancy between the textbook, which is classical, sharp, precise, oriented towards pure mathematics, build up with definitions, theorems and proofs, and the focus of the teaching: Problem oriented, project oriented and with a focus on applications.

It is her impression that being good at mathematics is well accepted in class, whereas having challenges in mathematics not really is accepted (Q1-F).

### DIDACTICAL CONTRACT

In relation to Mathematics at School, Grace states that she sees a need for improvement of the teaching (GRACE: Q1-D). Her own effort concerns doing her homework, and when she needs help, she contacts other students from class (GRACE: Q1-D).

Grace describes how she and the class talked to the teacher about the teaching. One part of the problem related to the fact that the teacher would explain some theory, and afterwards the class would go home and read up on it. And Grace did not appreciate that order. So now it has been changed, and they first read about the material, before it is explained in school (3.L). And she found this order much better.

She also mentions that the teacher has taken a less unbending approach in terms of accepting solution methods deviant from what he initially had in mind (3.L).

In the 1st year, we learned that Grace appreciated a more competitive ans task oriented teaching approach to the less traditional style she met in technical upper secondary school, which to a greater extend shared the elements of e.g. inquiry based education (see e.g. Blomhøj & Artigue, 2013, for a further elaboration of the concept). We see the same tendency here; that Grace demands a teaching approach closer related to the style she met in lower secondary school, and that turning things upside down makes her frustrated and makes her feel that she learns less.

In technical upper secondary school, mathematical proofs are not on the agenda too often, Grace explains. She regrets that, mainly from the rationale that it will be demanded at the final examinations (3.F).

### *MATHEMATICS AS A DISCIPLINE*

#### APPLICATIONS

As in the first year, Grace still indicates mathematics as relevant to her other subjects in her questionnaire answer (GRACE: Q3-B). In the interview she explains that in general in technical upper secondary school, mathematics is applied in and with many other subjects, especially she emphasises chemistry (3.G).

### *MATHEMATICS IN SOCIETY*

Grace finds mathematics to be highly useful outside school:"*It is an important subject to master*", she writes (GRACE: Q3-B). Also, she finds that mathematics is relevant at most places of employment (GRACE: Q3-C). But Grace also notes, that you may not experience very often that you need to compute the scalar product or vectors in Euclidean space outside a school setting (3.H).

### *MATHEMATICS AND ME*

Grace finds it to be a serious task to go to school – she does not identify herself as a party animal.

### CHALLENGES AND HELP

Grace still indicates that the challenges are most by a moderate level. It is new, that she finds 'remembering' a moderate challenge, next to finding solution strategies. Otherwise, the issues suggested only pose few challenges to her (Q1-E).

It is a major investment for her to choose A-level mathematics. She has a certain flair for it, but she cannot compete with the fastest in her class. And as we know from her ideas of good mathematics teaching, being fast is quite important to her feeling successful. She has to work in order not to loose enthusiasm. She wants it to be well decided that she chose A-level mathematics in technical upper secondary school.

However, Grace now gets help from a "professional teacher" as a supplement to getting help from her classmates. In class, two students are very fast in 'getting it', which makes it hard to contribute if you, like Grace, need time (Q1-F).

### RATING & TEACHING

Grace's rating of mathematics on the favourite subject scale has had its ups and downs during the three years (Illustration 2, p. 305).

Halfway through the third year, in the very late fall Grace now seems to appreciate mathematics less than earlier; her rating of it has dropped with four steps from [8] on the favourite subject scale to [4] (GRACE: Q1-A). At the interview, in the latter part of the third year, she elaborates on the reasons for this change. Grace tells that it related to her discontentedness with the teaching. It did not make sense to her,

she said, and, according to Grace, neither to the rest of the class, except for the three students with engineering parents, who could learn it at home. This influenced her view of the subject, which went from being a favourite subject to something for which one just had to pull oneself together to deal with (3.M).

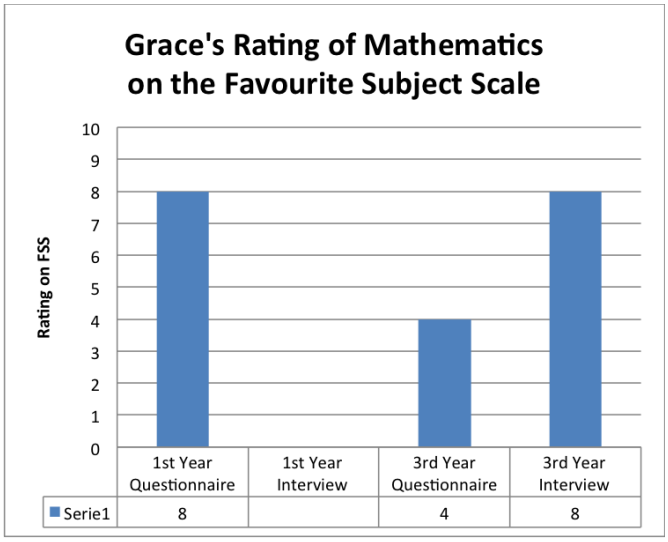


*Illustration 2: Grace's rating of mathematics as it developed from the 1st year questionnaire to the 3rd year interview.*

At the time of the interview, however, she now feels better about mathematics. Now she would rate it [8] again, just as in the 1st year questionnaire. She feels good about mathematics now. Better than in first and in second year. She says it just makes more and more sense to her (3.A).

Understanding mathematics is important to Grace; maybe even a fundamental need. This may explain the variation in her rating of mathematics over time: It could relate to a frustration that others understand it immediately.

In the third year questionnaire it did not seem as if she were too keen on answering the questions. It may relate to the fact that it was given during the study

programme project time, or maybe due to a down period in her appreciation of the subject.

### PLANS

At the end of the third and last year of upper secondary school Grace has now finally arrived at a conclusion concerning what she wants to study. She ended up deciding to study Food and Nutrition Engineering (3.J), which will involve some mathematical tools (3.K). It is a branch of chemical engineering, which, among other quite different ideas, was among her ideas since the first year (3.K., 1.L). She mentions the robustness of job opportunities as a ground for this choice, and she is fine with the fact that the study involves a good deal of mathematics.

These plans appear quite suddenly, just before the third year interview. At the time of the 3rd year questionnaire, she mentioned considering studying to become a veterinarian after graduation (Q3-G). This option was already mentioned in the fist year interview.

## GRACE'S 3RD YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics at school was subject to some turbulence the third year due to discrepancy between the students' and the teachers ideas of good mathematics teaching. A dialogue in class helps coming to an agreement everybody seemed to be able to accept. Given that proof will be required for the final exam, this should have a clear presence in the teaching.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is renewed and new sides of it is challenged; a dynamic view of mathematics. Being good at mathematics means being fast in grasping things

### *MATHEMATICS AND SOCIETY*

Mathematics is highly useful outside school as well.

### *MATHEMATICS AND ME*

Grace likes to understand mathematics, but that experience has been varying over the year. Her view of mathematics is very sensitive to the current state of affairs in school. But she is not scared away from mathematics, and it is part of her future plans in s STEM-study programme.

## GRACE'S BELIEFS TRANSPOSITION

*STABLE FOR GRACE:*

Mathematics is everywhere, and it is highly useful. Mathematics is not seen as a monument already there, but it is a discipline in continuous development. She is not scared off from mathematics. Her ideas of what Mathematics at School should be like was formed in the later years of lower secondary school.

*SUBJECT TO CHANGE:*

Grace's appreciation of the teaching at Technical upper secondary school is not stable. Her experience of being on top of things relates to her understanding of them. And if she does not understand, the teaching is held responsible.

# *THE CASE OF BRANDON*

*Brandon is a male student from BETA Upper Secondary School, in a mathematics-physics study programme which involves studying A-level mathematics. Brandon kept his rating of mathematics on [10] both in the 1st and in the 3rd year questionnaire. In the 1st year questionnaire he indicated medicine as an idea for further education and he mentioned it again in the third year questionnaire. Brandon is the only student amongst the case informants who seems to have a background as a second generation immigrant or something comparable to that.*

| Brandon | Date for Questionnaire | Date for interview |
|---|---|---|
| 1st Year | 25 November 2010 (+ supplement 16 December 2010) | 11 April 2011 |
| 3rd Year | 6 December 2012 | 6 March 2013 |

*Table 51: Dates for Questionnaires and Interviews*

## BRANDON'S 1ST YEAR QUESTIONNAIRE

| Q1-A | TRANSITION | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [10] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [415] | [Working on your own] |
| **3** | Did you like mathematics when you went to lower secondary school? [416] | [Yes, it was one of my favourite subjects]<br>Comment: "AWESOME" |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | It has become more exciting. Greater challenges, but not enough, still. It is far too easy so far.[417] |
| **4b** | Is there anything you liked better before?' | No, the mathematics part of lower secondary school was for too easy.[418] |
| **4c** | Is there anything you like better now? | I hope the future will offer more challenges.[419] |

*Table 52: BRANDON'S 1st year questionnaire, part A – TRANSITION*

415 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

416 Options:: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

417 Det er blevet mere spændende. Større udfordringer, men stadig ikke store nok. Det er alt for nemt, so far

418 Nej, folkeskolens matematik del var alt for let

419 Jeg håber at fremtiden vil byde på større udfordringer.

| Q1-B | FOR SCHOOL | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | An important subject that one can utilise everywhere in life. "We all yov math everyday"(sic.)[420] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **6** | What made you choose a study programme involving A-level mathematics? | My huge interest in mathematics made me realise that I wanted to keep up with it.[421] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer[422] | The question is poorly phrased, but mathematics and physics relate a lot to each other.[423] |

*Table 53: BRANDON'S 1st year questionnaire, part B - MATHEMATICS AND SCHOOL*

420 Et vigtigt fag man kan bruge alle stedet i livet. "We all yov math everyday" (sic.)

421 Min store interesse for matematik fik mig til at indse at jeg vil arbejde mere med det.

422 Begrund dit svar:

423 Spørgsmålet er dårligt formuleret, men matematik og fysik hænger meget sammen.

| Q1-C | BEYOND SCHOOL | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | Handling money, time is mathematics (time of day), planning, games (like lotteries in terms of probability) also in e.g. horse racing, here you can "calculate" which horse might win from examining their diets, exercise etc.[424] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? | [Both] |
| **10** | What do you think a professional mathematician at a university is doing? | Deals with mathematical methods that can be applied in everyday life. Some so-called algorithms, theories and other things.[425] |
| **11** | Would you have to be a genius in order to study mathematics in university? | [No] |

*Table 54: BRANDON'S 1st year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

424 Håndtere penge, tiden er matematik(klokken), Planlægning, spil (som lotto med hensyn til sandsynlighed) også i fx hestevæddeløb, her kan man bl.a. "regne" ud hvilken hest, der måske vil vinde ved at se på kost, træning mm.

425 Beskæftiger sig med matematiske metoder, som kan bruges i hverdagen. Nogle såkaldte algoritmer, teorier mm.

| Q1-D | IMRPOVING | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | The greatest "obstacle" is to take an interest in it. If one succeeds in that, everything else will succeed as well. Still, one has to toil.[426] |
| **12b** | What do you think is the best means for improving in mathematics? | Practice makes perfect[427] |
| **12c** | What do you do to improve in mathematics? | Study mathematics, do my homework, take notes. ← [I] like challenges[428] |
| **13a** | What do you do if you get stuck on a task at school? | I give it a try. You can also ask the teacher, but challenges are good for the brain.[429] |
| **13b** | What do you do if you get stuck on your homework? | I give it a try. You can read up on anything.[430] |

*Table 55: BRANDON'S 1st year Questionnaire, part D – STRATEGIES FOR IMPROVING*

426 Den største "hindring" er at interessere sig for det. Lykkes det, lykkes alt andet, dog skal man knokle.

427 Øvelse gør mester

428 Læser matematik , laver lektier, tager noter. ← kan godt lide udfordringer.

429 Jeg prøver mig frem. Spørge læreren kan man også, men hjernen har godt med udfordringer

430 Prøver mig frem. Man kan læse sig til alt.

| Q1-E | CHALLENGES & SUPPORT | | BRANDON |
|---|---|---|---|
| # | *Question* | | *Answer* [431] |
| **14** | What issues involve more challenges to you? | a) Remembering | [3] Moderate challenges |
| | | b) Computing | [5] The fewest challenges |
| | | c) Figuring out the purpose of a task | [4] Few challenges |
| | | d) Finding a way to solve a task | [5] The fewest challenges |
| | | e) Reading and understanding the textbook | [5] The fewest challenges |
| **15** | Where can you find support for mathematical activities? [432] | | [From classmates]; [Other places]: The Internet, the library → books |
| **16** | Did you parents take the Upper Secondary School Leaving Certificate? [433] | | [None of them did] |

*Table 56: BRANDON'S 1st year Questionnaire, part E – CHALLENGE AND SUPPORT*

431 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

432 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From classmates]; [Other places] If other places, from where or from whom?

433 [Yes, my mother did]; [Yes, my father did]; [Both my parents did]; [None of them did]

| Q1-F | IN CLASS | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? | [0] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [No] |
| **17c** | Are you content with that? | I am content with my effort, but my self-confidence is low.[434] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? | [More than 8 times] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [I do not know] |
| **18c** | Are you content with that? | One cannot answer that question. It depends.[435] |
| **19a** | In your class, is it okay to be good at mathematics? | Yes, of course one is allowed to be good. [436] |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Yes, it is human, everybody has difficulties in something – others maths, others languages[437] |

*Table 57: BRANDON'S 1st year Questionnaire, part F - MATHEMATICS IN CLASS*

434 Jeg er tilfreds med min indsats, men lav selvtillid

435 Det kan man ikke svare på. Det afhænger.

436 Ja, selvfølgelig må man være god.

437 Ja, det er humant. Alle har svært ved noget - andre math andre sproglige

| Q1-G | PLANS | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Medical doctor, but that requires a high average of grades. I also have a good mind to some mathematics or physics, but unfortunately I do not know what it should be.[438] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [Yes] |
| **21b** | Comments: | It should be nice to work with mathematics since it is something one takes an interest in.[439] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [No] |
| **22b** | Comments | No. "Math is awesome"[440](sic.)<br>Other comments: It should be made harder[441] |

*Table 58: BRANDON'S 1st year Questionnaire, part G – PLANS*

438 Læge, men det kræver et højt gennemsnit. Jeg kunne godt tænke mig noget om matematik og fysik, men ved desværre ikke hvad det skal være.

439 Det kunne være rart at arbejde med matematik, da det er noget man interesserer sig for.

440 Nej, Math is awesome (sic.)

441 Det skal gøres sværere

# BRANDON'S 1ST YEAR INTERVIEW

The interview took place at BETA upper secondary school in a vacant classroom and recorded on a digital recorder.

| Date | Duration (mm:ss) |
|---|---|
| 11 April 2011 | 24:55 |

### *1.A. TRANSITION*[442]

Q1: How do you think mathematics has changed from when you went to lower secondary school, till now, when you are in upper secondary school?

B1: At the beginning, it was merely repetition we had, so it was just the same. Well, I do not find it hard. I keep up and do my homework, so I am not challenged. Actually, I find it easy. I expected it to be harder. But it is not.

Q2: How come it is not harder? Is the teaching done in a way that makes it easier for you?

B2: I do not know, I just find it easy. It is not so much the teaching, it is rather just that it comes to me easily. And then, it is just formulae, in which you enter numbers and isolate. So it is not that hard.

442

Q1: Hvordan du synes, at matematik har ændret sig fra du gik i folkeskolen, og så til nu, hvor du går i gymnasiet.

B1: Fra starten var det bare repetition, vi havde, så det var bare det samme. Men senere, jeg ved ikke om, det er blevet sværere, det er nok det samme. Altså jeg synes ikke det er svært. Jeg følger med og laver lektier, så jeg har ikke svært ved det. Faktisk nærmere synes jeg, at det er let. Jeg havde regnet med, at det ville være sværere. Men det er det bare ikke.

Q2: Hvordan kan det være, at det ikke er sværere? Er det, har undervisningen, er undervisningen på en måde, så det gør det lettere for dig eller hvad er det, der?

B2: Jeg ved ikke, jeg synes bare jeg har let ved det. Det er ikke så meget undervisningen, det er bare mere med at jeg har let ved det. Og så er det jo bare formler, som man skal sætte tal ind og så isolere. Så det er ikke så svært.

***1.B. COOPERATION IN LOWER SECONDARY SCHOOL***[443]

Q3: Do you cooperate differently here, compared to lower secondary school?

B3: Well, here we work together more. Because in lower secondary school, it was more independent, where you solved the things yourself, and if you had problems, then you asked the teacher. Here, you can ask your neighbours and you work in groups, and when we did the STAR-project, we also worked in groups. Then you could ask if you wanted help, you could ask one from your group. So there is more cooperation here.

***1.C. PROOFS***[444]

Q4: Some people find that there is a difference, that in lower secondary school, it was more superficial, and now you go in depth, and that makes it easier to understand.

B4: Well, there are proofs here, there was not in lower secondary school. In lower secondary school, then it was merely: You have this [sound as if BRANDON hits the table] and that is how it is. Here, you prove why exactly it is this theorem you should calculate. So that is the difference, anyway.

Q5: How do you like that you also deal with proofs now?

B5: It is more exciting, because then we find out why it is exactly this theorem we are to apply.

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443

Q3: Arbejder I sammen på en anden måde i klassen nu, end i folkeskolen?

B3: Altså der er meget mere samarbejde her. Fordi i folkeskolen var det mere sådan selvstændigt, hvor man lavede tingene selv, og hvis man havde problemer, spurgte man læreren. Her kan man spørge sine sidekammerater, og man arbejder i grupper, og da vi lavede STAR-projektet, så arbejdede vi også i grupper. Så man kunne spørge, hvis man ville have hjælp, så kunne man spørge dem fra sin egen gruppe. Så det er mere samarbejde her.

444

Q4: Der er nogen, der synes, at der har været den forskel, at i folkeskolen så var det mere overfladisk, og nu kommer man mere i dybden, og det gør det nemmere at forstå.

B4: Altså der er beviser her, det var der ikke i folkeskolen. I folkeskolen, der var det bare: I har den her [lyd som om BRANDON slår i bordet] og sådan er det. Her, så beviser man, hvorfor det lige præcis er denne her sætning, man skal regne på. Så det er forskellen i hvert fald.

Q5: Hvad synes du om, at I også beskæftiger jer med beviser nu?

B5: Det er mere spændende, fordi så ser vi, så finder ud af, hvorfor det lige præcis er denne her sætning, vi skal bruge her.

### *1.D. COOPERATION IN UPPER SECONDARY SCHOOL*[445]

Q6: Have I understood correctly that you choose yourself whether you want to work with other people or you prefer to work on your own – most of the time?

B6: Yes, you can. Well, if you do not want to work with other people in mathematics class, then you can just sit by yourself and do things. But if you have problems, then you have to ask the teacher. I prefer to work with other people anyway. But on the other hand, if you work with somebody, then there are also those who just want the answers, from the class. So that does not work.

Q7: When you answered the questionnaire, you wrote that you preferred working on your own, but that has been a while...

B7: ...I think it has improved...

Q8: … in upper secondary school, so now you prefer working with others?

B8: Yes, especially if it is something I choose myself. For example, when we did the STAR-projects, then I did not choose myself, and I did not like that. Whereas, when you choose yourself, and you know how to do it, and you do not chit-chat, then it is fine.

445

Q6: Er det rigtig opfattet, at man sådan selv kan vælge, om man vil arbejde sammen med andre, eller om man bare vil sidde selv og arbejde med noget - for det meste?

B6: Ja, det kan man godt. Altså hvis du ikke gider arbejde sammen med nogen i matematiktimen, så kan du også bare sidde for dig selv og lave tingene. Men så hvis du har problemer, så er du nødt til at spørge læreren. Jeg foretrækker i hvert fald at arbejde sammen med andre. (...) Men på den anden side, så hvis man arbejder sammen med nogen, så der er også nogen, der bare vil have svarene, fra klassen. Så det går ikke.

Q7: Da du svarede på spørgeskemaet, der skrev du, at du var særlig glad for at arbejde alene. Men nu hvor at der er gået lidt mere tid...

B7: ...jeg synes, det er blevet bedre...

Q8: ... i gymnasiet, så kan du bedre lide at arbejde sammen med nogen?

B8: Ja, især hvis det er noget, jeg selv vælger. Fx da vi havde STAR-projekt, der valgte jeg ikke selv, og det var jeg ikke glad for. Hvorimod hvis man vælger selv, og man kan finde ud af det, og man ikke taler sammen, så er det godt.

*1.E. GETTING CHALLENGES*[446]

Q9: But all in all you think it has become more exciting because there are greater challenges, but you could manage even greater challenges, you say?

B9: Yes, they are not that tough, the challenges. Because, well, until now there has been one assignment with one task, that was difficult. The others have been some easy ones, in which you were to plug in a number and isolate and such.

Q10: Now, you say that you could need some harder challenges. How could you get that?

B10: I do not know, really. Well, the teacher could give us some harder assignments, we could deal with something that was harder, but that will not be until second or third year and such. So I just have to accept, that it this is the way it is.

*1.F. DEALING WITH CHALLENGES*[447]

Q11: What can one do to become even better at dealing with mathematical challenges? Then you write: "Practice makes perfect!"

B11: Yes, one can do different tasks all the time dealing with the same content. Then you improve. But at a some point one also gets tired of doing the same tasks. So...

446

Q9: Men alt i alt, så synes du det er blevet mere spændende, fordi der er lidt større udfordringer - men du kunne godt klare nogle endnu større udfordringer, synes du?

B9: Ja, de er ikke så store, udfordringerne. Fordi, altså, indtil videre har der kun været en problemregning, som hvor der har været en opgave, der har været svær. De andre, det har bare været nogle lette nogle, hvor man skulle indsætte tal og isolere og sådan.

Q10: Nu siger du, at egentlig kunne du godt bruge nogle større udfordringer. Hvordan kan du få det fx?

B10: Det ved jeg faktisk ikke. Altså læreren kan give sværere opgaver, vi kan beskæftige os med noget, der er sværere, men det er jo først i 2.g og 3.g og sådan. Så jeg må bare acceptere, det er sådan.

447

Q11: (…) Hvad man kan gøre for at blive endnu bedre til at klare matematiske udfordringer? Der siger du: "Øvelse gør mester!" (...)

B11: Ja, man kan hele tiden lave forskellige opgaver, der omhandler det samme. Så bliver man jo bedre til det. Men på et tidspunkt bliver man også træt af at lave de samme opgaver. Så...

### *1.G. Showing Interest*[448]

Q12: And then you say, what matters the most is to be interested in Mathematics.

B12: Yes, you have to be interested in Mathematics, because you chose this study programme. So if you are not interested in it, then why be here? And when you are interested in it, then I think you kind of improve. For, I am not interested in German, for example. And I am not as good at that compared to Mathematics and Physics, for example, which I am very interested in.

Q13: What difference does it make? Do you pose more questions, when you are interested?

B13: No, I do not thank that, I just think that it makes it exciting to work with what you like. That is why.

### *1.H. Strategies and Support*[449]

Q14: And then you say, that if you get stuck: Well you give it a try, but you can also ask the teacher, but challenges are good for the brain.

B14: Yes, if I cannot work it out, then I try myself, right? Otherwise I ask the teacher or the classmates. Because my parents, they cannot help me that much, because they do

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448

Q12: Og så siger du (...) det der betyder mest, det er at interessere sig for det.

B12: Ja, altså du skal interessere dig for matematik, for du har valgt den her linje. Så hvis du ikke interesserer dig for det, og du ikke kan lide det, så hvorfor være her? Og når man interesserer sig for det, så tror jeg også man bliver bedre-agtig. For jeg interesserer mig ikke for tysk fx. Og det er jeg fx ikke så god til i forhold til matematik og fysik, som jeg interesserer mig meget for.

Q13: Hvilken forskel gør det? Stiller man flere spørgsmål, når man er interesseret? (...)

B13: Nej, det tror jeg ikke, jeg tror bare, det gør det spændende at arbejde med, man kan godt lide det. Derfor.

449

Q14: Og så siger du, hvis du går i stå med noget: Jamen, du prøver dig frem, man kan også spørge læreren, men hjernen har det godt med udfordringer. (Der grines.)

B14: Ja, hvis jeg ikke kan finde ud af noget, så prøver jeg selv, ikke? Ellers spørger jeg læreren eller kammerater. For mine forældre, de kan ikke hjælpe mig så meget, fordi de har ikke uddannelser, og de kender det ikke, derfor.

Q15: Men du siger både på gymnasiet og derhjemme, altså du prøver altid først selv. Og så siger du: "Man kan læse sig til alt."

B15: Ja, man kan læse altså, det ved jeg nu ikke. Men altså man kan læse sig til det. Og du kan slå det op i nettet, hvis du har problemer, derfor.

not have any education, and they do not know about it. That is why.

Q15: But you also say that both at home and at school, you always try yourself first. And then you say: "*You can read up on everything*."

B15: Yes, one can read, well, I do not know. But you can read up on things. And you can look it up on the Internet, if you have problems. That is why.

***1.I. WHICH CHALLENGES***[450]

Q16: And then I ask which kind of different issues are more challenging to you. And one thing which is a bit lower than the others, but in the middle of the scale, is something about remembering things. It is a little more challenging?

B16: Yes, well, in lower secondary school, it was easy to remember things. Is has become a little harder here, but not something I would describe as a problem. I can actually remember the formulae, we have had.

Q17: But maybe you do not need to remember everything?

B17: No, because you can have a collection of formulae to look up in.

***1.J. SUPPORT***[451]

Q18: You also say, where you can get support for Mathematical activities: "With friends or in other places, for example the Internet or the library or books or something".

B18: Yes, what I do is, I have already mentioned it, but: When I have problems: My peers, or looking up the question on

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Q16: Og så spørger jeg til, hvad er det for nogle ting, man har flest udfordringer med. (...) og noget, der så ligger lavere end de andre, men som ligger midt imellem, det er noget med at huske ting. Det er der lidt flere udfordringer med. (...)

B16: Ja, altså i folkeskolen, der var nemt at huske ting. Det er blevet lidt sværere her, men det er ikke sådan noget, som jeg betragter som et problem. Jeg kan faktisk godt huske formlerne, som vi har haft.

Q17: Man behøver måske heller ikke at huske det hele?

B17: Nej, for man kan nemlig have en formelsamling og slå op i den.

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Q18: Du siger også, hvor kan man hente støtte til matematiske aktiviteter: "Hos kammerater og andre steder, fx internettet eller biblioteket eller bøger eller et eller andet" (...)

B18: Ja. Det, jeg gør, jeg har allerede sagt det, men: Når jeg har problemer: Kammeraterne eller søger spørgsmål på internettet, hvordan man regner det ud. Så finder, forstår jeg det. For man skal ikke bare søge efter resultatet og så kopiere det ind derfra.

the Internet, how to calculate it. Then I find, then I understand it. Because you only have to search for the result and copy-paste it from there.

*1.K. PARTICIPATION IN CLASS*[452]

Q19: You find that you answer questions more frequently than you ask them [in class]. You are content about it, but you do not think you have that much self confidence.

B19: It is no longer... it is not that much in Mathematics. It is rather in Danish and History. Because in mathematics, there is usually only one answer. So, I do not have any problems with that. It is rather in History, if I know the answer, then I am not sure if it is the right answer, so I do not give it.

Q20: No. One does not want to say something that is not correct.

B20: That is incorrect.

Q21: What would the others in class think then?

B21: I do not know... I just will not answer.

*1.L. SUCCESS AND DIFFICULTIES*[453]

Q22: Then I also ask how one is perceived if one is good at mathematics and if one have difficulties with mathematics. You say, there is no problem in being good at mathematics, you may well be that.

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Q19: Det er oftere at du svarer på noget end at du spørger om noget. Men du er tilfreds med din indsats, men du synes ikke, du har så stor selvtillid.

B19: Det er ikke mere, det er ikke så meget i matematik. Det er mere i dansk og historie. Fordi i matematik, så er der som regel bare et svar. Derfor, det har jeg ikke problemer med. Det er mere sådan i historie, hvis jeg kender svaret, så er jeg ikke helt sikker på, om det er det rigtige, så derfor siger jeg det ikke.

Q20: Nej. Man har ikke lyst til at komme til at sige noget, der ikke er rigtigt?

B20: Der er forkert.

Q21: Hvad tænker de andre i klassen så?

B21: Det ved jeg ikke, det, jeg vil bare ikke svare.

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Q22: Så spørger jeg også til, hvordan det bliver opfattet, hvis man er god til matematik, og hvordan det bliver opfattet, hvis man har svært ved matematik. Du siger, der er i hvert fald ikke nogle problemer med at være god til matematik, det må man meget gerne være.

B22: Ja, og hvis man har svært ved det, rigtig svært ved det, og man hele tiden spørger, når læreren forklarer, så kan det godt virke nogle gange irriterende, fordi så afbryder man jo hele tiden. Så man må faktisk vente til læreren har sagt det, han skal, og så bagefter spørge, når vi er i gang med at lave opgaverne.

B22: Yes, and if you have really, really big problems, and you ask all the time, when the teacher explains, then it can be a little annoying because then you interrupt all the time. So you should actually wait until the teacher has said what he wants to, and then ask afterwards, when we are working on the assignments.

*1.M. GROUP WORK*[454]

Q23: For example, if you are working in groups. Then you say: "Well, if I am teamed up with somebody...

B23: Who cannot work it out. Then they want the result, they do not want to know why, how I arrived at it, they just want the result, so they can write it in the report. Also the intermediate results, even though they do not understand them. So, you should read up on it, or ask the teacher or the classmates, and then ask the classmates or the teacher how they arrived at it.

Q24: So it means something, that one is interested in understanding things, or if one just wants to get them over and done with?

B24: If one is interested, then one wants to know why the answer is what it is. If you are not, then you just do not care, then you just want it in the report and hand it in.

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Q23: (...) fx hvis man skal lave gruppearbejde. Så siger du: "Jamen, hvis jeg bliver sat sammen med nogen..."

B23: Der ikke kan finde ud af det. Så vil de godt sådan have resultatet, de vil ikke sådan have hvorfor, hvordan jeg kom frem til det, de vil bare sådan have resultatet, og så kan de skrive det i rapporten. Også mellemregningerne, selv om de ikke forstår det. Så er det ikke godt. Hvis man ikke kan finde ud af det, så skal man finde ud af det. (Brandon ler.) Så skal man læse sig frem til det, eller spørge læreren eller kammeraterne, og så spørge kammeraten eller læreren, hvordan du kom frem til det.

Q24: Så det betyder noget, om man er interesseret i at forstå det, eller om man bare gerne vil blive færdig?

B24: Hvis man er interesseret i det, så vil man vide, hvorfor svaret er det. Hvis man ikke er, så er man bare ligeglad, så vil man bare have det med i rapporten og så have det afleveret.

***1.N. PLANS***[455]

Q25: Then I ask about your preliminary plans for tertiary education, where you consider becoming a medical doctor.

B25: It is more, if I get the necessary average of marks I would really love to become a doctor, because it is something that has my interest. But if I cannot get that average, then it could be something within mathematics, but I just do not know what. I would like to work with mathematics. But, on the other hand, I do not know what it should be, then. But if I can get the grades for being admitted to study medicine, then that it should be.

***1.O. MATHEMATICS FOR WHOM?***[456]

Q26: Now, I will return to some previous questions concerning who should learn mathematics. If everybody should learn mathematics, for example?

B26: Well, it differs. One can use something simple as the formula of [compound] interest, everybody could use that. And there is also something that we learn in the first year, so everybody learns it. But when we get to the second year, differential calculus and such, then you need not, if you are not interested in it. So one just choose the study programme one wants. Yes, that is it.

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Q25: Så spørger jeg også til foreløbige planer efter gymnasiet, hvor at du i hvert fald tænker på, at du gerne vil være læge.

B25: Det er mere, hvis jeg kan få gennemsnittet, så vil jeg virkelig være læge, fordi det er noget, der interesserer mig. Men hvis jeg ikke kan få gennemsnittet, så kunne det godt være noget inden for matematik, men jeg ved bare ikke helt hvad. Jeg vil gerne arbejde med matematik. Men på den anden side, ved jeg ikke hvad det skal være så. Men hvis jeg virkelig kan få karaktererne til at komme ind og studere medicin, så skal det være det.

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Q26: Nu vil jeg lige vende tilbage til nogle spørgsmål, (…) og det handler om, hvem der skal lære matematik. Om alle skal lære matematik fx.

B26: Altså det er forskelligt. Man kan godt bruge sådan noget simpelt, sådan noget som renteformel, det kunne alle godt bruge. Og det er også noget, man lærer i 1.g, så det er alle der lærer det. Men fx når vi kommer i 2. g, så differentialregning og sådan noget, så behøver man ikke, hvis man ikke interesserer sig for det. Så man kan bare tage den linje, man har lyst til. Ja, det er det.

Q27: Så matematik er noget, som alle skal lære, men noget der passer til, hvad de gerne vil, og hvad de skal bruge.

B27: Ja, om de også skal bruge det senere i deres liv. Hvis du skal studere medicin, så skal du have matematik A, så der kan godt være nogen, der har lyst til at studere medicin, men de kan ikke lide matematik, så det er ikke så godt.

Q27: So mathematics is something everybody should learn, but something that relates to what they want and what they will need?

B27: Yes, if they will need it later in their life. If you are to study medicine, you need it on A-level. Then there may be somebody, who wants to study medicine, but does not like mathematics; That it is not so good.

***1.P. STUDY PROGRAMME***[457]

Q28: And you yourself chose a study programme because you were interested in Mathematics and you wanted to continue having it. In your school, was it only this Physics-Chemistry study programme, that involved A-level Mathematics?

B28: I think it was this one [study programme], with Physics and Chemistry, that were with A. There were some others with B-level, which could be upgraded to A, but then you had to take Social Science A, I do not care about that. And then it was an advantage, that in Medicine, you need both Mathematics A and Physics B, so that was good.

Q29: So it matched what you wanted do do afterwards? You did know that you dreamed about becoming a doctor, before you chose your study programme?

B29: Yes, since sixth grade, I think.

***1.Q. MATHEMATICS APPLIED IN OTHER SUBJECTS***[458]

Q30: What you learn in mathematics, can you apply that in your other subjects?

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Q28: Og du har selv valgt en studieretning, fordi du har en stor interesse for matematik, og du gerne vil arbejde videre med det. (…) Var der kun den her med fysik og kemi, hvor der var matematik på A-niveau?

B28: Jeg tror faktisk, der kun var den her med fysik og kemi, som var A. Der var nogle andre med matematik B, som man kunne opgradere til A, men så fik man også samfundsfag A, og det gider jeg ikke. Og så var der en fordel, at man i medicin skulle både have matematik A og fysik B, så det er godt.

Q29: Så det passede med hvad du gerne ville senere? Du vidste godt, at du drømte om at blive læge, inden du valgte linje?

B29: Ja, siden 6. klasse tror jeg.

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Q30: (…) Det du lærer i matematik, kan bruge det i dine andre fag?

S30: Matematik og fysik, det hænger sådan meget sammen. Og det er lidt det samme med kemi, hvor med sådan formler. Ikke så meget i kemi, men mere matematik og fysik. Det kan man godt på en måde relatere hinanden til. Men ikke så meget de andre fag. Så. Så hvis man har matematik A her, så har man også lettere ved fysik på en måde.

B30: Mathematics and Physics, they relate a lot to each other. And also somehow with Chemistry, with formulae. Not just as much in Chemistry, but more Mathematics and Physics. You can somehow relate them. But not that much to the other subjects. So, if you have Mathematics A here, then you are helped in physics, kind of.

### *1.R. MATHEMATICS IN SOCIETY*[459]

Q31: And then I also ask, what is mathematics used for "out in the World"?

B31: There some simple things. E.g. when you buy things, then it is mathematics. So is iime, definitely. Then I do not know much more.

Q32: You write: Dealing with money and time, planning and games, both probability and horse racing, when one can calculate which horse would be more likely to win, by examining nutrition and exercise, etcetera.

B32: It may be, I do not know. I was just something I wrote.

### *1.S. THE NATURE OF MATHEMATICS*[460]

Q33: One question I ask, is whether mathematics is discovered by human beings, well if it existed already and then human beings discovered it, or if it is something invented

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Q31: Og så spørger jeg også til, (...) hvad man bruger matematik til ude i Verden?

B31: Altså der er noget simpelt. Fx når man skal købe ting, det er matematik. (utydeligt). Det er tiden, i hvert fald. Så ved jeg ikke meget andet.

Q32: Du skriver: Håndtere penge og tiden, planlægning og spil, både med sandsynlighed og i hestevæddeløb, så kan man regne ud hvilken hest, der måske kan vinde ved at se på kost og træning osv.

B32: Det kan godt være, jeg ved det ikke. Det er bare noget, jeg har skrevet.

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Q33: Et af de spørgsmål, jeg stiller, det er om matematik er opdaget af mennesker, altså noget, der fandtes i forvejen, men som mennesker har opdaget, eller om det er noget, som mennesker har opfundet. Om det er opdaget eller opfundet.

B33: De har ikke opfundet det, de har ikke opfundet det. Altså det har været der i rigtig lang tid fx. Der har altid været noget med handel. Så der bruger man det fx. Og så er der med tiden kommet sådan nogle sværere ting som nu kan jeg ikke komme i tanke om noget, som fx cosinus og sinus, det har der nok ikke været for... Altså noget, der er svært, som man har i 3. g, det har der nok ikke været for 1.000 år siden fx. Så man har opdaget nogle nye ting, nogle flere ting med tiden, men jeg tror bare, det altid har været der.

Q34: Sådan noget matematik har altid været der, det er ikke noget mennesker har opfundet?

B34: Nej. Men så er der kommet sværere ting gennem tiden, som nok nogle matematikere har opdaget eller lavet, hvorfor det lige præcis er det, så er det det.

by human beings. Whether Mathematics is discovered or invented.

B33: They did not invent it, they did not invent it. Well, it has been there for a long, long time, for example. There has always been something about trade. So there it is used, for example. And then, over time, some more difficult things, which I cannot think of right now, like for example cosine and sine, it may not always have been there, because...well, something which is hard, that you deal with in the third year, it may not have been there one thousand years ago, for example. So one has discovered new things, some more things over time, but I just think it always has been there.

Q34: So, some mathematics has always been there, it is not something human beings have invented?

B34: No. But then some harder things over time have emerged, which some mathematicians have discovered or made, why it is exactly that. So, that is it.

***1.T. MATHEMATICS AT UNIVERSITY***[461]

Q35: What is a Mathematician in a university doing? Or what do you imagine a Mathematician in a university might be doing?

B35: I do not know. It is hard formulae. Tough things. At least I have seen some formulae and calculations, and they look complicated, so that must be it.

Q36: You write here: "*Deals with mathematical methods, which can be used in everyday life. Some so called algorithms, theories and so on.*"

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Q35: Hvad laver en matematiker på et universitet? Eller hvad forestiller du dig en matematiker på et universitet kunne gå rundt og lave?

B35: Det ved jeg ikke. Det er svære formler. Svære ting, jeg ved ikke. Jeg har i hvert fald set nogle form- (formler?), (ud?-) regninger og de ser ret indviklede ud, så det må være det.

Q36: Og du skriver her: "*Beskæftiger sig med matematiske metoder, som kan bruges i hverdagen. Nogle såkaldte algoritmer, teorier mm.*"

B36: Det er jeg ikke sikker på, det er bare noget, jeg har skrevet.

Q37: Men i hvert fald nogle svære ting?

B37: Ja. Yes.

Q38: Og formler, sagde du?

B38: Ja. Det kunne være noget inden for computere, som man kunne bruge. Nogen, der interesserer sig for matematik, kunne også lave noget inden for teknologi, computere og sådan noget.

B36: I am not sure about that. It is just something I wrote.

Q37: But some tough things?

B37: Yes. "Yes" (sic.).

Q38: And formulae, you said?

B38: Yes, it could be within computers, one could use. Somebody interested in mathematics could also do something within technology, computers and such.

***1.U. GENIUS***[462]

Q39: Then I ask if one should be a genius to study mathematics at university, but that is not necessary?

B39: No, one should be interested in it, but one does not have to be a genius.

Q40: Do you think that anyone from your school class might choose to study mathematics at university?

B40: Do not think so.

***1.V. CHOOSING UPPER SECONDARY SCHOOL PROGRAMME***[463]

Q41: We talked about why you chose general upper secondary school, why it was not a commercial school programme or something completely different. How come it was BETA upper secondary school?

B41: I was in a special bridging programme here, and I thought it was good. On the other hand, I think that in technical upper secondary school, I think it is, if you choose a study programme there, then you do not have the other subjects

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Q39: Så spørger jeg også, om man skal være et geni for at læse matematik på et universitet, men det behøver man ikke?

B39: Nej, man skal interessere sig for det, man behøver ikke at være et geni.

Q40: Tror du der var nogen fra din klasse, der kunne finde på at læse matematik på universitetet?

B40: Tror jeg ikke.

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Q41: Vi snakkede ikke om, hvorfor du valgte det almindelige gymnasium, hvorfor det ikke var HHX eller HTX eller noget helt andet. Hvordan kan det være, at det var "BETA upper secondary school"?

B41: Jeg var i brobygning her, og der synes jeg, det var godt. På den anden side, så tror jeg at man i HTX, tror jeg det er, hvis man vælger en studieretning der, så har man ikke så meget de andre fag. Så det kunne faktisk være bedre, hvis man interesserer sig meget for matematik, fysik, kemi og så vælge det derhenne. Men jeg ved ikke helt om, man kan, hvis man nu skal studere medicin, så skal man også have historie, tror jeg nok. Så jeg er (ikke?, red.) helt sikker. Men jeg var i hvert fald i brobygning her, og der synes jeg, det var godt. Og det var jeg også andre steder, hvorimod de steder ikke var så gode.

as much. So it could actually be better, if you are very interested in Mathematics, Physics and Chemistry, to choose it in such a school. But I do not know if one can, if you are to study medicine, that it is required to take History, I think so. So I am not completely sure. But I was in a bridging programme here, and then I thought it was good. And I also went some other places, and those places were not so good.

## THEMES IN BRANDON'S 1ST YEAR

First there will be given some highlights from the questionnaire answers and the interview transcripts, which will be followed by an analysis of who Brandon is, what is driving him, and how the four aspects of beliefs may interact.

### MATHEMATICS AT SCHOOL

Upper secondary school mathematics seems much easier than Brandon expected. He does not ascribe the easiness to the teaching but rather to his own person. He finds it to concern putting in numbers to formulae and the isolate (1.A.). Participation in class is no problem to Brandon, if it is about mathematics, because them there is only one answer. In other subject, such as Danish or History, he might know the answer without feeling comfortable in saying it in class, because he would not know if he were right (1.G.).

It is new in upper secondary school mathematics, that you deal with proofs. This enables the students to find out why a theorem can be applied in a certain context (1.C.).

#### *COOPERATION & UNDERSTANDING*

One contrast between mathematics in lower secondary school, and now in upper secondary school, seems to be the group work, which is more common now (1.B.). Brandon prefers cooperating with others, except if his group members only want the results, without seeking to understand (1.D.). If Brandon is put in a group he has not chosen himself, he often experiences that his group fellows are only interested in the result, and maybe just enough calculations for it to appear nice in the report, even though they do not understand it (1.H.).

In this sense there is somehow a discrepancy between Brandon's norms for

mathematics learners at school and what he experiences from some peers. It is clear that Brandon values understanding and that it is something you should seek actively yourself.

### MATHEMATICS AS A DISCIPLINE

Brandon suggests mathematics to be both invented and discovered, and his ideas of the activities of mathematician in university relate to the application of mathematics in everyday life. He also mentions "algorithms" and "theories".

Mathematics is also useful also when dealing with physics, (BRANDON: Q1-B). *"Mathematics is a great help, especially for physics, but also for chemistry, but not that much for other subjects"*, Brandon says *(1.K.).*

### MATHEMATICS IN SOCIETY

According to Brandon, mathematics can come in useful anywhere in life, and it is something everybody should learn. It is applied in society for handling money, planning, and what could be interpreted as mathematical modelling (BRANDON: Q1-B), but also for e.g. trade and counting time (1.L.).

Everybody should learn some mathematics, Brandon says, but mainly related to what could be relevant for people's lives in the future. Not everybody should necessarily study A-level mathematics, but the formula of interest rates would be relevant to anybody, he says (1.J.).

### MATHEMATICS & ME

#### *RATING*

Brandon seems to be on top of the situation in mathematics according to his questionnaire; Mathematics is rated with a [10] on the favourite subject scale, he finds it has become more exciting now in the 3rd because of greater challenges, but

that does not stop him from requesting even more challenges (*BRANDON: Q1-A*).

*CHALLENGES & SUPPORT*

To Brandon, the greatest obstacle to improve in mathematics is to take an interest in it – if you do that, the rest will follow, as long as you are willing to work hard to achieve it (BRANDON: Q1-D). If Brandon meets a challenge, he seems to deal with it himself. Otherwise he asks the teacher or his classmates, because his parents does not have any education (1.G.). Even though no challenges seems to be of too great trouble to Brandon, remembering is indicated to involve a moderate challenge to him (BRANDON: Q1-D, Q1-E). From his family, at home, Brandon cannot find help for mathematical activities, since his parents do not have any education. Instead, he can ask friends, but he also consults the Internet, and in the library he can find books. In class he is quite active in answering questions, but he rarely asks any. He does not seem to feel any discrimination in terms of whether you are good at mathematics or not in class (BRANDON: Q1-E). The fact that mathematics seems easy to Brandon is perceived as something he just has to live with – and wait until the 2$^{nd}$ or 3$^{rd}$years for more challenges to come (1.E.).

*PLANS*

In terms of plans for tertiary education, Brandon wishes to study medicine which demands a high average of grades. Otherwise, he says he might consider something related to mathematics or physics, without having any specific ideas, though. He will not try to avoid mathematics, since he thinks it is an "awesome" subject (BRANDON: Q1-G). During the 3$^{rd}$ year interview, it turns out that Brandon literally does not have an ideas of the role of mathematics in professions in society – at least not in terms of a specific education leading to a job he could find interesting. But if he does not obtain the required average of grades for admission to medical school, he might consider something related to mathematics, but he has no idea of

what it should be then(1.I.).

## BRANDON'S 1ST YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics at school is easy – it mainly involved applying formulae.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is both invented and discovered. Mathematician in university deals with hard formulae and maybe computers.

### *MATHEMATICS IN SOCIETY*

Mathematics is used for many thing in society – dealing with money and time, for example. And a certain level of mathematics should be learned by everybody.

### *MATHEMATICS AND ME*

Mathematics is fun, because Brandon is good at it. Brandon chose A-level Mathematics due to his plans of studying medicine.

## BRANDON'S 3RD YEAR QUESTIONNAIRE

| Q3-A | TRANSITION | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [10] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [464] | [Working on your own]; [Working in pairs]; [The whole class together] |
| **3** | Did you like mathematics when you went to lower secondary school?[465] | [Yes, it was one of my favourite Subjects] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | The level, of course. You have become capable of combining more things. [466] |
| **New 4d** | In which ways has mathematics changed from first year to third year of upper secondary school? | You are capable of combining more things; e.g. vectors and differential calculus.[467] |

*Table 59: BRANDON'S 3rd year questionnaire, part A – TRANSITION*

464 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

465 [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

466 Niveauet selvfølgelig – man er blevet i stand til at holde flere ting sammen

467 Man kan holde flere ting sammen fx vektorer og diff regning

| Q3-B | FOR SCHOOL | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | A part of everyday life – simple mathematics in terms of trade and the like[468] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | E.g. physics – where you can use differential calculus and integral calculus – connects to physics, e.g. when you deal with mechanics.[469] |

*Table 60: BRANDON's 3rd year questionnaire, part B - MATHEMATICS AND SCHOOL*

468 En del af hverdagen – simpel matematik i form af handel og lign

469 F.eks. fysik – hvor man kan bruge diff regning og integralregning – sammenhæng med fysik når man f.eks. beskæftiger sig med mekanik

| Q3-C | BEYOND SCHOOL | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | Trade - money[470] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? [471] | [Both] |
| **10** | What do you think a professional mathematician at a university is doing? | Possibly mathematics seen also in relation to everyday life[472] |
| **11** | Would you have to be a genius in order to study mathematics in university? [473] | [No] |

*Table 61: BRANDON's 3rd year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

470 Handel – penge

471 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]

472 Evt matematik set også i sammenhæng med hverdagen

473 [Yes]; [No]; [I do not know]

| Q3-D | IMRPOVING | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | - |
| **12b** | What do you think is the best means for improving in mathematics? | Interest – that one has to take an interest to the subject[474] |
| **12c** | What do you do to improve in mathematics? | Do my homework and make sure I understand it.[475] |
| **13a** | What do you do if you get stuck on a task at school? | Try some more – but otherwise I ask friends and eventually the teacher[476] |
| **13b** | What do you do if you get stuck on your homework? | Try to examine similar tasks, but otherwise I ask friends first and eventually the teacher[477] |
| **13c** | What do you do if you get stuck on your written assignments? | Same as previous |

*Table 62: BRANDON's 3rd year Questionnaire, part D – STRATEGIES FOR IMPROVING*

474 Interesse – at man skal interessere sig for faget

475 Læser lektierne og er sikker på at jeg forstår det

476 Prøver lidt ekstra – men ellers spørger jeg venner og i sidste ende læreren

477 Prøver at kigge på lign opgaver men ellers spørger jeg først venner og derefter læreren

| Q3-E | CHALLENGES & SUPPORT | | BRANDON |
|---|---|---|---|
| # | *Question* | | *Answer* [478] |
| **14** | What issues involve more challenges to you? | a) Remembering | [5] The fewest challenges |
| | | b) Computing | [4] Few challenges |
| | | c) Figuring out the purpose of a task | [3] Moderate challenges |
| | | d) Finding a way to solve a task | [4] Few challenges |
| | | e) Reading and understanding the textbook | [4] Few challenges |
| **15** | Where can you find support for mathematical activities? | | [From classmates]; [Other]: The teacher, the Internet, the textbook, the collection of formulae[479] |
| **New 16a** | Do you find that you meet some special challenges related to succeeding in mathematics compared to your classmates? | | No |
| **New 16b** | Do you find that you have some some special strengths related to succeeding in mathematics compared to your classmates? | | I am extremely good at remembering formulae – and also of finding a way of solving a task[480] |

*Table 63: BRANDON's 3rd year Questionnaire, part E – CHALLENGE & SUPPORT*

478 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

479 Læreren – internettet – mat-bog – formelsamling

480 Jeg er ekstrem god til at huske formler – er også god til at finde en måde at løse en opgave på

| Q3-F | IN CLASS | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? [481] | [1-3 times] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [No] |
| **17c** | Are you content with that? | Yes |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? [482] | [More than eight times] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [Yes] |
| **18c** | Are you content with that? | "Yes" |
| **19a** | In your class, is it okay to be good at mathematics? | Yes, it is fine[483] |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Yes, fine as well[484] |

*Table 64: BRANDON's 3rd year Questionnaire, part F - MATHEMATICS IN CLASS*

481 Options: [0]; [1-3]; [4-8]; [More than 8 times]

482 Options: [0]; [1-3]; [4-8]; [More than 8 times]

483 Ja – det er helt fint

484 Ja – ligeledes helt fint

| Q3-XA | UNDERSTANDING | BRANDON |
|---|---|---|
| # | *Question* | *Answer*[485] |
| **X.1.** | Have you recently experienced to *understand* what you worked with in mathematics? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.3.** | Have you, during upper secondary school experienced that you *understood* what you worked with and then s*ubsequently learned it by heart*? | [No] |
| | If yes, on which occasion? | - |
| **X.5.** | Have you during upper secondary school experienced *understanding something* but *never learning it by heart?* | [No] |
| | If yes, on which occasion? | - |

485 Options: [Yes], [No] or [I do not know]

| Q3-XB | LEARNING BY HEART | BRANDON |
|---|---|---|
| # | *Question* | *Answer*[486] |
| **X.2.** | Have you recently experiences having to *learn something by heart*? | [No] |
| | If yes, on which occasion? | - |
| **X.4.** | Have you, during upper secondary school experienced that you had to *learn something by heart*, and then, *subsequently understood it*? | [Yes] |
| | If yes, on which occasion? | I learn by heart quickly – even though I do not understand it[487] |
| **X.6.** | Have you during upper secondary school experienced *learning something by heart without ever understanding it*? | [Yes] |
| | If yes, on which occasion? | Especially concerning Euclidean space, there are types of tasks that I did not learn by heart; I therefore need to consult earlier examples[488] |
| **X.7.** | Additional comments on understanding or rote learning in mathematics | - |

486 Options: [Yes], [No] or [I do not know]

487 Jeg lærer hurtigt udenad – selvom jeg ikke forstår det

488 Især indenfor rumgeometri er der typer af opgaver som jeg ikke har lært udenad og må derfor kigge på gamle eksempler

| Q3-XC | A-LEVEL EXAMINATION | BRANDON |
|---|---|---|
| **X.8.** | *Topic* | *Answer*[489] |
| **(a)** | Parabola | [Readily] |
| **(b)** | Exponential | [Okay] |
| **(c)** | Pythagoras | [Readily] |
| **(d)** | Sine and cosine relations | [Readily] |
| **(e)** | Definition of differentiability | [Okay] |
| **(f)** | Sum and product of differential functions | [Readily] |
| **(g)** | Indefinite integral | [Readily] |
| **(h)** | Volume of solid of revolution | [Okay] |
| **(i)** | Differential Equations and their solutions | [Rather not] |
| **(j)** | Vectors in the plane, including scalar product | [Readily] |
| **(k)** | Lines and planes | [Readily] |
| **X.9.a.** | Which topic is your favourite?<br>- and why? | Geometry in arbitrary triangles[490] |
| **X.9.b.** | Which topic would you rather avoid?<br>- and why? | Differential equations – we just had them and I am not 100% on top of it[491] |

*Table 65: BRANDON: Topics for oral and written examinations in A-level Mathematics for Upper Secondary School Programmes in Denmark 2013*[492]

489 Options: [Readily], [Okay], [Rather not], [I do not know]

490 Geometri i vilkårlige trekanter

491 Diff ligning – vi har lige haft det og jeg har ikke 100 % styr på det

492 Topics common to both the Technical (htx) and the General (stx) upper secondary programme examinations are suggested.

| Q3-G | PLANS | BRANDON |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Medicine or something similar – physiology?[493] |
| **20b** | What kind of influence would your final marks in mathematics have on your plans (or hope for plans) after upper secondary school? | Great influence – since it brings up the average of marks such that I can get a mark that will allow my admittance to medical school[494] |
| **20c** | What kind of influence would your experiences with mathematics in upper secondary school have on your plans after upper secondary school? | No major influence[495] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [Yes] |
| **21b** | Comments: | Yes, should be interesting, but I cannot imagine [many] exciting[496] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [No] |
| **22b** | Comments | Mathematics is exciting[497] |

*Table 66: BRANDON's 3rd year Questionnaire, part G – PLANS*

493 Medicin el lign – fysiologi?

494 Stor betydning – da det trækker gennemsnittet op således at jeg kan få en karakter – der Giver adgang til medicinstudiet

495 Ikke stor betydning

496 Ja ville være interessant men kan ikke forstille mig (mange?) spændende

497 Matematik er spændende

# BRANDON'S 3RD YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 6 March 2013 | 22:49 |

### *3.A. MATHEMATICS IS EASY* [498]

Q1: How do you feel about mathematics right now?

B1: Mathematics? It is probably the best subject in school, I would say. It is going fine, also in terms of marks, and I think the tasks, they are rather easy, for example compared to Physics and Chemistry. So it is going really well.

Q2: Are the assignments different now compared to the first year?

B2: Well, new topics have been introduced, task-wise, but the level is the same. It may be that there are other tasks, and that they have become a little harder, but otherwise, the level is the same.

498

Q1: Hvordan har du det med matematik lige nu?

B1: Matematik? Det er nok det bedste fag i gymnasiet, vil jeg mene. Det går meget fint, sådan, også karaktermæssigt og sådan, og jeg synes opgaverne, de er rimelig lette, f.eks. i forhold til Fysik og Kemi. Så det går faktisk meget dejligt.

Q2: Er det nogle andre opgaver I får nu i 3.g end I fik i 1.g?

B2: Altså, der er kommet nye emner til, men opgavemæssigt, altså niveau, jeg synes sådan set det er det samme. Det kan godt være der er andre opgaver, og at de er blevet lidt sværere, men ellers, niveauet, det er det samme. Men... Opgaverne, jeg synes også mere man er blevet i stand til at koble flere forskellige ting sammen, her i 3.g. For eksempel når man har med vektorer, så kan man koble flere forskellige ting sammen. Og det er sådan set det, ja.

***3.B. STUDY PROGRAMME PROJECT (SRP)***[499]

Q3: Did you incorporate Mathematics in your study programme project (SRP)?

B3: Well, I wrote in Mathematics and History. And what I wrote, the mathematics part was relatively easy. We had some formulae, and then it was just to plug in and...and then it was about proving something, but...it was relatively easy, yes.

Q4: How could History contribute? Did it contribute something to have it?

B4: What Mathematics was used for in relation to the SRP, was that you used Mathematics to do an ENIGMA-machine, a complicated coding-machine, and then you used it in the war, the Second World War. So you used the mathematics for building this machine, which was used in war.

Q5: Okay. I may be interested in whether it changed your view of mathematics to do this project?

B5: What do you mean?

Q6: If you think you saw some new facets of mathematics in this projects, that you did not know before?

B6: No. I do not think so. You just connected it a little bit, and then there were some other formulae, but more or less the same.

499

Q3: Havde du matematik med i dit SRP projekt?

B3: Altså, ja, jeg skrev i Matematik og Historie. Og det jeg skrev, matematikken, det var statistik, matematik-delen, og det var forholdsvis let. Vi havde nogle formler, og så var det bare at indsætte og så... Og så var det at bevise nogle ting, men... Det var forholdsvis let, ja.

Q4: Hvad kunne historie bidrage med? Gav det noget at have det med?

B4: …Det man sådan set brugte matematikken til den forbindelse med SRP, det var at man brugte matematikken til at lave en ENIGMA-maskine, en kompliceret kodningsmaskine, og den brugte man så i selve krigen, 2. Verdenskrig. Så man brugte matematikken til at lave selve maskinen, som man brugte i krig.

Q5: Okay. Jeg er måske interesseret i (…) om det ændrede ved din opfattelse af matematik at lave det her projekt.

B5: Hvordan mener du?

Q6: Om du synes du så nogle nye sider af matematik, som du ikke kendte til før?

B6: Nej, det synes jeg ikke. Man koblede det bare lidt sammen, og så var det med nogle andre formler, men forholdsvis det samme.

### *3.C. INTEREST IN MATHEMATICS*[500]

Q7: Well, your interest in Mathematics had been stable, pretty high, both in the first year and now, you rate with 10, on a scale from 1 to 10, on which 10 is your favourite subject.

B7: Yes, it is true. I do not know, I just always had an interest in Mathematics. I think it is so...it is relatively easy, when you understand it, and it is not complicated and... it is enticing.

Q8: For example, it contains proofs for theorems, and then I thought: Is there not a lot to learn by heart?

B8: I do not know, there is, yes, but I do not know, I just think it is really easy, with mathematics. I am able to learn it by heart, I am also, like, really good at remembering formulae and such. So it comes easily...proofs and formulae come easily to me. So that is why I think mathematics is awesome, so...

500

Q7: Altså, din interesse ser jo ud til at have ligget meget stabilt, højt... både i 1.g og nu, der giver du det en 10'er på en skala fra 1 til 10 på, med dit yndlingsfag.

B7: Ja, det passer også (...) Jeg ved ikke, jeg har bare altid haft interesse for matematik. Jeg synes også det er sådan - Det er forholdsvis let når man forstår det, og det er ikke kompliceret og... Det er spændende.

Q8: For eksempel så indgår der jo beviser for sætninger. Og der tænker jeg, er der ikke en masse at lære udenad?

B8: Jeg ved ikke, det er der jo, men jeg ved ikke, jeg synes bare det er rigtig let, med matematik. Jeg kan godt lære det udenad, jeg er også sådan, rigtig god til at huske formler og sådan. Så jeg har faktisk let ved... Jeg har let ved de her beviser og let ved de her formler. Så derfor synes jeg også matematik er ret fedt, så...

### *3.D. REMEMBERING & UNDERSTANDING*[501]

Q9: What happens to you at first? Well, do you remember first? Or do you understand first? Or can you even separate those two things?

B9: I remember first, before I understand things. Well, some tasks may come, for which I do not understand the point, but since I have some formulae, I can solve the task. So I remember things first, and then I understand them. It is still, with vectors in Euclidean space, there is a lot that I have not understood, but I can still use the formulae, and so on. And solve the tasks and get them right.

Q10: Let us see, it also says here: What you say is: *"Especially within spatial geometry there are tasks, that I have not learned by heart"*, and accordingly you look at old examples.

B10: I feel like, the tasks we get, they resemble each other a lot, and for example, some of the first I did, then I probably used some formulae, and when I go back to solve a task, which resembles one I did previously, then it is just to go back and look at the old task, and then it is just new figures. Then it is just to plug them in, and then I get it right. Even...sometimes in spatial geometry, there are some weird tasks, where you are supposed to draw and such, and I do not get it. But because I have some formulae, I can solve it. And plug in and such.

501

Q9: Hvad sker først for dig? Altså, husker du det først? Eller forstår du det først? Eller kan du overhovedet skille de to ting ad?

B9: Jeg husker det først, før jeg forstår det. Altså, der kan godt komme nogle opgaver hvor jeg slet ikke forstår opgaven, men fordi jeg har nogle formler, så kan jeg godt løse opgaven. Så jeg husker tingene først, og så er det jeg forstår dem. Det er stadigvæk sådan med vektorer i rummet, der er der meget jeg ikke har forstået, men jeg kan stadigvæk bruge formlerne og sådan. Og løse opgaverne og have det rigtigt.

Q10: Skal vi lige se, det står nemlig også her. Det du siger er: især indenfor rumgeometri er der opgaver som jeg ikke har lært udenad og derfor kigger på gamle eksempler.

B10: Jeg har det sådan at de opgaver vi får, de minder meget om hinanden, og for eksempel nogle af de første jeg har lavet, der har jeg nok brugt nogle formler, og når jeg skal tilbage og lave en opgave der minder om den opgave som jeg lavede tidligere, så er det bare tilbage og kigge på den gamle opgave, og så er det bare nye tal. Så er det bare at indsætte, så får jeg det rigtigt. Selvom... Nogle gange i rumgeometri, så kommer der også nogle underlige opgaver hvor man skal tegne og sådan, og jeg fatter det ikke. Men fordi jeg har nogle formler, så kan jeg godt løse det. Og indsætte og sådan.

### *3.E. TOPICS FOR EXAMINATION*[502]

Q11: I ask about some topics that you can draw for the exam: There is a single one, for which you, when you got the questionnaire, answered "*rather not*", and that is differential equations.

B11: I am actually fine with that now. I may be... when was it?

Q12: Just before Christmas, I think.

B12: Yes, differential equations, that is as easy as pie, I think, but what should have been there, if it says something about spatial geometry, then it should be here somewhere. But otherwise, the others are fine, it is more or less the same.

Q13: So spatial geometry and such, they should be in the "Rather not" category?

B13: Well, those with vectors and such like, they should be in here, while the others should have been over there.

Q14: But is that because you did not learn it until the third year?

B14: Yes, it is exactly why I, well, with the other things, then I kind of went deeply into them, for example...those proofs, we have had, I have written them down in a notebook and such. And by doing that you understand them better, right? And I have not done that for spatial geometry, because that is something we just finished and such. But

502

Q11: Jeg spørger jo til nogle emner, som man kan trække til eksamen: (…) Der er en enkelt en som du, da du fik spørgeskemaet sagde "helst ikke" til, og det er en differentialligning.

B11: Det har jeg det faktisk ret fint med nu. Det kan godt være... Hvornår var det her fra?

Q12: Det var lige før jul, tror jeg.

B12: Ja ... Differentialligning det er røv-let, synes jeg ... men det der måske burde have stået - hvis der står noget med rumgeometri, så burde det nok stå her et sted. Men ellers, de andre er det fint nok, det er nogenlunde det samme.

Q13: Så Rumgeometri, det er snarere dem der burde være i "Helst ikke"-rækken?

B13: Altså, ja, det dér med vektorer i rummet og sådan, de burde nok ligge her, hvorimod de andre burde ligge hér, så...

Q14: Men er det fordi at I først har fået det i 3.g?

B14: Ja, det er netop derfor at jeg... Altså med de andre ting så har jeg ligesom også sat mig i dybden med det, jeg har for eksem... De beviser vi har haft, dem har jeg for eksempel skrevet ned i et hæfte og sådan. Og ved at man gør det, så forstår man det også mere, ikke. Og det har jeg så ikke gjort med rumgeometri, for det er noget vi lige har afsluttet og sådan. Men når jeg gør det, så er jeg sikker på jeg kommer til at forstå det. Og det kommer jeg jo også til, indtil mundtlig eksamen, og skriftlig og sådan noget. Men ellers, det er fint nok.

when I do that, I am sure I will understand it. And I also will, for oral and written exams and so on. But otherwise, it is fine.

### *3.F. OBSTACLES?*[503]

Q15: Then I ask: "*What is the greatest hindrance for improving in Mathematics*?" There is no answer. Are there no hindrances?

B15: I cannot think of anything. Well, I do not know, I cannot think of any.

Q16: Not to you? There are no hindrances to you? For improving in Mathematics?

B16: I do not know. I cannot think of any.

Q17: But the best means for learning Mathematics, then you say "*interest*", one should take an interest in things?

B17: Yes, I think so. What is of interest to me, namely Mathematics, Physics and Chemistry, they are the subjects I am best at, whereas a subject that I actually hate is Danish, that is also the toughest, that is also what I score least in. So you have to take an interest in it, and then you have to practice, and do your homework and such, then it will be fine. But the most important is the interest. Because it is the interest that motivates you to do these things. So, without that, it does not work. So Danish homework, that is something I rarely do.

503

Q15: Så spørger jeg også om, hvad er den største forhindring for at blive bedre til matematik? Der er ikke noget svar. Der er ikke nogen forhindringer?

B15: Jeg kan ikke komme i tanker om noget. Altså, jeg ved det ikke, jeg kan ikke komme i tanker om noget.

Q16: Ikke for dig. Der er ikke nogen forhindringer for dig? For at blive bedre til matematik?

B16: Jeg ved det ikke. Jeg kan ikke komme i tanker om noget.

Q17: Men det bedste middel til at blive bedre til matematik, der siger du interesse, man skal interessere sig for tingene.

B17: Ja, det synes jeg. Det jeg interesserer mig for, som jo er matematik og fysik og kemi, det er også de fag jeg er bedst til, hvorimod et fag jeg sådan set hader som dansk, det er også sværest, det er også det jeg sådan, scorer mindst i. Så du skal interessere dig for det, og så skal du også øve dig, og lave dine lektier og sådan, så går det fint. Men det største, vil jeg mene, det er interessen, faktisk. For det er interessen der motiverer dig til at lave de her ting. Så uden den, så går det ikke. Så dansklektier, det er noget jeg sjældent laver.

### *3.G. WHAT TO LIKE ABOUT MATHEMATICS*[504]

Q18: So it is not that all subjects get a 10?

B18: No, it is probably just mathematics. So...

Q19: What do you think is special about mathematics that makes it...

B19: I do not know, it is just, things like numbers and such. You do some tasks, where you have one concrete answer at the end, whereas in other subjects, then you can have like...then you have to argue for everything conceivable and do all sorts of other things, when in Mathematics, there is just one answer, and it is the same with Physics and Chemistry, they also have my interest. And then you do not have to argue for the results and such. There is just one answer.

### *3.H. MATHEMATICS APPLIED IN OTHER SUBJECTS*[505]

Q20: Do you still have Physics and Chemistry?

B20: Yes, I have A-level Physics and Chemistry as well.

Q21: So you can still apply Mathematics for to other subjects?

B21: Yes, sometimes you can apply Mathematics in Physics, there are some times, we… arh… I cannot think of anything. Something about motion, when for example you use a golf ball for... use a golf ball, for example. When you shoot, in golf. Then one applies integral calculus, for

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504

Q18: Så det er ikke fordi alle fagene ligger på en 10'er?

B18: Nej, det er nok kun matematik. Så...

Q19: Hvad synes du der er særligt ved matematik, der gør det...

B19: Jeg ved ikke, det er bare, sådan noget med sådan, tal og sådan... Man laver nogle opgaver, hvor man har ét konkret svar til sidst, hvorimod i andre fag, der kan man have sådan... Der skal man argumentere for alt muligt og lave alt muligt andet, hvor i matematik, der er der bare ét svar, og det er det samme med fysik og kemi, og det er det der ligesom interesserer mig. Og så skal man heller ikke argumentere for svarene og sådan noget. Det er bare, ét svar.

505

Q20: Har du stadigvæk fysik og kemi?

B20: Ja, jeg har det på A, fysik og kemi, også.

Q21: Så du kan stadigvæk bruge matematik til de andre fag?

B21: Jo, man kan godt bruge matematikken i for eksempel fysik, der er nogle gange vi har nogle... Arh, jeg kan ikke komme i tanker om noget - Noget med nogle bevægelser, når man for eksempel bruger en golfbold til at... Bruger en golfbold for eksempel. Når man skyder, sådan, i golf. Så er det man bruger integralregning, sådan til for eksempel at se hvor langt den kom, eller noget andet. Så man bruger matematikken i fysik, men ikke så meget i kemi, mest fysik.

checking how far it went, or something. So you use Mathematics in physics, but not that much in Chemistry, mainly in Physics.

*3.I. MATHEMATICS IN SOCIETY*[506]

Q22: Can you use Mathematics outside school? What does it mean to our society?

B22: Well, it is such things like finances I think of, which may be something with money or interest. Otherwise I cannot think of anything else.

*3.J. MATHEMATICS AS A SCIENCE*[507]

Q23: What about the Mathematicians, what are they doing with this Mathematics?

B23: I do not know, really. I do not know, well, it cannot just be tough tasks and such...they may see it in relation to society, what it is exactly, I do not know.

Q24: Is Mathematics something human beings discover, or is it something they invent?

B24: They probably discover it, because it is something which has always been there. It is just to discover it, and then... you cannot invent it really, well you can invent some

506

Q22: Kan man bruge matematik udenfor skolen? Hvad betyder matematik for vores samfund?

B22: Altså, så er det sådan noget som økonomi jeg tænker på, som måske noget med nogle penge og noget rente-agtigt noget. (…) Ellers kan jeg ikke lige komme i tanker om noget mere.

507

Q23: Hvad med matematikerne? Hvad laver de med den dér matematik dér?

B23: Det ved jeg faktisk ikke. Jeg ved det ikke, altså, det kan jo ikke bare være at lave sådan nogle svære opgaver og sådan... De ser det nok i forhold til samfundet, men hvad det præcist er, det ved jeg ikke.

Q24: Er matematik noget mennesker opdager, eller er det noget mennesker opfinder?

B24: De opdager det nok, for det er noget der altid har været der. Det er bare at opdage det, og så... Man kan jo heller ikke finde på det, på dén måde. Altså, man kan finde på nogle formler og sådan, men ellers er det der, og så er det bare...

Q25: Er der overhovedet flere formler at finde på? Er de ikke alle sammen fundet?

B25: Det ved jeg ikke, det kan godt være der er flere. Det kan også være de allerede er fundet. Jeg ved det ikke. Altså, der er jo ikke en grænse, er der? Så det er bare... Ja, der er nok flere, vil jeg mene.

Q26: Du skriver, "hvad laver en professionel matematiker på et universitet?", "Eventuelt matematik, set i sammenhæng med hverdagen".

B26: Det var bare lige det jeg kom i tanker om jeg ved. Altså, det er ligesom det jeg allerede har sagt, det er sådan noget som renter og som revisor eller noget lignende, men ellers kan jeg ikke lige komme i tanker om noget.

formulae and such, but otherwise, it is there, and then it just is there...

Q25: Are there any more formulae to invent at all? Have they not been found, all of them?

B25: I do not know, there may be more. It may be that they have already been found. I do not know. Well, there is probably not a limit, is there? So it is just...yes, there are more, I would say.

Q26: You answer to "*What is a professional mathematician at a university doing?*": "*Possibly Mathematics seen in relation to everyday life*"

B26: I was just what came to my mind, that I knew. Well, it is like I said already, it is things like interest and as accounting or something like that, but otherwise I cannot think of anything.

***3.K. CHALLENGES***[508]

Q27: I asked you to tick off what issues involve more challenges to you, and remembering stuff is what involves the least challenges to you, and then slightly more in terms of finding out what a task is about.

B27: Yes, it is like I just said. I may not understand a task, but I can remember, for example, I can remember how I solved it earlier. So, consequently I can solve the same sort of task, just with some other figures now. And then I also remember formulae. But sometimes, to understand a task it is rather the new tasks we get now in the third year. Tasks from the first or the second year, I remember those,

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Q27: Jeg beder jer sætte nogle krydse ved hvilke ting I har flest udfordringer med, og (…) at huske ting, det er det der er færrest udfordringer med. og så er der en lille smule mere med at finde ud af hvad en opgave går ud på.

B27: Ja, det er ligesom det jeg lige har sagt. Det kan godt være jeg ikke forstår en opgave, men jeg kan godt huske hvordan, for eksempel- Jeg kan for eksempel også husker hvordan jeg har lavet den tidligere. Så derfor kan jeg godt lave den samme, bare med nogle andre tal nu. (...) Og så kan jeg også huske formler. Men det nogle gange at forstå en opgave, det er mere de nye opgaver vi får nu i 3.g. Opgaver fra 1. og 2.g, det kan jeg godt huske, men nogle nye som vi lige har haft, det har jeg lidt svært ved, men det kommer nok her.

Q28: Hvad er det der er nyt ved de nye opgaver?

B28: Det er bare fordi jeg ikke har arbejdet med dem så meget, så derfor kan jeg ikke huske det. Hvorimod, de andre ting, dem har jeg arbejdet med så meget at jeg bare kan huske formlerne, så.... Så det handler bare om at arbejde mere med det og at øve sig, sådan, så kan man også huske det.

but some new, that we just had, they are tougher to me, but it will come along.

Q28: What is it about the new tasks?

B28: It is just because I have not worked with them that much, then I do not remember it. Whereas the other things, those I worked with so much that I just remember the formulae, so... so it is just a matter of working with it and practice, so that you remember it.

***3.L. MATHEMATICS IN FUTURE LIFE?***[509]

Q29: After upper secondary school, now you are about to say farewell to your class and to Mathematics. How do you feel about...

B29: ...Mathematics? Well, I am fine with that. And then...but I do not think that I will, like, keep working with it. Because I cannot, as I said earlier, I cannot really see any jobs in it. Otherwise it is actually what interests me the most. So if I saw any potential jobs in it, then I had probably...then I might have chosen it. Who knows, maybe it will be Mathematics?

Q30: What kind of jobs do you find interesting? What could be exciting to work with?

B30: I thought...It has not so much to do with Mathematics, but it is more like, helping people and such, concerning health and...yes, health, medicine, something like that. To help people, be in contact with people all the time. To have a varying work schedule and such...Yes!

509

Q29: Efter gymnasiet, nu skal I jo snart til at sige farvel til klassen og matematikundervisningen og det hele. Hvordan har du det med at skulle slutte fra...

B29: ...Matematik? Altså, jeg har det fint nok med det. Og så... Men jeg tror ikke jeg kommer til at sådan, arbejde videre med det. For jeg kan ikke ligesom sådan, ligesom jeg sagde tidligere, jeg kan ikke rigtig se nogle job i det. Ellers er det nok det der interesserer mig mest. Hvis jeg så nogle potentielle gode job i det, så havde det nok... Så kunne det godt være jeg valgte det. For jeg tager også et sabbatår for at finde ud af hvad jeg ligesom skal. Så kommer jeg nok til at undersøge nogle ting. Hvem ved, det kan godt være det bliver matematik.

Q30: Hvilke slags job synes du lyder interessante? Hvad kunne det være spændende at arbejde med?

B30: Jeg tænkte... Det har ikke så meget med matematik at gøre , men det er mere sådan, at hjælpe mennesker og sådan, hvad angår sundhed og... Ja, sundhed, medicin, noget i dén retning. At hjælpe mennesker, være i kontakt med mennesker hele tiden. Have vekslende arbejdstider og sådan... Yes...

### *3.M. GRADES*[510]

Q31: So your marks in Mathematics, it has a great influence, because it is the average of grades, that determines whether one is admitted to medical school, for example?

B31: Yes, precisely. So the mark in Mathematics, right now they are on an A or a B. And it pulls up the average, because other things draw it down. So the marks in Mathematics are important to me, yes. And the same in other subjects. This is what eventually may enable me to choose the tertiary education I wish, but anyhow, one might as well have a good average of marks.

### *3.N. USE OF MATHEMATICS*[511]

Q32: But otherwise, the mathematics you learn, can you use it afterwards? Can you use what you have learned, for something?

B32: Not in everyday life, I do not think so.Well, of course I can sit in a car and find out, okay, I drove this and that per hour, and then when will I be there and when there. But... otherwise not. It would rather be physics, maybe, and...yes.

510

Q31: Så din karakter i matematik, den har stor betydning, fordi det er gennemsnittet der gør om man kommer ind på for eksempel medicinstudiet?

B31: Ja, præcis. Så den karakter fra matematik, lige nu, p.t. så er den på 12-10. Og den trækker gennemsnittet op, fordi der er nogle andre ting der ligesom trækker det ned. Så karakteren i matematik, den er vigtig for mig, jo. Og det samme i nogle andre fag. Så er det også det der gør at jeg i sidste ende kan vælge det studie jeg sådan, ønsker. Jeg er endnu ikke sikker på hvilket studie jeg ønsker, men man kan ligeså godt have et godt karaktergennemsnit.

511

Q32: Men ellers, den matematik du lærer, kan du bruge den bagefter? Kan du bruge det du har lært til noget?

B32: (...) Ikke sådan hverdagsmæssigt, det tror jeg ikke. Altså, jeg kan selvfølgelig godt sidde i en bil og så finde ud af, okay, jeg har kørt så meget i timen, og så hvornår er jeg dér og hvornår er dér. Men... Ikke sådan ellers, nej. Det er mere sådan, fysik måske, og... Ja.

### *3.O. WHAT IS FUN ABOUT MATHEMATICS?*[512]

Q33: When you think Mathematics is fun, can you say what is fun about it?

B33: I did mention it a little. It is more like – I just like to work with numbers and such, formulae and so...like calculate something and get a precise answer. And...yes

Q34: Well, you think it is fun, but is it so much fun that if you are in a bad mood, then it cheers you up?

B34: No, when I say fun, it is more in relation to the other school subjects. It is not like, something I would do voluntarily in my spare time. Because there I have other things. Compared to the other things in school, then it is probably the most exciting and such, most interesting. But if I am grumpy or something, and I do Mathematics, it does not cheer me up.

### *3.P. CREATIVITY IN MATHEMATICS*[513]

Q35: Are you ever creative in Mathematics?

B35: How? Seen how? Which connection? Well, it may be that you get a task, and then you cannot remember any formulae, and then you can put together some things and then achieve it. Or, I had, for example, sometimes in the second year, when I could not remember the formula for...I do not remember completely, but then I remembered the proof, and then I deduced the formula, and finally I used

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512

Q33: Når du synes matematik er sjovt... Kan du sige hvad der er sjovt ved det?

B33: Jeg har allerede været lidt inde på det. Det er mere sådan- Jeg kan bare godt lidt at arbejde med tal og sådan, formler og så... Ligesom regne noget ud og så få præcist svar. Og... Ja.

Q34: Altså, du synes det er sjovt, men det er ikke sådan så det er så sjovt at hvis du er i dårligt humør bliver du glad af det?

B34: Nej, når jeg siger det er sjovt, så er det mere i forhold til de andre ting her i skolen. Det er ikke sådan, noget jeg frivilligt har tænkt mig at lave i min fritid. For der har jeg andre ting (...) I forhold til de andre ting her i skolen, så er det nok det mest spændende og sådan, mest interessante. Men hvis jeg er sur eller noget, og jeg laver matematik, så bliver jeg ikke glad af det.

513

Q35: Er man nogensinde kreativ i matematik?

B35: Hvordan? Set i hvordan? Hvilken sammenhæng? Altså, det kan godt være man får en opgave, og så er det lige at man ikke kan huske nogle formler, og så kan man sætte forskellige ting sammen, og så kommer man frem til det. Eller, jeg havde for eksempel en gang i 2.g, hvor jeg ikke kunne huske formlen for... Jeg kan ikke helt huske det, men så kunne jeg huske beviset, og så lavede jeg beviset, og så kom jeg frem til formlen, og så brugte jeg den. På den måde kreativt. På den måde kan man godt være kreativ.

it. In that sense creative. In that sense you may well be creative.

*3.Q. EXPECTATIONS IN MATHEMATICS CLASS*[514]

Q36: Have the expectations to you as students changed during upper secondary school, do you think?

B36: From the point of view of the teacher?

Q37: From the point of view of the teacher.

B37: I do not know, honestly. It, well, it is very important when you do a proof, that you remember some specific things, especially compared to the first year, then it was not that important, if you remembered this symbol, but now in third year, it is important to remember it, and our teacher says that you should also remember it for the oral exam and such. In that sense, it may have changed. They have probably tightened up, or for our sake, they have tightened up a little, so you remember for the oral exam.

514

Q36: Har forventningerne til jer som elever ændret sig her undervejs i gymnasiet, synes du?

B36: Ud fra lærernes synspunkt?

Q37: Fra lærerens synspunkt.

B37: Det ved jeg s'gu ikke. (…) Altså, det er meget vigtigt når man for eksempel laver et bevis, at man lige husker nogle bestemte ting, for eksempel i forhold til, da vi gik i 1.g, der var det ikke så vigtigt at man lige huskede det her tegn, men her i 3.g, så er det vigtigt at man husker det, og vores lærer siger også at man skal huske det til den mundtlige eksamen og sådan. På den måde har det nok ændret sig. De er nok blevet lidt mere skrappe, eller for vores skyld lidt mere skrappe, så man kan huske det til den mundtlige eksamen.

## THEMES IN BRANDON'S 3RD YEAR

### MATHEMATICS AT SCHOOL

The most remarkable change [from the 1st year to the 3rd year] seems to be the issue of combining different sub fields of mathematics (3.A.).

[In the 3rd year, they are] Combining more things, which Brandon exemplifies with vectors and differential calculus (BRANDON: Q3-A).

For the 3rd year interdisciplinary study programme project (SRP), Brandon wrote in Mathematics and History. He found the mathematics, or rather statistics part to be pretty easy; it was just to use some formulae, substitute something, and then prove something, but relatively easy, he says. He found it to mainly be dealing with combining different things, but it did not appear to add anything new to his idea of mathematics, according to what he says.

The tasks they get [in mathematics at school] resemble each other a lot, and often one can just go back and read some of the previous tasks, and then change the numbers that are to be inserted into the formula (3.D.).

To Brandon, Mathematics is about solving tasks giving one specific result at the end. There is only one answer. And that goes for Physics and Chemistry as well. In other subjects, you have to argue for your answer (3.G.).

### MATHEMATICS AS A DISCIPLINE

Mathematics, in terms of e.g. differential calculus, is applied in physics (BRANDON: Q3-B).

Brandon suggests that mathematics is both invented and discovered, which was the same as in his first year questionnaire. Mathematicians in university might

deal with mathematics in relation to everyday life (BRANDON: Q3-C).

Mathematics can be used for describing movements, e.g. of a golf ball. Then you need to use integral calculus in order to find out how far the ball went. So mathematics is applied in physics, Brandon explains, but not that much in chemistry.

Brandon does not have a specific idea of mathematician in university might be doing for a living. They cannot just deal with difficult tasks, he reasons, so they might deal with mathematics in relation to society, he thinks, but without having a specific idea of how, exactly. And since mathematics is already there, they cannot just invent it. But at the same time, there cannot be an upper bond of the number of formulae, so there might be more of them (3.J.).

In some sense, Brandon finds that you can be creative in mathematics; once, in the 2nd year, he did not remember the formula he needed, but he could recall the proof of it. Then he carried out the proof and arrived at the formula. So in that sense, one can be creative in mathematics (3.N.).

### MATHEMATICS IN SOCIETY

Since mathematics is a part of everyday life, e.g. in trade, it is important that everybody learns mathematics, Brandon writes (BRANDON: Q3-B).

Outside an educational setting, Brandon mentions that mathematics is applied in economy, in terms of interest rates and money, (3.I.).

### MATHEMATICS AND ME

#### *RATING*

In 3rd year, Brandon keeps his rating of mathematics on the highest possible level, giving it [10] out of [10], the same as in 1st year. As opposed to his 1st year questionnaire, he is no longer in favour of working on his own only, but he now

adds working in pairs and the whole class together. As a change from earlier, both lower secondary school and 1st year in upper secondary school, Brandon mentions that one has become capable of combining more things, which he exemplifies with vectors and differential calculus (BRANDON: Q3-A).

Brandon seems to like Mathematics primarily because it is easy to him (3.C.).

*MATHEMATICS LEARNING*

The best means to improve in mathematics according to Brandon is to take an interest in the subject. His own tactics is to do his homework and make sure he understands it (BRANDON: Q3-D).

Brandon sees himself to be quite active in class in terms of answering questions, and less active when it comes to asking (BRANDON: Q3-F).

Brandon primarily seems to learn by heart immediately and then subsequently develop understanding, but the reverse order seems to have occurred as well (BRANDON: Q3-XB).

Brandon seems rather confident with most subjects for the oral and written examinations in A-level mathematics, except perhaps "Differential Equations and their solutions".They just had this topic recently, and he does not yet feel that he is on top of it. Geometry in arbitrary triangles, on the other hand, might be a favourite (BRANDON: Q3-XC).

Mathematics is still enjoyable to Brandon now in the 3rd year. He still emphasises that he finds the tasks easy, and he also mentions that he gets good grades in the subject (3.A.).

*UNDERSTANDING & REMEMBERING*

Usually Brandon remembers new things before he understands them; he may be able to solve some tasks, even though he does not understand what it is about, due to his ability to apply formulae. It is like that for vectors in euclidean space, but he still succeeds in solving the tasks. The tasks they get resemble each other a lot, and often he can just go back and read some of his previous tasks, and then change the numbers that are to be inserted into the formula. For some of the tasks in Euclidean space, they are supposed to draw, which Brandon is not good at. But, then, because he has some formulae, he can solve the tasks anyway (3.D.).

The proofs they had earlier, Brandon wrote down in a notebook. And by doing that he finds that he understands better. But he has not done that for geometry in Euclidean space yet, because they just had it, but when he does, he is most certain he will come to understand it (3.E.).

*CHALLENGES & SUPPORT*

Now in the 3rd year, Brandon finds [R*emembering]* to involve the fewest challenges to him compared to the other categories in the questionnaire. Instead, [*Figuring out the purpose of a task*] now involves moderate challenges (BRANDON: Q3-E).

The sources of support for mathematical activities are still friends and the Internet, but Brandon also mentions the teacher and the collection of formulae. He does not perceive himself as having any special challenges in mathematics compared to his classmates, but he finds himself to be really good at remembering formulae, but also at finding a way to solve a task(BRANDON: Q3-E).

*PLANS*

Medicine is still Brandon's preferred plan for tertiary education, but something

comparable might do as well. The final marks in mathematics could have a great influence on these plans, since it demands a high average of grades to be admitted to medical school. Brandon would not mind opting for an education containing a good deal of mathematics, but he is unable to imagine any exciting in that. Which is peculiar, since his last comment is that mathematics is exciting (BRANDON: Q3-G).

Brandon is quite fine with quitting mathematics after upper secondary school, and he does not seem to expect to keep up with it. He cannot really see any jobs in it. Otherwise, mathematics is what interests him the most. But if there had been some good jobs in mathematics, he would have been fine in carrying on with mathematics. Jobs concerning helping people appeal, such as medicine, appeals the most to Brandon. So if he gets the grades for studying medicine, that is what he wants to do (3.L.).

## BRANDON'S 3RD YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics is slightly more demanding now.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is bot invented and discovered – or maybe just discovered.

### *MATHEMATICS IN SOCIETY*

Trade and the like uses simple mathematics. And everybody should learn it.

### *MATHEMATICS AND ME*

Brandon is not just as much on top of things as he were in the first year. But he is confident that he will catch in before the exam. Brandon likes mathematics, but he does not love it.

## BRANDON'S BELIEFS TRANSPOSITION

Brandon's beliefs about mathematics does not seem to change much. He still likes it, he still chooses it due to its role for admittance to studying medicine, and there is little change in his ideas of mathematics outside school. Also his idea of the nature of mathematics corresponds fairly well to his beliefs in the first year.

# *THE CASE OF ADELE*

*Adele is a female student from Alfa Upper Secondary School, in a mathematics-physics study programme which involves studying A-level mathematics. In the 1st year, she considered either university or the military as possible options after graduation. In the 3rd year, her plans are to take a university degree in the humanities to avoid mathematics. Her rating of mathematics has been on [7] both times she answered the questionnaire.*

| Adele | Date for Questionnaire | Date for interview |
|---|---|---|
| 1st Year | 24 November 2010 (+ supplement 10 January 2010) | 4 May 2011 |
| 3rd year | 28 November 2012 | 4 March 2013 |

*Table 67: Dates for Questionnaires and Interviews*

## ADELE'S 1ST YEAR QUESTIONNAIRE

| Q1-A | TRANSITION | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [7] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [515] | [Working in pairs]; |
| **3** | Did you like mathematics when you went to lower secondary school? [516] | [It was not really me] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | I have become more fund of the subject. It seems more professional and the teaching, homework and written homework works well.[517] |
| **4b** | Is there anything you liked better before?' | No |
| **4c** | Is there anything you like better now? | Yes, there is order in the classroom and the teacher is able to teach. In this way, mathematics becomes a pleasant subject.[518] |

*Table 68: ADELE'S 1st year questionnaire, part A – TRANSITION*

515 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

516 Possible responses: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

517 Jeg er blevet mere glad for faget. Det virker mere professionelt, og der er styr på undervisningen, lektier og afleveringer.

518 Ja, der er ro i timerne, og læreren får lov til at undervise. På den måde bliver matematik et hyggeligt fag.

| Q1-B | FOR SCHOOL | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | I believe that mathematics strengthens ones sense of logic, such that you can improve.[519] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **6** | What made you choose a study programme involving A-level mathematics? | At the end of lower secondary school, I became fund of the subject, and that is why I wanted to give it a chance in upper secondary school. [520] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | In physics we compute with different units, where mathematics definitely is involved. Also in chemistry, there is units, you should compute, etc.[521] |

*Table 69: ADELE'S 1st year questionnaire, part B - MATHEMATICS AND SCHOOL*

519 Jeg tror at matematik styrker ens logiske sans, så man kan bliver dygtigere.

520 I slutningen af folkeskolen blev jeg glad for faget, derfor ville jeg give det en chance på gymnasiet.

521 I fysik regner vi med forskellige enheder, hvor matematik klart er involveret. I kemi er der også enheder, man skal kunne regne om osv.

| Q1-C | BEYOND SCHOOL | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | All kinds of shops (markets), banks, business world.[522] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings?[523] | [None of these options] |
| **10** | What do you think a professional mathematician at a university is doing? | Probably, one concentrates on a single "topic" within mathematics, and immerse in that.[524] |
| **11** | Would you have to be a genius in order to study mathematics in university? [525] | [I do not know] |

*Table 70: ADELE'S 1st year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

522 Alle slags butikker (markeder), banker, erhvervslivet.

523 Options: [Mathematics is invented]; [Mathematics is discovered]; [Both]; [None of these options]; [I do not know]

524 Man koncentrerer sig nok om et enkelt "emne" indenfor matematik, og så går man i dybden med det.

525 Options:[Yes]; [No]; [I do not know]

| Q1-D | IMRPOVING | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | That I need a substantial amount of time on a task.[526] |
| **12b** | What do you think is the best means for improving in mathematics? | Practice[527] |
| **12c** | What do you do to improve in mathematics? | I practice[528] |
| **13a** | What do you do if you get stuck on a task at school? | I ask my classmates for help, and if they cannot help, then I ask my teacher.[529] |
| **13b** | What do you do if you get stuck on your homework? | Then I ask my father or my brother[530] |

*Table 71: ADELE'S 1st year Questionnaire, part D – STRATEGIES FOR IMPROVING*

526 At jeg skal bruge rimelig lang tid på en opgave.

527 Øvelse

528 Jeg øver.

529 Jeg spørger mine kamerater om hjælp, og hvis de ikke kan, så spørger jeg min lærer.

530 Så spørger jeg min far eller bror.

| Q1-E | CHALLENGES & SUPPORT | | ADELE |
|---|---|---|---|
| # | *Question* | | *Answer* [531] |
| **14** | What issues involve more challenges to you? | a) Remembering | [3] Moderate |
| | | b) Computing | [5] The fewest |
| | | c) Figuring out the purpose of a task | [2] Several challenges |
| | | d) Finding a way to solve a task | [4] Few challenges |
| | | e) Reading and understanding the textbook | [4] Few challenges |
| **15** | Where can you find support for mathematical activities? [532] | | [From siblings]; [From classmates] |
| **16** | Did you parents take the Upper Secondary School Leaving Certificate? [533] | | [Both of them did] |

*Table 72: ADELE'S 1st year Questionnaire, part E – CHALLENGE AND SUPPORT*

531 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

532 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From classmates]; [Other places] If other places, from where or from whom?

533 [Yes, my mother did]; [Yes, my father did]; [Both my parents did]; [None of them did]

| Q1-F | IN CLASS | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? | [0] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [No] |
| **17c** | Are you content with that? | No, I should improve in asking for help[534] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? | [0] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | No, I should dare to ask some more[535] |
| **19a** | In your class, is it okay to be good at mathematics? | Yes. |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Maybe a bit[536] |

*Table 73: ADELE'S 1st year Questionnaire, part F - MATHEMATICS IN CLASS*

534 Nej, jeg skal blive bedre til at spørge om hjælp.

535 Nej, jeg skal turde svare noget mere

536 Måske en smule.

| Q1-G | PLANS | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | University or the Defence[537] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [I do not know] |
| **21b** | Comments: | I do not know what I want[538] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [I do not know] |
| **22b** | Comments | I do not know what I want |

*Table 74: ADELE'S 1st year Questionnaire, part G – PLANS*

537 Universitetet eller måske forsvaret.

538 Jeg ved ikke hvad jeg vil være.

# ADELE'S 1ST YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 4 May 2011 | 33:58 |

### *1A. MATHEMATICS AT SCHOOL I*[539]

Q1: The first thing I will address is how you think mathematics has changed since lower secondary school?

A1: Well, definitely harder, it is. You are introduced to more things. For example sine, cosine and the equation for the circle and such things. So it is a lot of new things, you are introduced to. And that is of course a little hard.

Q2: Is there any difference in the way you work with it?

A2: Yes, there is. There is. Well, in my school, lower secondary school, it was rather slacky. It was one of those schools not performing optimally. Whereas here, we get a topic, we work with it, we go over it on the blackboard and then we get a report or an exercise in it, and then it is corrected at the blackboard afterwards, so we know what has been calculated correctly, so it is really, really good teaching we get up here compared to lower secondary school.

539

Q1: Det første jeg sådan vil komme ind på, det er, hvordan du synes matematik har ændret sig siden du gik i folkeskolen.

A1: Jo, det er helt klart sværere - det er det. Det er sådan mange flere ting, man bliver præsenteret for. For eksempel sinus og cosinus og cirklens ligning og sådan nogle ting. Så det er mange nye ting, man bliver præsenteret for. Og det (utydeligt) selvfølgelig lidt svært.

Q2: Er der også forskel på den måde, I arbejder med det?

A2: Ja, det er der. Det er der. Altså på min skole, folkeskole, der var det meget meget løst. Det var en af de skoler, der ikke rigtig sådan fungerede fuldstændig til tops. Altså her der får vi et emne, vi arbejder med og får det gennemgået på tavlen og så får vi en rapport eller en opgave for i det, og så retter vi det på tavlen bagefter, og så bliver vi ligesom klar over hvad der skal regnes rigtig og så videre, så det er rigtig, rigtig god undervisning vi får heroppe i forhold til folkeskolen.

***1.B. FAVOURITE SUBJECT SCALE***[540]

Q3: I asked that on a scale from 1 to 10, on which 10 would be your favourite subject, where is mathematics? And at that time, you ticked off [7]. Is that still valid?

A3: Maybe it has dropped a little. But it is more because the other subjects have become more interesting.

Q4: Yes.

A4: Hmm.

Q5: Are any specific subjects for which you have increased your interest?

A5: Yes, but it is because, in the introductory term, we were not taught German, but we are now, and I find it really exciting. And physics, I prefer that to mathematics. And...but mathematics, if it is not a "7", then it is at least around "6". Well, it is not a poor rating, it has. It is a good rating, and I will say that I am not too good at it. Because I am a little below average, trying as well as I can. But it is just the way in which we work, and the thing about that you sometimes, well, sometimes it can be hard but at other times you can have success, and then you actually become happy.

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Q3: Jeg spurgte til på sådan en skala fra 1 til 10, hvor 10 er dit yndlingsfag, hvor ligger matematik (…) og på det tidspunkt, hvor du fik spørgeskemaet, så satte du kryds ved 7. Er det stadig du synes det passer?

A3: Hmm, ja... (Trækker på det.) Nej, måske faldet lidt. Men det er mere fordi at så er de andre fag blevet mere interessante.

Q4: Ja.

A4: Øhm...

Q5: Er der nogle bestemte fag, som du har fået mere interesse for?

A5: Ja, men det er fordi, at i grundforløbet der havde vi ikke tysk, men det har vi så fået nu her, og det synes jeg er rigtig spændende. Og fysik, det kan jeg bedre lide end matematik. Og... Men matematik, hvis det ikke lige er en 7'er, så ligger det i hvert fald på en, omkring 6. Altså, det er ikke en lav placering, det har. Det har en god placering, og jeg vil sige, jeg er ikke særlig god til det. For jeg er sådan lidt under middel, prøver så godt jeg kan i hvert fald, ikke? Men det er bare måden, vi arbejder på, og det med at man nogle gange godt, altså, nogle gange kan det godt være svært, men andre gange så kan man godt få en succes og så bliver man faktisk glad.

### *1.C. SUPPORT AT HOME*[541]

Q6: Who can help you, if you run into something for which you do not know how to proceed?

A6: Well, I enjoy that enormous privilege that my father is teacher [at an upper secondary school]. He teaches physics and [another subject], so he also knows a little mathematics. So I can ask him for help. But my brother, because he has the same study programme as me, just in 3rd year. And otherwise, then we have the homework café, which is really good. Yes, where you can get help from the teacher there. But otherwise, it is primarily my father, that I talk to.

Q7: Do you need your father more now, compared to earlier? Did you draw on your father at all in lower secondary school?

A7: Yes, I did. A little. But, yes, he knows about the new things, for example what I mentioned before: Sine and cosine and the circle and functions and all that. So, I draw on him more now. Yes, I do.

### *1.D. SUPPORT AT SCHOOL*[542]

Q8: But when you work here, then you can draw on each other and the teacher?

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Q6: Hvem kan hjælpe dig, hvis du løber ind i noget, hvor du ikke selv ved hvordan du kommer videre.

A6: Altså, jeg har jo det kæmpe privilegium i at min far han er lærer [at an upper secondary school]. (Sif ler.) Han er lærer i fysik og [another subject], så han kan jo også lidt matematik. Så ham kan jeg spørge om hjælp. Men også min bror, fordi han har samme linje som mig, bare i 3.g. Og så ellers, så har vi jo lektiecafé, som er sådan rigtig rigtig god. Ja, hvor man kan jo få hjælp af læreren, der er her. Men ellers så er det primært farmand, som jeg går hjem til.

Q7: Har du mere brug for din far nu, end før? Brugte du overhovedet din far, da du gik i folkeskolen?

A7: Ja, det gjorde jeg, det gjorde jeg. Lidt. (...) Men jo, han ved jo noget om de nye ting, for eksempel om det jeg sagde før, sinus og cosinus og cirklen og funktioner og alt det der. Så der - jeg bruger ham mere nu. Ja, det gør jeg

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Q8: Men når I arbejder her, altså, så kan man måske bruge hinanden og læreren?

A8: Ja, det kan man. Altså, det kan man helt klart. Men jeg synes også bare vores undervisning, den er simpelt hen så skøn, fordi at der bliver taget hånd om det hele. Og der bliver gennemgået rigtig godt. Og så synes jeg også at vi har sådan nogle dejlige venner i klassen, at hvis man nu ikke rigtig kan finde ud af det, så er det lovligt og uskyldigt at spørge: "*Ih, jeg kan ikke finde ud af det, vil du ikke lige forklare mig, hvordan det er?*" Så det er rigtig dejligt. (...)

A8: Yes, you can. Well, definitely. But I also think that our teaching, it is just so great, because everything is taken care of. And everything is went over really well. And I also think that we have some lovely friends in class, so if you do not know how to work out something, then it is legitimate and innocent to ask: "*Oh, I cannot work it out, could you explain to me how it is?*" So that is really wonderful.

***1.E. ON KINDS OF MATHEMATICAL CHALLENGES***[543]

Q9: I also ask to things you do when you do mathematics, and what you find to involve the most and the fewest challenges. You tick off fewest challenges for "*Computing*" and the one ticked off as giving several challenges is "*Figuring out the purpose of a task*". Could you say some more about that?

A9: Yes I can, definitely. It is actually a tough challenge to me. Quite often, when I read a task, then I could read it wrongly, if, and I have to ask: "*What is it I am supposed to find out?*" Because when I know what I am supposed to do, then it is just to find a formula from our textbook, and then I can say: "*Oh, but it is that.*" But sometimes I cannot really work out what I am supposed to do.

Q10: Is this when you reach out for a friend, or?

A10: Yes, then I reach out for a friend and say: "*What are we supposed to do, really?*" "*Oh, but you should do this and that and such...*" Then I find my book and find the formula or something like that.

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Q9: Jeg spørger også til (...) ting, man laver, når man laver matematik, og hvad man så synes der er flest og færrest udfordringer med (…) Du sætter "*Færrest udfordringer*" med at "*regne ting ud*". (...) Og så den, du har sat til, at der er lidt flere udfordringer med, det er "*At finde ud af hvad en opgave går ud på*" (…) Kan du sige noget mere om det?

A9: Ja, det kan jeg, helt klart. Det er faktisk en stor udfordring for mig. Det er tit, når jeg læser en opgave, så kan jeg læse den forkert, hvis, og bliver nødt til at spørge: "*Hvad er det, jeg skal finde ud af?*" Fordi når jeg så finder ud af, hvad jeg skal, jamen så kan jeg jo bare hive en formel frem fra vores bog, og så kan jeg sige: "*Nå, men, det er det*." Men nogle gange så kan jeg ikke rigtig finde ud af, hvad det er jeg skal. (...)

Q10: Er det der hvor du så griber fat i enten kammerater eller?

A10: Ja, så griber jeg fat i mine kammerater og siger: "*Hvad er det egentlig vi skal*?" Eller også så griber jeg fat i far derhjemme, hvis det er en opgave, vi får for derhjemme og siger: "*Hvad er det, jeg skal?*" "*Nå, men du skal gøre sådan og sådan...*" Så tager jeg min bog frem og finder formlen frem eller sådan noget, ikke?

### *1.F. CHOOSING STUDY PROGRAMME*[544]

Q11: Were you in doubt of which study programme to choose, when deciding your upper secondary education?

A11: Well, just before we to decide, I had become fond of mathematics in lower secondary school, because we got a good teacher the last year. I did not always like Mathematics, but suddenly I started to like it. And my brother, he has the same study programme, he said I could just try it out for the introductory term. One can always change. And then I tried it. And then I have actually become fond of it.

### *1.G. STUDY PROGRAMME AND PLANS*[545]

Q12: Is it something you need for further education? Or is it: "*No, I thought it would be fun*"? Or: "*I would like to keep my possibilities open*"?

A12: Well, yes, back then I thought I might want to become a doctor. But then I found out, here at upper secondary school, that it might be a little hard. You have to be top of the pop, to get admitted to those educations. Si that was why I chose this study programme, because I knew I could use it for something more. But otherwise, well, yes.

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Q11: Var du i tvivl om hvilken studieretning du skulle vælge, da du skulle vælge ungdomsuddannelse?

A11: Altså, lige inden vi skulle til at vælge, der var jeg blevet lidt glad for matematik i folkeskolen, fordi vi fik jo en god lærer i det sidste år. Jeg har ikke altid været god til matematik, men lige pludselig blev jeg sådan glad for det. Og min storebror, han har den samme linje, han sagde at jeg bare kunne prøve det, der var jo grundforløb. (…) Man kan altid skifte. Og så prøvede jeg det. Og så er jeg egentlig blevet meget glad for det.

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Q12: Er det noget du tænker du skal bruge til en uddannelse? Eller er det: "*Nej, jeg synes det var sjovt*"? Eller: "*Jeg vil gerne holde mulighederne åbne*"?

A12: Ja, jo, dengang der tænkte jeg at jeg måske gerne ville være læge. Men så fandt jeg ud af, herude på gymnasiet at det nok er lidt svært. Fordi der skal man nok være toppen af poppen, ikke, for at kunne komme ind på de studier der. Så det var også en af grundene til at jeg valgte den her linje, det var fordi jeg vidste, at den kunne man bruge til noget mere. Men altså, ja... Så. Ja.

### *1.H. STUDY PROGRAMME AND OTHER SUBJECTS*[546]

Q13: And the other subjects in the study programme, Physics and Chemistry. Was it your interest in them that made you choose this line of study? Or was it: "*Oh, good to have them, to have those subjects, then I am sure that I...*"?

A13: It was a mix, because physics-chemistry in lower secondary school, it was not on our, it was not what I see up here. But I was curious about what it was. Because my brother said that it was completely different. And my father also said, he is a physics teacher, he said: "*It is not what you are doing down there, try and have a look of what it is like in upper secondary school.*" And then eventually I have become incredibly fond of physics. Chemistry is a little complicated, sometimes. I think it is about working it out, and know what it is about. Then you can.

### *1.I. MATHEMATICS FOR ALL?*[547]

Q14: I also ask if mathematics is something everybody should learn. Is there some of what you learn now, for which you think: "*No, it may be that not everybody should take A-level mathematics, but everybody would benefit of knowing exactly this*"?

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Q13: Og de andre fag på linjen, altså fysik og kemi. Er det også sådan, er det også interessen for dem, der gjorde, du har valgt den her linje? Eller det også "*Ej, det er godt at have dem, at have de her fag, fordi så er jeg sikker på, at jeg...*"

A13: Ja, det var sådan lidt blandet, fordi fysik-kemi i folkeskolen, det er jo ikke i hvert fald på vores, det var ikke lige sådan det jeg ser heroppe. (…) Men jeg var meget nysgerrig på at se hvad det var. Fordi min storebror sagde jo, at det var noget helt andet. Og min far sagde jo også - han er jo fysiklærer - han sagde: "*Det er altså ikke sådan noget du går rundt og laver dernede, så du kunne jo prøve og se hvad det var oppe på gymnasiet.*" Og så er det endt med, at jeg er blevet utrolig glad for fysik. Kemi, det kan godt være lidt indviklet nogle gange. (...) Jeg tror, det handler bare om at selvfølgelig bare at kunne finde ud af det, og kunne finde ud af, hvad det går ud på, ikke? Så kan man godt.

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Q14: Jeg spørger også til, om matematik er noget, som alle bør lære. (...) Er der noget af det, du lærer nu, hvor du tænker: "*Nej, men det kan godt være, at alle ikke skal have matematik på a-niveau, men alle ville da egentlig have glæde af at kunne lige præcis det her*."

A14: Altså, nogle gange taler jeg med mine venner, der har valgt fx handelsskolen og sådan noget. Og så synes jeg jo, at det er vildt spændende at kunne sige til dem: "*Jeg har fundet ud af, hvordan man kan fx det med cirklens ligning, hvordan man placerer en cirkel og jeg kan finde ud af, hvordan man finder ud af, hvor meget energi en elkedel bruger*." Men det synes de jo ikke er interessant. Så jeg vil sige, at jeg synes ikke, at det er noget som alle skal lære - kun nogen, som er interesseret i det. Men jeg vil mene, at man skal kunne noget matematik til husbehov. Altså kunne regne nogle små enkle ting ud. Og så hvis man får børn fx, så skal man også kunne kunne hjælpe dem. Men der er jo også mange der har svært ved det.

A14: Well, sometimes I talk to some of my friends who chose e.g. business college or something. And then I find it really exciting to tell them: "*I found out how one can, e.g. this with the equation of the circle, or I now know how to find out the energy consumption of an electric kettle*". But they do not find that interesting. So I would say, I do not think it is something everybody should learn, only those interested in it. But I do think that one should know some mathematics for household use. Well, be able to calculate simple things. And if you have children, for example, then one should also be able to help them. But there are many who struggle with it.

#### *1.J. CHOOSING STUDY PROGRAMME II*[548]

Q15: Did you deliberately go for a study programme with A-level Mathematics, or was it rather: "*Oh, I want this science study programme, and okay, it involves A-level Mathematics, then I just go for it*"?

A15: Yes, well, there was this study programme with Physics A and Chemistry A and Mathematics A, and then I said that I did not have to make it any harder than it had to be[549]. Because, what I wanted was something that could fulfil my dreams of medical school. Because the other option would be to take this biotechnology, but then I though: "*That is not for me*". I did not go for Mathematics on A-level. But I did want to try to have some more mathematics after I had become fond of it in lower secondary school.

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Q15: Gik du efter, at det skulle være en linje hvor der var matematik på a-niveau? Eller var det mere: "Ej, jeg vil gerne have den her naturvidenskabelige linje og fint nok, så er matematik på a-niveau, så tager vi bare det"?

A15: Ja, altså, der var jo en linje der også havde fysik A og kemi A og matematik A, og så sagde jeg, at jeg skulle ikke gøre det sværere end det var. Fordi det jeg egentlig var ude efter, det var jo at få en der kunne opfylde fx noget med noget lægestudie, ikke? (...) Fordi en anden udvej det ville være sådan noget bioteknologi, men så tænkte jeg: "*Det skal jeg slet ikke over i.*" (...) Jeg gik ikke efter, at matematik skulle være på A. Men jeg ville rigtig gerne prøve at have noget matematik, fordi nu var jeg blevet glad for det i folkeskolen.

549 *Adele's study programme involves Mathematics A, Physics B and Chemistry B, but interested students could upgrade Physics and/or Chemistry to A-level as well.*

### *1.K. MATHEMATICS IN OTHER SUBJECTS*[550]

Q16: Can you apply the mathematics you learn in your other subjects?

A16: Yes, because, now we just had, or we still have, about vectors. And our Mathematics teacher, who also teaches Physics, that you can apply it in Physics, when you measure force. How much force is there. And in which direction. So it is going to be exciting, when we in physics starts to apply it.

### *1.L. SOCIAL CONTEXT AT SCHOOL*[551]

Q17: You tick off that you typically raise your hand zero times, both in terms of asking and answering and that you actually: "*No, I am not content, I should dare some more*".

A17: I went to the school-home interview at the beginning of the term. And then exactly my Physics and Mathematics teachers, it is subjects that are new to me, because the only reason I chose them was that I was curious. And so I do not always dare to say what I (unintelligible). But often I catch myself in, when the others answer, that then it was the same as I would have said. So I can see that sometimes I am actually right.

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Q16: Ja. Kan du bruge det, du lærer i matematik, i dine andre fag?

A16: Ja, fordi, nu har vi lige haft, eller det har vi jo stadig væk, om vektorer. Og der har vores matematiklærer, som også er fysiklærer, siger jo at det kan man jo bruge i fysik, når nu man skal måle kræfter. Hvor meget kraft der er. Og i hvilken retning. Så det bliver jo sjovt, når vi så i fysik skal til at bruge det.

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Q17: Du sætter kryds ud for, at du typisk rækker hånden i vejret nul gange og typisk både med svar og spørgsmål og at du sådan faktisk: "Nej det er jeg egentlig ikke tilfreds med, jeg burde faktisk turde noget mere.

A17: Jeg var jo til forældresamtale i starten af året. Og der var de jo, netop min fysik- og matematiklærere, og det er jo nogle fag som jeg er lidt ny til, fordi den eneste grund til, at jeg valgte dem, det er jo fordi jeg var nysgerrig. Og derfor så tør jeg ikke altid lige sige, hvad det er, jeg (utydeligt). Men tit kan jeg også tage mig selv i, at når de andre svarer, så var det også det jeg selv ville have svaret. Så jeg kan godt se, at nogle gange, så er jeg faktisk rigtig på den. (…)

### *1.M. SOCIAL CONTEXT AT SCHOOL II*[552]

Q18: One takes into consideration what the others might think, when you ask or answer?

A18: Well, you would, if you have an answer of which you are unsure, you would not always say it. But it is also the atmosphere in class, I am afraid of how it would be perceived. If I am completely sure, I raise my hand. But I also often observe some of my able peers in class, some of the most skilful, that you know are good, they scuttle around in class, "*What did you get in this and that?*" and such things, marks for the year, everything. Some of the smartest students, they sometimes raise their hand and say something relatively unsure, and then the teacher must say no. But everybody still knows, "*Oh, he meant it in this and that way*" and then he is still intelligent. So everybody are allowed to say something which is not completely right.

### *1.N. GROUP WORK I*[553]

Q19: How much does it mean if one is active, when working in groups? What do you emphasise?

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Q18: Man tænker over, hvad de andre tænker, når man sådan spørger eller svarer?

A18: Altså, man vil jo - hvis man har et svar, man er usikker på, så svarer man ikke altid. Så ville jeg ikke svare. (…) men også det der, stemningen i klassen, jeg er bange for hvordan det bliver modtaget. Hvis jeg er rigtig sikker, så rækker jeg hånden op. Men jeg ser også tit nogle af mine kammerater i klassen, nogle af de dygtigste, som man ved er dygtige, de render rundt i klassen, "*Hvad har du fået i den og den*?", og sådan noget, årskarakterer, alt. Altså nogle af de klogeste elever, de rækker nogle gange hånden op og siger noget, der er sådan halv-usikkert og så må læreren sige nej. Men alle er stadig klar over, "*Nå, han mente det på den og den måde*" og han er stadig rigtig klog. Så alle har jo lov til at sige noget, der ikke er helt rigtigt.

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Q19: Hvor meget betyder det om man er aktiv, når man laver gruppearbejde (...)? Hvad lægger du vægt på der?

A19: (...) Altså der har jeg prøvet at være aktiv og så ikke aktiv, og at være aktiv det er jo også, når man kan finde ud af det. Og når man så kan finde ud af det, så går det lige pludselig stærkt. Og så dem, som måske ikke er blevet færdige, de kan ikke følge med. Og det er jo dejligt, at man kan sidde der og sige: "*Nu gør vi sådan og sådan, så gør vi sådan og sådan, og så isolerer der og så bla bla bla.*" Men jeg har jo også selv været i den situation, at man sidder der og man har ikke rigtig fattet hvad det helt går ud på. Og så lige pludselig, så går det bare hurtigt, fordi der er nogen, der har fattet det på ingen tid. Og så er det rigtig, rigtig godt, hvis man er sammen med nogen, der måske ikke er så hurtige, men forstående og tålmodige. Sådan så man også har en chance for at være aktiv selv. (..) Man bliver jo ikke rigtig klogere af at den anden sidder og laver al arbejdet. Så det er rigtig vigtigt, at man er aktiv.

A19: Well, I have tried being active and less active, and to be active is also when you can work it out. And when you can work it out, then suddenly it goes fast. And those, who might not have completed, they cannot follow. And then it is nice, if you can sit there and "*Now we do this and that, and isolate here, and so on...*"But I have also tried to be in that situation in which you just sit there and do not get what is going on. And then, suddenly, then it just goes fast, because someone figured it out in no time. And then it is really, really nice, if you are with somebody, who may not be that fast, bit who are understanding and patient. Such that one gets the chance to be active oneself. One does not improve, by somebody else doing the work. So it is really important that you are active.

***1.O. GROUP WORK II***[554]

Q20: And then you say, that if you are with someone, who quick as lightening has an idea of what to do, and compute ahead, then group work does not work that well?

A20: No, because then maybe five are sitting, working, and then two people suddenly has an idea, and then three people are just gazing around – then it is not optimal for them. I think it is so amusing, when I suddenly realise: "*Oh, I worked it out!*" And then you can see it is correct, right? Then you go ahead, but the other, they just sit there and cannot follow.

***1.P. ACCEPTANCE FROM FRIENDS***[555]

Q21: Do you consider, when you team up with someone, how it is about engagement and level and such?

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Q20: Og så siger du, at hvis man er sammen med nogen, som lynhurtigt har en idé til, hvordan det skal gøres, og regner derudaf, så det er egentlig ikke altid at gruppearbejdet fungerer så godt?

A20: Nej, fordi så kan det være at der sidder, at der er fem, der arbejder og der er to, der lige pludselig har fundet ud af en idé, og der sidder tre og sådan kigger lidt frem og tilbage - det er jo ikke optimalt for dem (…) Jeg synes det er skidesjovt, når nu at jeg lige pludselig tænker: "*Ej, jeg har fundet ud af det*," og man kan se, at det er rigtigt, ikke? Så kører man jo på en bølge, men de andre, de sidder jo der og kan ikke rigtig komme med. (...)

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Q21: Tænker du over, når du lige skal finde nogen at være sammen med, hvordan sådan noget som engagement og sådan noget som niveau, hvordan det ligger?

A21: (…) det ender med at være nogle af mine tætte venner, fordi de ved - jeg ved at de vil være tålmodige over for mig. Fordi nogle gange kan det godt tage lang tid for mig at forstå noget. Så jeg ved, at hvis jeg er sammen med nogle af dem, der er tålmodige over for mig og taler pænt til mig og sådan, så skal det nok gå godt.

A21: It may end up being some of my close friends, because they know – I know they will be patient with me. Because sometimes it takes me a while to understand things. Then I know that if I team up with people who are patient with me, and speak nicely to me and such, then I will be alright.

***1.Q. INFLUENCES ON PLANS AFTER GRADUATION***[556]

Q22: Yes! Then there is this about preliminary plans after upper secondary school. Before you started, medical doctor was an idea you had. And here, you wrote "*University*" or maybe the "*Military*". What does it look like now?

A22: The Military, that was more like, because I heard you could go in for half a year and then just run around (unintelligible) and I find that amusing. Just a short year off. But it is still university. Not so much doctor any more, because now I have realised that I may be relatively bright, but I am not in the top. And then I think those in the top should be allowed to become doctors. But otherwise just university. I do not know whether it will be science or not. It may eventually be, I do not know, Religion or Danish or something. Because that is where I am stronger. I was probably always stronger there. But it has only been two years since I started to take an interest to mathematics and such. So.

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Q22: Jo! Så er det dét der med foreløbige planer efter gymnasiet (…) inden du startede på gymnasiet, så var læge i hvert fald en idé. Og her, der har du skrevet universitetet eller måske forsvaret. Hvordan ser det ud nu?

A22: Forsvaret, det var mere sådan, fordi jeg havde hørt at man kunne komme sådan et halvt år ind og sådan få lov til at løbe rundt i (utydeligt) og det synes jeg er sådan meget sjovt. Bare sådan et lille friår. Men det er stadig universitetet. Og ikke så meget læge mere, fordi nu har jeg indset, at det kan godt være, at jeg er nogenlunde begavet, men jeg er ikke dér hvor det ringer. Og så synes, så skal de andre, der hvor det ringer, de skal have lov til at blive læger. Men ellers så er det bare universitetet. Jeg ved ikke hvorvidt naturvidenskab det bliver. Det kan godt være det ender med noget, jeg ved ikke, religion eller dansk eller et eller andet. Fordi det er nok i virkeligheden dem, jeg er stærkest til, fordi det har jeg nok altid været stærkest til. Men det er jo først for altså to år siden at jeg begyndte sådan at interessere mig for matematik og dét. Så.

***1.R. INFLUENCES ON PLANS AFTER GRADUATION II***[557]

Q23: Well, Religion or Danish, it may be what you feel stronger at, and it sounds like what is important for you when choosing an education is whether you feel strong at it?

A23: Yes, because, when you... it is not always such that if you are strong in a subject, then you like that subject. But if you like a subject, then you become good at that subject. And, as I said, I have always been good at the subjects in the humanities, right? Because there you are sung the praises of. So it may be something I want to study, because I imagine, at university, you only deal with one subject, or two subjects, and then you have to choose, so there you have the chance of choosing something, such that you only work with things you like. But I definitely think that when you are at university you should choose something you like and are good at.

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Q23: (…) Jamen, religion eller dansk, det er måske noget af det, du føler dig stærkest til, og det lyder som om det måske er det, der er vigtigt, når du skal vælge en uddannelse, at du føler dig stærk til det?

A23: Ja, fordi når man - det er ikke altid, når man er stærk i et fag, at man kan lide faget. Men det er tit, at når man kan lide et fag, så bliver man også stærk i faget. Og jeg er jo, som sagt, jeg har altid været sådan god til de lidt mere humanistiske fag, ikke? (...) for dér får man kastet ros efter sig. Så det er nok noget, jeg vil studere, for jeg kan forestille mig på universitetet, der får man jo kun ét fag eller to fag, man beskæftiger sig med, og det er jo, så skal man da vælge noget, så har man jo en chance for at vælge noget, sådan at man kun laver noget, man kan lide. Men jeg synes helt klart, at når man så er kommet på universitetet, så skal man vælge noget man kan lide og noget man er god til.

*1.S. MATHEMATICS IN SOCIETY*[558]

Q24: One thing is that mathematics is important, because it is a teaching subject, and you need it as part of your education. But where else is it important in society?

A24: We briefly studied something, these, what are they called, those numbers... bio-, bino-, something numbers that are used in computers, right?

Q25: Yes, binary numbers.

A25: Yes, binary numbers. And there I could imagine, that this is something, well, technology develops all the time, and mathematics is definitely applied there, and physics and that...

Q26: Yes, are there other examples?

A26: Yes, I thought about something on "*News*", where sometimes there is a section called "*Money*", right? Where it is about stocks. "*Now they have decreased by two percent and increased by three*" and all that. So, in general about prices. And how much something is worth, and such. Inflation.

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Q24: En ting er, at matematik er vigtigt, fordi man underviser i det, og man skal bruge det på sine uddannelser. Men hvor er det ellers vigtigt henne i samfundet?

A24: Vi havde kort om noget, de der, hvad er det nu det hedder, de dér tal der... Bio-, bino-, et eller andet tal, som man bruger i computere, ikke?

Q25: Ja. Binære tal.

A25: Ja, binære tal, det var det, det hed. Og der kunne jeg forestille mig, at det er jo noget, altså teknologien udvikler sig jo hele tiden. Og der skal man jo helt klart bruge matematik og også fysik og det...

Q26: Ja, er der andre eksempler, (...)?

A26: Ja, jeg tænkte lige på det der, der er noget på noget "*News*", hvor det nogle gange går på et emne, hvor det hedder "*Penge*", ikke? Hvor det er noget med aktier. "*Nu er det faldet med de der to procent og steget med tre*," og alt det der. Sådan generelt med priser. Og hvor meget noget er værd, og sådan noget. Inflation (...)

***1.T. ONTOLOGY OF MATHEMATICS***[559]

Q27: One thing I asked was if you imagine that mathematics is discovered or invented?

A27: I would say it is a mix, because it is something you work out how to, yes, you find some formulae, for example; circles have always been there, you have been able to draw a circle in the sand. But, and then suddenly you find out, that if we use this formula, then we can find its circumference or we can find its centre and so on, right? So, in a way, it has, then it is just some numbers we enter, but in some way they have always been there.

***1.U. MATHEMATICS AT UNIVERSITY***[560]

Q28: Another thing, I ask, is what you imagine a mathematician in a university is doing?

A28: Yes, well, you attend lectures and have tutorials, I think. And I think that you do some tasks, sometimes. But then I also think some of the teaching concerns that you do a project yourself and that you yourself are supposed to investigate something within mathematics, which you find tremendously interesting. Which you write some kind of good report about. And then there is the last enormous report to hand in. I do not know how many pages, but I

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Q27: En ting, jeg spørger om, det er om du forestiller dig, at matematik (...) er opdaget eller opfundet. (...)?

A27: Ja, det er - jeg vil sige, at det er blandet, fordi det er jo noget, man finder ud, af hvordan man - ja, man finder ud af nogle formler, fx der har jo altid været cirkler, man har jo altid kunne tegne en cirkel i sandet. Men, og så finder man lige pludselig ud af, at hvis vi bruger den formel, så kan vi finde ud dens omkreds eller vi kan finde ud dens centrum og så videre, ikke? Så på en måde, så har det jo, det er jo bare sådan nogle tal, vi selv har puttet ind, men de har jo altid været der på en måde.

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Q28: En anden ting, jeg spørger om, hvad du forestiller dig, at en matematiker på et universitet laver?

A28: (...) Ja. Altså man går jo selvfølgelig til sådan noget forelæsninger og har timer, tror jeg. Og så tror jeg også, man kan lave nogle opgaver, nogle gange. Men så tror jeg også, at meget af undervisningen det går ud på, at man selv laver et projekt og man selv skal undersøge noget inden for matematikken, som man synes er enormt spændende. Som man så kan skrive en eller anden god opgave om. Og så er der selvfølgelig den sidste kæmpestore opgave, man afleverer. Jeg ved ikke hvor meget den skal fylde, jeg synes engang jeg har hørt noget med 100 sider, men jeg ved ikke om det er rigtig.

Q29: Den må godt.

A29: Ja, men, 100 sider, som man skal aflevere til sidst. Men ellers, så tror jeg det er meget, man har en smule undervisning og man kan lave nogle opgaver måske, nogle gange. Ellers så meget projekt og man har gang i et eller andet.

heard something about 100 pages, but I do not know if it is true.

Q29: It may well be.

A29: Well, but, 100 pages, that you hand in eventually. But otherwise, I think it is mainly that you have some teaching and that you can do some tasks now and then. Otherwise especially project and having something going on.

***1.V. ACTIVITIES IN UNIVERSITY MATHEMATICS***[561]

Q30: Well, in the questionnaire, I ask: "*What do you think a mathematician in a university is doing?*" And you write: "*One concentrates probably on a topic and explores it in depth.*"

A30: Well, yes, about that topic. Well, one takes a topic within mathematics, e.g. triangles or Pythagoras with the triangles and then one can write, or explore it in depth and write something about it.

***1.W. GENIUS?***[562]

Q31: Should one be a genius to study mathematics at university?

A31: No, you should not. My older brother, he is very, very fond of mathematics. So he might want to study it at university. But he is not completely sure. But he says there are no requirements for admission grades. It is just, if you have passed. So I think, that even if you do not get straight A's, but maybe around C's or D's, then you can be admitted nevertheless. And then, then you may have to struggle somewhat more with it, compared to one who

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Q30: Altså, i spørgeskemaet, der spørger jeg: "*Hvad tror du en professionel matematiker på et universitet laver*?" Og siger du: "*Man koncentrerer sig nok om et enkelt emne inden for matematik og så går man i dybden med det*."

A30: Ja, altså, (utydeligt) med det der projekt der. Altså, at man tager et eller andet emne i matematikken, fx trekanter og så eller Pythagoras med trekanterne og så kan man skrive, eller fordybe sig i det og skrive noget om det. (...)

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Q31: Skal man være et geni for at læse matematik på universitetet?

A31: Nej, det skal man ikke. Min storebror, han er blevet meget, meget glad for matematik. Så han vil måske gerne ind og læse det på universitetet. Men han er ikke helt sikker, han er ikke sikker på noget egentlig. Men han siger, at der er ikke noget karaktermæssigt krav. Det er bare, hvis man har det bestået. Så jeg tænker, at selv om man ikke lige får 12 i det hele, men måske ligger på sikker 4-7, så kan man nok godt komme ind og så. Og så, selvfølgelig kommer man måske til at bakse lidt mere med det, end en der har nemt ved det - men jeg tror ikke, at man behøves at være et geni for at læse det på universitet. Overhovedet.

picks it up easily, but I do not think you have to be a genius to study it at university. Not at all.

***1.X. MATHEMATICS AT SCHOOL II***[563]

Q32: This thing about how mathematics has changed, and whether there is something you like better now. Well, you say: "*There is order in the classroom and the teacher is able to teach.*" and "*In this way, mathematics becomes a pleasant subject*"

A32: Yes. Well, but that is true. Now, I think it is really pleasant to be taught mathematics, because now it is quiet, and the teacher is able to teach calmly and quietly. And I think, it may be because it is exactly ALFRED, who is our teacher. He does it in a really, really good manner, in which there is room for some humour now and then. I also find that important, because, like, then it is not too heavy, and because one should also include those students who may have other things on their mind.

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Q32: Det der med hvordan faget matematik har ændret sig, og om der er noget, du bedre kan lide nu. Altså, hvor du siger: Ja, der er noget, du bedre kan lide nu: "*Der er ro i timerne, og læreren får lov til at undervise*". Og "*På den måde, så bliver matematik et hyggeligt fag.*"

A32: Ja. Jamen, det er rigtigt. Nu, jeg synes, det er rigtig dejligt at have undervisning i matematik, fordi nu er her stille, og læreren underviser i ro og mag. Og jeg synes, og det kan jo fx være fordi det lige netop er [Teacher ALFRED], der er vores lærer. Han gør det på en rigtig, rigtig god måde, hvor der også kan være plads til lidt humor engang imellem. Og det synes jeg også er vigtigt, for ligesom, at det ikke bliver så tungt, og fordi man også skal have de elever med, som måske går med nogle andre ting i hovedet.

### *1.Y. TRANSITION FROM LOWER SECONDARY SCHOOL*[564]

Q33: You said, when I asked: "*Did you like Mathematics when you went to lower secondary school?*", "*No, it was not really me.*" Except at the end, you say now?

A33: Yes, except at the end. I have not always been good at it. I do not know, I think, I did not always concentrate on it, did not do my assignments. It was all the way back from second grade and such, right? And, as I told you, in ninth grade, then the principal said: "*Well, now I am your Mathematics teacher, so we can have some calm and order.*" And then I fell completely for e.g. geometry and such like. I think it was incredibly pleasant and exciting to sit and calculate on little marbles, quadrilaterals and triangles, how they could fit a box and everything, right? And there was a little more structure. Not as much as in upper secondary school, but there was still some more structure in things. And sometimes we got an assignment and such. And then I began thinking: "*It may be, that this is a pleasant subject, Mathematics.*" But it was not until then I realised that it could be a pleasant subject. So that is why I often excuse myself, that I am a little behind. But, well. Yes.

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Q33: Du sagde, da jeg spørger til: "Kunne du lide matematik, da du gik i folkeskolen?" "Ej, det var ikke lige mig." Undtagen til sidst, siger du så.

A33: Ja, undtagen til sidst. Jeg (…) har ikke altid været så god til det. Jeg ved ikke lige, jeg tror, jeg koncentrerede mig ikke altid om det, lavede ikke altid mine opgaver. Det var helt tilbage i anden klasse og sådan noget, ikke? Og så, som sagt, i niende klasse, der - skoleinspektøren, han sagde: "*Nå, nu er jeg jeres matematiklærer, så der kan komme lidt ro og mag over det her*." Og så faldt jeg pladask for fx geometri og sådan noget. Jeg synes det var utrolig hyggeligt og spændende at sidde og kunne regne med sådan nogle små kugler, firkanter og trekanter, hvordan de kunne passe i en kasse og alt muligt, ikke? Og der var lidt mere struktur. Ikke så meget som på gymnasiet, men der var også lidt mere struktur over det. Og nogle gange fik vi et lille opgavesæt og sådan noget. Så det var der, hvor jeg sådan begyndte at tænke: "*Det kan da godt være, at det er et sjovt fag, det her*." Men det var allerede, det var først dér, at det ligesom slog mig, at det kunne være et sjovt fag. Så derfor så undskylder jeg også tit med, at jeg måske er lidt bagud. Men altså, ja.

## THEMES IN ADELE'S 1ST YEAR

The questionnaire answers and the interview transcripts forms the basis from which my interpretation and analysis of Adele's account of her experiences with mathematics in the 1st year of upper secondary school. These are organised in themes relating to the four aspects of beliefs.

### *MATHEMATICS AT SCHOOL*

A prerequisite for successful mathematics teaching is peace and order in class (ADELE: Q1-A). Also thorough feedback on homework and written assignments is important (ADELE: 1.A.).

In class, being good at mathematics seems to be more okay than the opposite, according to Adele (*ADELE: Q1*-F).

### *MATHEMATICS AS A DISCIPLINE*

Adele sees mathematics as a tool for other sciences, as it is useful for physics and chemistry, for computing units (*ADELE: Q1-C*).

The content taught in mathematics is highly applicable in physics; Adele mentions vectors, and her teacher can help them see these things, since he also teaches physics [not her class, though] (*ADELE:* 1.I.).

Circles have always been there, but then suddenly we found out that by means of a formula we could find its circumference or its centre. In that sense, Adele finds that even though we found a way to add some numbers to it, in some sense mathematics has always been here. Adele is also asked what she thinks a mathematician in a university is doing, and she answers by mentioning what she thinks a mathematics student is doing. So in that sense it is indicated that to her

mathematics is primarily a teaching subject – which might imply that doing mathematics is primarily about learning what is already there. (1.N.).

From her brother, Adele knows that there are no specific requirements of average grades for being admitted to studying mathematics in university. She thinks that it may not be necessary to be a genius to study mathematics, but she anticipates that it will demand some hard work, in that case (1.O.).

### *MATHEMATICS IN SOCIETY*

Adele also see the societal purposes of learning mathematics, In society, in the business world, mathematics is used. (*ADELE: Q1-C*).

Not all her friends from out of school share her enthusiasm for the equation of a circle or the techniques for calculating the energy consumption of an electric kettle, which is part of her high-level mathematics programme. Nevertheless, Adele thinks that everybody should be able to compute simple stuff and be able to help their children with their school homework (*ADELE:* 1.G.).

Mathematics is used all the time, Adele says; Binary numbers are applied in computer technology in society, and mathematical techniques in terms of interest rates and shares are important to the financial sector (1.L.).

### *MATHEMATICS & ME*

The rating of mathematics may have fallen from [7] around the time of answering the questionnaire, and now Adele would rate it around [6], perhaps. She primarily substantiates this by saying that the other subjects have gone up a bit, but she also mentions that she find herself to be a little below average in her class, and that she sometimes finds it hard to learn mathematics (*ADELE:* 1.B.).

DIDACTICAL CONTRACT

Humour is quite welcome in mathematics teaching, according to Adele. This makes it less "heavy" and she finds it to have a motivating influence on some students (1.P.).

In last year of lower secondary school, Adele suddenly got her principal as a mathematics teacher, and she appreciated it a lot. She fell for geometry and she adored the peace and order in class. This is when she started to like mathematics, but she also contends that this may be the reason that she is sometimes behind; that she did not have this kind of experience earlier (*ADELE:* 1.Q.).

Mathematics as it is run in upper secondary school appeals much more to Adele, than mathematics in lower secondary school [before she the last year of lower secondary school]. She enjoys the calm and order in class (ADELE: Q1-A).

Adele seems to be fond of the way teaching is organised in her class; there is clam and order such that the teacher can actually teach, and with thorough feedback on homework and written assignments (*ADELE:* 1.A.).

Adele is rather quiet in class, but she is not content about it (*ADELE: Q1*-F).

Typically Adele is rather quiet in mathematics class, both in terms of answering and asking. She says she only chose this study programme due to her curiosity, which is why she may not feel secure in participating more actively. However, she sometimes realises that what she would have answered, also would have been correct (1.J).

Adele values cooperation in which she can be active herself. Also a supportive environment is important to her. Otherwise, she would not have the chance of developing her understanding (1.K.).

#### CHALLENGES AND SUPPORT

The greatest challenge amongst those suggested in the questionnaire was [Figuring out the purpose of a task]. Sometimes Adele does not know what she is supposed to do, she says. When she knows what she is supposed to do, then it is just a matter of finding the right formula, she adds (*ADELE:* 1.E.). Figuring out the purpose of a task was indicated as harder to Adele compared to remembering, but even fewer challenges were suggested to be involved in understanding the textbook, finding a way to solve a task, and the fewest challenges seemed to be involved in actual computing (ADELE: Q1-E).

Adele needs time and practice to develop her mathematics skills, and she can get support at home, from her father and her brother, if needed (*ADELE: Q1-D*).

Adele's father is her primary source of help. He is an upper secondary school teacher in physics, but also her brother can help her. He is in the same study programme as she, only two years ahead of her (*ADELE:* 1.C.).

#### PLANS

No specific career idea was mentioned in the questionnaire for choosing a study programme involving A-level mathematics. A-level mathematics But a feature of general educational value is mentioned; Adele sees mathematics as strengthening your sense of logic (*ADELE: Q1-B*). However, in the interview Adele explains that when choosing this study programme (Mathematics A, physics B, chemistry B), she considered opting for studying medicine for tertiary education. Nevertheless, now, in upper secondary school, she has realised that only the few very best students can achieve that. But her study programme is useful for many purposes, she says. Her experiences with mathematics in her last year of lower secondary school also had an influence, when she suddenly started to like mathematics due to her new teacher, the principal of the school (*ADELE:* 1.F.).

A-level mathematics was not a purpose in itself for Adele, but the study programme would qualify her for studying medicine, without taking the "extreme" science study programme, which would involve high-level chemistry and physics as well, in stead of those two subjects on B-level (*ADELE:* 1.H.).

University or the military are the two options Adele has in mind for her plans after graduation, even though she is not sure what she wants to do (ADELE: Q1-G).

Whether it will be science or studies within the humanities in university after graduation is not certain to Adele yet, but she has realised that she will not be amongst those students to be admitted to medical school. She mentions that she feels stronger in the humanities, in subjects such as Religion or Danish. And she emphasises that she only discovered recently that she also likes Mathematics (1.K.).

## ADELE'S 1ST YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

A-level mathematics in upper secondary school is quite hard and speed is a key factor. It is not entirely okay not to be good at mathematics.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is neither discovered, nor invented. Her ideas of the activities of a mathematician in university relate more to the teaching subject of mathematics than it relates to the discipline. Adele does not indicate any perception of mathematics as a discipline under development, so mathematics may be a static discipline to her.

### *MATHEMATICS & SOCIETY*

Mathematics is useful everywhere, and everybody should learn it.

### *MATHEMATICS & ME*

Adele values order and calm in class and that the teaching is highly structured.

However, she does not feel too competent as a mathematics learner. She is dependent on friends or family to explain thing to her nicely and to help her figuring out what a task is about. She seem to excuse her lack of expertise to herself and others by referring to the poor mathematics teaching she received in the majority of her lower secondary school years. It is important to her to be good at what she is doing.

Adele's dreams of studying medicine led her to choose a study programme involving A-level mathematics. However, she soon finds that this possibility is not for her, since the average of grades it requires does not seem realistic for her to

obtain. Instead, she chooses to emphasise the elements of the general educational values of learning mathematics and science to herself.

## ADELE'S 3RD YEAR QUESTIONNAIRE

| Q3-A | TRANSITION | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [7] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [565] | [Working on your own]; [Working in pairs]; [Group Work]; [The whole class together] |
| **3** | Did you like mathematics when you went to lower secondary school? [566] | [No, I did not like it at all] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | It has become harder, but the teaching is better and there is order in class.[567] |
| **New 4d** | In which ways has mathematics changed from first year to third year of upper secondary school? | It has become harder[568] |

*Table 75: ADELE's 3rd year questionnaire, part A – TRANSITION*

565 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

566 [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

567 Det er blevet sværere, men undervisningen er bedre og der er ro i timen

568 Det er blevet sværere

| Q3-B | FOR SCHOOL | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | It is good to know, a good basis[569] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | Physics, chemistry[570] |

*Table 76: ADELE's 3rd year questionnaire, part B - MATHEMATICS AND SCHOOL*

569 Det er en god ting at kunne, god basis

570 Fysik, kemi

| Q3-C | BEYOND SCHOOL | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | Physics, statistics, data processing, banks, economists[571] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? [572] | [Mathematics is invented] |
| **10** | What do you think a professional mathematician at a university is doing? | Conducts research, computes, dances[573] |
| **11** | Would you have to be a genius in order to study mathematics in university? [574] | [i do not know]<br>Easy to apply for admission, hard study programme![575] |

*Table 77: ADELE's 3rd year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

571 Fysik, statistik, databehandling, banker, økonomer

572 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]

573 Forsker, regner, danser

574 [Yes]; [No]; [I do not know]

575 "Let at søge ind – hårdt studie!"

| Q3-D | IMRPOVING | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | If the level is too high[576] |
| **12b** | What do you think is the best means for improving in mathematics? | Practice, practice makes perfect[577] |
| **12c** | What do you do to improve in mathematics? | Pay attention to the teaching, make my written assignments[578] |
| **13a** | What do you do if you get stuck on a task at school? | Proceed to the next task or ask the teacher[579] |
| **13b** | What do you do if you get stuck on your homework? | Ask the teacher or friends, the textbook[580] |
| **13c** | What do you do if you get stuck on your written assignments? | (Same as previous) |

*Table 78: ADELE's 3rd year Questionnaire, part D – STRATEGIES FOR IMPROVING*

576 Hvis niveauet er for højt

577 Træning, øvelse gør mester!

578 Følger med i timen, laver mine afleveringer

579 Går i gang med den næste el. spørger læreren

580 Spørger læreren eller venner, matbog

| Q3-E | CHALLENGES & SUPPORT | | ADELE |
|---|---|---|---|
| # | *Question* | | *Answer* [581] |
| **14** | What issues involve more challenges to you? | a) Remembering | [4] Few challenges |
| | | b) Computing | [4] Few challenges |
| | | c) Figuring out the purpose of a task | [2] Several challenges |
| | | d) Finding a way to solve a task | [3] Moderate |
| | | e) Reading and understanding the textbook | [3] Moderate |
| **15** | Where can you find support for mathematical activities? | | From: [Parents]; [Siblings]; [Classmates] [Other]: "The teacher"; "The Homework Café" |
| **New 16a** | Do you find that you meet some special challenges related to succeeding in mathematics compared to your classmates? | | I often feel that the fundamentals are missing[582] |
| **New 16b** | Do you find that you have some some special strengths related to succeeding in mathematics compared to your classmates? | | Then it should be formulae[583] |

*Table 79: ADELE's 3rd year Questionnaire, part E – CHALLENGE & SUPPORT*

581 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

582 Føler tit at det basale mangler

583 Så skulle det være formler

| Q3-F | IN CLASS | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? [584] | [1-3] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [No] |
| **17c** | Are you content with that? | No[585] |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? [586] | [1-3] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | No |
| **19a** | In your class, is it okay to be good at mathematics? | Yes |
| **19b** | In your class, is it okay to have difficulties in mathematics? | No. Some are understanding, some are downright mean[587] |

*Table 80: ADELE's 3rd year Questionnaire, part F - MATHEMATICS IN CLASS*

584 Options: [0]; [1-3]; [4-8]; [More than 8 times]

585 Nej.

586 Options: [0]; [1-3]; [4-8]; [More than 8 times]

587 NEJ. Nogen er forstående, andre direkte ondskabsfulde

| Q3-XA | UNDERSTANDING | ADELE |
|---|---|---|
| # | *Question* | *Answer*[588] |
| **X.1.** | Have you recently experienced to *understand* what you worked with in mathematics? | [Yes] |
| | If yes, on which occasion? | Vector functions[589] |
| **X.3.** | Have you, during upper secondary school experienced that you *understood* what you worked with and then s*ubsequently learned it by heart*? | [yes] |
| | If yes, on which occasion? | Do not remember[590] |
| **X.5.** | Have you during upper secondary school experienced *understanding something* but *never learning it by heart?* | [I do not know] |
| | If yes, on which occasion? | - |

| Q3-XB | LEARNING BY HEART | ADELE |
|---|---|---|
| # | *Question* | *Answer*[591] |
| **X.2.** | Have you recently experiences having to *learn something by heart*? | [Yes] |
| | If yes, on which occasion? | The same, but usually i also understand[592] |
| **X.4.** | Have you, during upper secondary school experienced that you had to *learn something by heart*, and then, *subsequently understood it*? | [yes] |
| | If yes, on which occasion? | Do not remember[593] |
| **X.6.** | Have you during upper secondary school experienced *learning something by heart without ever understanding it*? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.7.** | Additional comments on understanding or rote learning in mathematics | - |

588 Options: [Yes], [No] or [I do not know]
589 Vektorfunktioner
590 Kan ikke huske
591 Options: [Yes], [No] or [I do not know]
592 Samme, men jeg forstår oftest også
593 Kan ikke huske

| Q3-XC | A-LEVEL EXAMINATION | ADELE |
|---|---|---|
| **X.8.** | *Topic* | *Answer*[594] |
| **(a)** | Parabola | [Readily] |
| **(b)** | Exponential | [Okay] |
| **(c)** | Pythagoras | [Readily] |
| **(d)** | Sine and cosine relations | [Readily] |
| **(e)** | Definition of differentiability | [Readily] |
| **(f)** | Sum and product of differential functions | [Readily] |
| **(g)** | Indefinite integral | [Readily] |
| **(h)** | Volume of solid of revolution | [Okay] |
| **(i)** | Differential Equations and their solutions | [Rather not] |
| **(j)** | Vectors in the plane, including scalar product | [Okay] |
| **(k)** | Lines and planes | [Okay] |
| **X.9.a.** | Which topic is your favourite?<br>- and why? | Trigonometry - you can envision it – differential calculus - easy and with a good basis from physics. Integrals – the same. Polynomials of second degree – you can envision it[595] |
| **X.9.b.** | Which topic would you rather avoid?<br>- and why? | Probability – not proficient[596] |

*Table 81: ADELE: Topics for oral and written examinations in A-level Mathematics for Upper Secondary School Programmes in Denmark 2013*[597]

594 Options: [Readily], [Okay], [Rather not], [I do not know]

595 Triginometri – man kan se det for sig. Differentialregning – let og med en god basis fra fysik. Integral – samme. 2. Gradspolynomier – kan se det for sig

596 Sandsynlighed – knap så god til

597 Topics common to both the Technical (htx) and the General (stx) upper secondary programme examinations are suggested.

| Q3-G | PLANS | ADELE |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Humanities[598] |
| **20b** | What kind of influence would your final marks in mathematics have on your plans (or hope for plans) after upper secondary school? | It can improve my average of grades – besides that, nothing[599] |
| **20c** | What kind of influence would your experiences with mathematics in upper secondary school have on your plans after upper secondary school? | It has been fun, but [i] am stronger in the other subjects[600] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [No] |
| **21b** | Comments: | - |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [I do not know] |
| **22b** | Comments | Depends on the job[601] |

*Table 82: ADELE's 3rd year Questionnaire, part G – PLANS*

598 Humanistisk

599 Kan gøre mit snit godt – ellers ikke noget

600 Det har været sjovt, men er stærkere i de andre fag

601 Kommer an på jobbet

# ADELE'S 3RD YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 4 March 2013 | 21:12 |

### *3A. UPGRADING HUMANITIES IN STUDY PROGRAMME*[602]

Q1: What has happened since last time? That may be a big question?

A1: Oh, a lot of things. I learned more mathematics. And I do not take physics and chemistry anymore, because I decided not to upgrade it [to A-level]. So now I take A-level English and I got a lot of nice C-level humanistic disciplines: Classical Studies and then I chose Philosophy, so I may finally have worked out that I may be… may have a passion for the Humanities too. Even if I think that Mathematics is super nice. And then I decided to take A-level English. I could have let go of that and taken [A-level] Chemistry or Physics instead, but I chose English, because it comes easy to me, and that was just what I could cope with, now in the third year.

602

Q1: Hvad er der sket siden sidst? Det er måske et lidt stort spørgsmål.

A1: Uha, der er sket mange ting. Jeg har lært mere matematik. Og… jeg har ikke fysik og kemi mere, fordi det valgte jeg ikke at opgradere. Så nu har jeg engelsk på A-niveau og har fået en masse af de der skønne C-niveau humanistiske fag, oldtidskundskab og så har jeg valgt filosofi, så jeg har nok fundet ud af, at jeg måske er… har en forkærlighed for de humanistiske fag også. Selvom jeg synes matematik, det er super hyggeligt. (...) Og så har jeg valgt engelsk på A-niveau. Det kunne jeg også have valgt at smide, og så tage fysik eller kemi i stedet for, men jeg valgte engelsk, for det har jeg let ved og det var lige det, jeg kunne overskue her i 3. g. Så…

### *3.B. MATHEMATICS AND THE FUTURE*[603]

Q2: The Mathematics, you have learned, can you use it for something?

A2: Yes, I think that has been my approach all the time, that it is good to know some Mathematics for kind of being able to function in real life, right? It would also be nice, if you have to help your children with it one day. But also because it gives a lot of possibilities. If I wanted to do something with languages, then I might chose some kind of business orientation. And then I might use a little Mathematics there. So I just think it is really nice, that you have some foundation in Mathematics and mathematical understanding for some things. So it will not be completely new to you, when you proceed.

### *3.C. MATHEMATICS AS SUCH I*[604]

Q3: What is mathematics?

A3: What is mathematics? Mathematics is a way of describing the world. It is a place where you can, well, define a movement, a shape (…) all kinds of things. And it is kind of a beautiful thought, I think, that you can describe so many things by means of some mathematical formulae and calculations. And I absolutely adore that one can do that.

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603

Q2: Det matematik, du har lært, kan du bruge det til noget? (…)

A2: Ja (…) det synes jeg hele tiden også min holdning har været, at det er godt at kunne en smule matematik til at ligesom at (...) kunne fungere ude i det virkelige liv, ikke? (…) Det kunne også være rart hvis man skulle hjælpe sine børn med det en dag (...) Men også fordi det giver mange muligheder (...) hvis nu jeg ville noget med sprog, så kunne det jo også være jeg vælger en eller anden form for handelsvej. Og så kunne det jo være, at jeg skulle bruge lidt matematik der. Så det, men jeg synes bare, at det er rigtig lækkert, at man har sådan en, altså, en basis af noget matematik og matematisk forståelse for nogle ting. Sådan så det ikke er totalt nyt for én, når man kommer ud og videre.

604

Q3: (…) Hvad er matematik?

A3: Hvad er matematik? Matematik det er en måde at beskrive verden på. (…) Det er et sted hvor man kan… altså… Ja, man kan definere en bevægelse, en form (…) alle mulige forskellige ting. Og det er jo faktisk en meget meget smuk tanke synes jeg, at det, at man kan beskrive så mange ting ved hjælp af nogle matematiske formler og udregninger. Og det kan jeg super godt lide, at man kan det.

### *3.D. THE VALUE OF SUBJECTS AT SCHOOL*[605]

Q4: Is it okay to be good at mathematics and is it okay to have difficulty with mathematics...your answer is not unambiguous in terms of how well accepted that is?

A4: In our class, which is a purely Mathematical stream with Physics and Chemistry; some people still take Physics and Chemistry; there I think it is important to be good at mathematics. And sometimes Humanistic disciplines are looked askance at, right? If you cannot work Mathematics out, there will often be prompted a comment. Because; in a Mathematical study programme, then Religion, that is a subject to put-down, because "*It has nothing to do with our study programme*." Well, we are good friends in class and such, but otherwise I think one can, like... (you are) not as accepted, if you are not as good as the others.

### *3.E. EFFORT*[606]

Q5: What about your investment of energy in mathematics, now when you have made up your mind, that you go for something else afterwards?

A5: Well, I think it is the same as with he other subjects. Well, the other subjects come more easily to me. But I still try eagerly to keep up in Mathematics. And I want to be able to do some good assignments. Also for understanding it better and such things. So I will, I try a lot to keep up and do some good assignments and good notes and such like.

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605

Q4: (…) Er det okay at være god til matematik og er det okay at have svært ved matematik (…) dit svar er ikke sådan helt entydigt med at, ih hvor er der bare plads til det?

A4: (…) I vores klasse, hvor det jo så er rent matematisk linje og fysik, kemi, der er nogen der stadig har fysik og kemi, ikke, der tror jeg at det er meget meget vigtigt, at man også er god til matematik. Og man nogen gange godt kan se lidt ned på de lidt humanistiske fag, ikke? (...) Der kommer tit en kommentar, hvis nu man ikke lige kan finde ud af matematik (...) fordi, naturvidenskabelig linje, så religion, det er et fag, der skal trædes på, (...) fordi at "*det har jo slet ikke noget med vores linje at gøre*" (…) altså vi er jo gode venner i klassen og sådan nogle ting, men… Men ellers så synes jeg godt man kan sådan… Ikke accepteres så godt, hvis man ikke er så god, som de andre er (…)

606

Q5: Hvad med, altså, hvad med din energi til at lægge i matematik, nu hvor du sådan et eller andet sted har sagt til dig selv, jeg sigter efter noget andet bagefter?

A5: Altså, den tror jeg er den samme som i de andre fag. Altså… Jeg har måske lidt nemmere ved de andre fag. Men jeg prøver stadig ihærdigt at følge med i matematik, (...) og jeg vil godt kunne lave nogle gode afleveringer (...) Også for bedre at kunne forstå det og sådan nogle ting, så. Så jeg vil, så jeg prøver meget på at følge med og få lavet nogle gode afleveringer og nogle gode notater og sådan.

### *3.F. SPEAKING IN CLASS*[607]

Q6: You are not content about how often you answer questions in class?

A6: No, I am not. I am kind of worried, if I should say something that is not completely right, that someone will react like this: "*No, no, no, no, no, this is completely idiotic*!" And maybe also due to...well, I would like to make a good impression on the teacher, right? Right now, after the winter holiday [Mid-February] I think I have been kind of better in asking: "*Was it not like this and that?*". And I will try to keep it like that, because that is how you learn best. And also to give the impression that you are following the teaching and that you want to do something, instead of just being completely passive.

### *3.G. ONESELF AS MATHEMATICS LEARNER*[608]

Q7: Is there enough time for you to understand what you are supposed to learn?

A7: Yes, it does depend on which topic it is, because sometimes I can envision it clearly, well, this thing about integrals, it was spot on for me. And then at other times, I know that I can, I know that I can manage a lot of things, but I am a little more slow that the others to kind of understand. I am a little more slow in kind of turning things a little and say, this is what we should do. But otherwise I think I understand it reasonably well, and

607

Q6: Du er ikke tilfreds med, hvor mange gange du svarer på noget i klassen?

A6: Nej, det er jeg ikke. (...), jeg er lidt bange for, hvis jeg nu skulle sige noget, der ikke er helt rigtigt, så sidder nogen bare sådan her: "Ej ej ej ej ej, det er fuldstændigt åndssvagt!" Og måske også for at det… altså jeg vil også godt give et godt indtryk til læreren [Teacher ALPHRED] ikke? (…) Lige her (...) efter vinterferien, så synes jeg jeg har været lidt bedre til ligesom at spørge ind til, var det ikke sådan og sådan det hang sammen og kan man ikke sige sådan og sådan… Og det vil jeg prøve ligesom at…holde ved fordi det er jo sådan man, altså, lærer bedst. Og også give et indtryk af at man, altså man er med i undervisningen og at man også godt vil noget, i stedet for at man bare sådan er helt passiv.

608

Q7: (...) Er der tid til at man når at forstå det I skal lære?

A7: Ja det kommer så an på hvilket emne det er, fordi nogen gange så kan jeg se det tydeligt for mig, altså, det der med integraler… det var lige noget for mig. Og så er der nogen gange hvor (...) jeg ved godt jeg kan, jeg ved godt jeg kan lave mange ting, men jeg er lidt langsommere end de andre til ligesom at forstå. Jeg er lidt langsommere end de andre til lige at sige.. altså lige at dreje den lidt og så, sådan her skal vi gøre. (...) Men jeg synes ellers at jeg forstår det sådan nogenlunde, og så ellers så går jeg jo hjem og så spørger jeg min far eller en anden eller spørger mine veninder bagefter (...) Men jeg synes ellers [ALFRED], han er faktisk meget god til ligesom at forklare det. (...)

otherwise I go home and ask my father or someone else, or my friends afterwards. But otherwise I think ALFRED, he is actually quite good at explaining it.

### *3.H. ON MATHEMATICAL TOPICS*[609]

Q8: I listed some topics, you could tick off for what you would prefer to draw for exam questions. There is "*Differential equations*", for example, where you think: "*Rather not*". Can you say something about why it is placed in that category?

A8: It is probably because I am not completely in control of it. I think it is something about substitution and something you put in, instead, well, but I cannot do it at all. For example, something like trigonometry and cosine and sine, I am completely fascinated by it, but precisely this, this I do not like as much as the other topics.

### *3.I. UNDERSTANDING AND REMEMBERING*[610]

Q9: I also ask about understanding and remembering. And you say, for example, well, lately, you have had to learn something by heart. But usually, you also understand it?

A9: Yes. I feel like this about Mathematics, if, for example, one should, yes, learn something, a line between two points or something. If you then just do it, and maybe do not understand it in the first place, but just learn how to do it, purely mechanically, then I think that you, little by little, learn and understand how you do it. I think, quite often, it

609

Q8: Jeg listede nogle emner op, man kunne krydse af, med hvad man helst ville trække til eksamen. (…) Der en differentialligning for eksempel, hvor du tænker, ej helst ikke (…) kan du sige noget om, hvorfor ligger den der?

A8: Det er nok fordi at det var det jeg sådan ikke helt har fået helt styr på. Jeg tror der er noget med noget andet substitution og noget man sætter ind i stedet for, jamen jeg kan slet ikke. (...) Som for eksempel, altså jeg, sådan noget som trigonometri og cosinus og sinus, det er jeg helt pjattet med, men lige præcis, lige præcis det der, det… Det er jeg ikke lige så glad for som de andre.

610

Q9: Ja, og så spørger jeg til med at forstå og huske. Og du siger for eksempel, jamen du har på det seneste været nødt til at lære noget udenad. Men du forstår det som regel også?

A9: Ja.(...) jeg har det sådan at matematik, hvis man for eksempel skal, ja, lære et eller andet, en linje mellem to punkter eller et eller andet. Hvis man så bare gør det, og man måske ikke lige umiddelbart forstår det første gang, men man bare lærer hvordan man gør det, sådan rent mekanisk, så synes jeg lige efterhånden, man lærer og forstår hvordan man gør. (...) Det synes jeg, det er tit at det er sådan, at man bare lærer hvordan det er, og så lige pludselig så, nåååå ja okay, jeg kan godt se hvorfor vi gør det her nu, fordi så får vi det og det, ikke? Så det… Det er nogen gange det er sådan i matematik, at man bare lærer sådan mekanisk udenad, så lige pludselig siger man, nå ja for filan, nu ved jeg hvordan det hænger sammen.

is like this; that you just learn how it is, and then, suddenly, then, ah, yes, okay, now I can see why we do this, because then we get this and that, right? So...it is often like this in Mathematic;, that you just learn it mechanically, by heart, but then suddenly you say, oh yes, now I know how things are related.

### *3.J. ON MATHEMATICS AT UNIVERSITY I*[611]

Q10: What is a Mathematician in a university doing?

A10: Well, a Mathematician in university, yes, what is he doing? He, yes, now I can answer it by cheating a little, because my brother is studying Mathematics now. And then he does assignments. Well, he reads really, really, really, really a lot, I know. And then he solves tasks. What does...lectures, and then one reads and one does assignments and then sometimes one get kind of a project, right? And then you take examinations in terms, I know. But that is the conception, I have. A lot of reading and a lot of lectures and tutorials in small groups in university.

### *3.K. ON MATHEMATICS AT UNIVERSITY II*[612]

Q11: And what about those who teach him, when they do not teach?

611

Q10: Hvad laver en matematiker på et universitet? (...)

A10: Altså en matematiker på et universitet, ja, hvad laver han. Han… Ja nu kan jeg jo svare på det lidt snyd, for min storebror læser matematik nu, men jeg ved da, at min storebror han går til forelæsninger. Og så laver han opgaver. … Altså han læser rigtig rigtig rigtig rigtig meget, ved jeg. Og så laver han de der opgaver som sagt. Hvad laver… Forelæsninger, og så læser man og så laver man nogle opgaver og så kommer der nogen gange sådan noget, en form for projekt, ikke? Og så går man til eksaminer i sådan nogle blokke, ved jeg. Men det er den opfattelse jeg har. En masse læsning og forelæsning og sådan nogle opgaveregnetimer, man har sammen i nogle små grupper på universitetet.

612

Q11: Ja. Og hvad laver dem der underviser ham, når de ikke underviser ham?

A11: Uha når de ikke forelæser, ja hvad laver de så? Det kan være de har deres eget lille projekt de går op i, eller deres… et eller andet de skriver en eller anden fin afhandling om eller… Det kan være de er blevet smidt ud til gymnasiet som censor eller et eller andet, hvad ved jeg.

Q12: (…) nu kan jeg lige så godt kigge hvad du svarede her. Forsker og regner og… danser (…)

A12: Hvad har jeg dog skrevet der? (…) Jeg har nok været lidt fjollet på det tidspunkt, jeg har skrevet danser. (…) Det skal vi ikke tage os af. Ja, forsker og regner (…) Ja, så har jeg jo så fået et bedre indtryk af det nu, fordi man storebror han laver det. (…) Og jeg har jo også været derude fordi (…) vi skulle møde min bror på H.C. Ørsted, og der synes jeg det var en rigtig lækker stemning der var derude (…) så det ser ud som om de hygger sig rigtigt meget. Og det er ikke kun de der nørdede typer som man, altså, sådan rigtigt stereotypisk, ikke? Det er jo både de flotte blondiner, og de seje drenge og så er der også et par nørder eller to. Men altså et virkede super fedt derude.

A11: When they are not lecturing, what are they doing then? It could be their own little project, they practice, or their...something they write a nice thesis about or...maybe they are sent out to upper secondary schools as external examinors or something, what do I know?

Q12: Now I might as well look up what you wrote. Conducts research, calculates and dances.

A12: What have I written there? I must have been a little silly there, I wrote dances. We should not consider that. Yes, conducts research and calculates. Yes, I have a better perception of it now, because my older brother is doing it. And I have been there because, we were meeting him at the department, and I think there was a great atmosphere there. I seems as if they are having a great time. And it is not only the nerdy types... like really stereotypical, right? There are both the gorgeous blonds and the cool boys, and then there is a nerd or two. But it seems super nice out there.

### *3.L. PRELIMINARY PLANS*[613]

Q13: Are you taking up studies directly after graduation or are you going to take a break?

A13: Everybody gets that question. I do not know, because I do not really know what I want, so it may be a year off. But I did consider something related to languages. Or some Philosophy. But it also concerns what possibilities, well, what one could do with those disciplines afterwards. And if I studied Philosophy, for example, and a language, then you could only become an upper secondary school teacher, right? Where if one, I do not know, if you study something in science, then many doors are open, right, to many things. So I do not really know. It is hard. Yes, so it will probably be a year of.

613

Q13: Ja. Vil du direkte i gang med noget efter gymnasiet? Eller vil du holde pause?

A13: Ja, det var det alle bliver spurgt om. Jeg ved det ikke, fordi at jeg ved ikke rigtigt hvad jeg vil, så det bliver jo nok et sabbatår. Men jeg har overvejet ellers det kunne blive noget sprog. Det kunne også blive noget filosofi. Men det handler jo også om hvad man, altså, hvad man kan med faget bagefter. Og hvis jeg læste filosofi for eksempel og et sprog, så ville du jo kun blive gymnasielærer, ikke? Hvorimod hvis man, jeg ved jo godt, hvis man læser sådan noget lidt naturvidenskabeligt, så kan man jo, altså så er dørene mange, ikke, til forskellige ting. Så jeg ved det ikke helt. Det er svært. Ja, så det bliver nok et sabbatår.

### *3.M. MATHEMATICS AND ME*[614]

Q14: And then how do you feel about mathematics yourself?

A14: Well, but I am fine with mathematics, I am glad that I chose it, because, as I said, it is a good thing to have and to know. And then, because you can almost enjoy sitting and doing little tasks at home and have a good time with that. And this thing about, that one can describe, well, all kinds of things in everyday life by some calculations. And then sometimes it can be a little difficult, but that is life. I think a lot of people feel like that about mathematics. Yes.

### *3.N. DIFFERENT ROLES IN DIFFERENT SUBJECTS*[615]

Q15: Do you feel that you have a different role when you are in your English class, compared to your Mathematics class?

A15: Yes, well, I feel that I speak up a bit more in the English class and in Philosophy, because I know it is something that I find that I am rather good at. And also because I am really interested in Philosophy and English, I do not know, but my mother says that I have a talent for languages, and I have [always] been good at it. So, there I think I really speak up a lot, at least compared to Mathematics, when I think about it. So there I might have another role, yes. There I am not as passive. Yes.

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614

Q14: (...) Og så hvordan du selv har det med matematik?

A14: Ja, men jeg har det godt med matematik, jeg er glad for at jeg har valgt det, fordi jeg synes som sagt, at det er bare en god ting at have og kunne. Og så fordi at man nærmest kan nyde at sidde og lave nogle små matematikopgaver derhjemme og hygge sig med det. Og det der med at man kan beskrive, altså alle mulige ting i hverdagen ved nogle beregninger. Og så, altså, ja selvom jeg ikke er the shooting star inden for matematik, men så synes jeg alligevel det er sjovt at kunne de der ting. (…) Og så nogle gange så kan det selvfølgelig godt være en lille smule svært, men sådan er det jo. Sådan tror jeg der er mange der har det med matematik. Ja.

615

Q15: Føler du, du har en anden rolle, når du er så er på dit engelskhold eller på dit filosofihold, end du har når du for eksempel har matematik?

A15: Ja altså jeg føler at, jeg føler at jeg får sagt lidt mere i timerne i engelsk og filosofi, fordi det ved jeg, det er noget som jeg, som jeg synes at, jeg er ret god til. Og fordi at jeg jo også blevet pokkers interesseret i filosofi og engelsk det, det ved jeg ikke, det, min mor siger jeg har sprogøre, men det har jeg været god til. Så der synes jeg, jeg siger rigtig meget, i hvert fald i forhold til matematik, når jeg lige kommer til at tænke på det. Så der har jeg nok en lidt anden rolle ja. Der er jeg ikke så passiv. Ja.

## THEMES IN ADELE'S 3RD YEAR

In the following we will be summing up what has been expressed in the 3rd year questionnaire and in the interview.

### *MATHEMATICS AT SCHOOL*

The classroom environment in Adele's does not support activity from a person who is insecure of her mathematical skills; Adele finds that some classmates are downright mean, if you are not good at mathematics, whereas others are nice and supportive (ADELE: Q3-F).

In her class, amongst her classmates, Adele finds that subjects from the Humanities are not valued as highly as subjects from the Natural Sciences. She finds, that if you are not good at mathematics, you may receive less kind comments (ADELE: 3.D.).

### *MATHEMATICS AS A DISCIPLINE*

Adele still sees Mathematics as related to her other subjects, Chemistry and Physics, which may indicate that she has some kind of perception of the role of mathematics in other sciences.(ADELE: Q3-B).

Now, in the 3rd year she indicates mathematics as something invented by human beings, rather than discovered. Mathematicians seems to be quite human to Adele; besides envisioning mathematicians in university to conduct research and computing, she also answers that they dance! Nevertheless, she does not seem to underestimate the effort that might be needed for studying it (ADELE: Q3-C)

Adele emphasises mathematics as a way of describing phenomena; she mentions forms and movements. But it is not evident that it is the pure usefulness of

it, that appeals to her, or whether it may be the mere fact that these descriptions are possible. It almost seems to thrill her (ADELE: 3.C.).

Adele does have an idea of what is going on around Mathematics in university, since her brother is studying it. But her answers relate to the activities of studying it, or teaching it, rather than the aspects of developing new mathematics (ADELE: 3.J., 3.K.).

### *MATHEMATICS IN SOCIETY*

Adele still finds mathematics useful for everybody and she sees it as useful for professions as well(ADELE: Q3-B).

### *MATHEMATICS & ME*

#### RATING

Adele stays on [7] for her rating of Mathematics on the Favourite Subject Scale, in both the 1st and the 3rd year. Mathematics has become harder, but she values the teaching in upper secondary school, since there is peace and order in class (ADELE: Q3-A).

#### DIDACTICAL CONTRACT

In class, Adele is not comfortable with speaking up; she is worried how her ideas will be received amongst her peers, but also of the impression she will leave with her mathematics teacher (ADELE: 3.F.).

#### CHALLENGES & SUPPORT

A too high level is the greatest obstacle in learning mathematics, according to Adele. As a strategy for improving in mathematics, Adele mentions repeated practice (ADELE: Q3-D).

Figuring out the purpose of a task is still in the 3rd year more challenging to Adele than other types of challenges (ADELE: Q3-E).

Adele has a wide palette of possibilities for support for mathematical activities; both at home and at school. She seems to be slightly more active in class compared to the 1st year; instead of being completely quiet in class she now indicates to ask or answer questions at least once in a while (ADELE: Q3-F).

### "ME" AS A MATHEMATICS LEARNER

She, herself, feels that the fundamentals are sometimes missing, but she might have an advantage in being good at formulae (ADELE: Q3-F).

For most of the suggested topics for examination, Adele seems to feel rather confident. Only differential equations and their solutions do not seem to appeal, and besides those topics on the list, also probability is mentioned as something she wishes to avoid (ADELE: Q3-XC).

It does not appear as if Adele does not understand Mathematics at all, but she may feel that she needs a little more time than her classmates to come to understand. And if she is in doubt, she can ask her father or her friends (ADELE: 3.G.).

For Adele it is not uncommon just to learn how to do something without necessarily understanding why from the beginning; however, quite often this kind of understanding evolves later on (ADELE: 3.I.).

For Adele, working with mathematical tasks seems rather enjoyable, and she repeatedly emphasises the power of describing things which Mathematics provides (ADELE: 3.M.).

In Language class and in Philosophy, Adele is more active and more confident,

compared to her role Mathematics class, in which she describes herself as having a more passive role (ADELE: 3.N.).

### PLANS

The experiences with mathematics in upper secondary school has convinced Adele that she should opt for an education within the humanities, since she feels stronger in that field compared to those involving a good deal of Mathematics (ADELE: Q3-G).In line with this, in the 3rd year Adele has made use of the greater freedom of choice for subjects by choosing A-level English and C-level philosophy in stead of upgrading Chemistry and Physics from B-level to A-level. In this way she has made a more humanistic turn to her otherwise science-oriented study programme (ADELE: 3.A.). In this way she has deselected the subjects involving a good deal of mathematics for those that do not.

As Adele's plans appear right now, she is not likely to need A-level mathematics to be admitted to the study programmes she now finds appealing. Despite that, she seems content with the fact that she has some fundamental knowledge of mathematics both in terms of the general purpose of managing your life and potentially for raising your children, but also if she might combine languages with some kind of business perspective in her future career (ADELE: 3.B.).

Adele seems more keen on studying Languages or Philosophy, but the idea of a career with these subjects does not appeal to her to the same extent as a career with the point of departure in the Natural Sciences. Instead of deciding on a specific tertiary education immediately, she considers taking a year off before she decides (ADELE: 3.L.).

### *WHO IS ADELE, AND WHAT DRIVES HER?*

Adele want to be good at what she is doing, she wants to be a nice person to

others and she wants others to treat her nicely. She may feel slightly discriminated in class due to her performance which she describes as less than the average. This does not seem to discourage her effort in mathematics, but it influences on her long-term plans and also on her possibilities for deselecting to upgrade mathematics-related subjects such as Physics and Chemistry. And in line with this; in Language class and in Philosophy, she is sung the praise of for her performance which influence her when selecting these subject in her third year of upper secondary school, and inviting these subjects into her plans for the future instead of the mathematics-related ones.

## ADELE'S 3RD YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics is still a challenge to Adele. She still appreciates her teacher and the way mathematics is taught; it seems to involve many elements from the task discourse.

### *MATHEMATICS AS A DISCIPLINE*

Adele appreciates mathematics for its ability to describe things. Even though she now knows a mathematics student in university, her ideas of the discipline is mainly relates to its role as an educational subject, rather than a discipline developing in itself.

### *MATHEMATICS AND SOCIETY*

Mathematics is useful and everybody should learn it. Mathematics can be used for describing things.

### *MATHEMATICS AND ME*

Mathematics is no longer part of Adele's future plans. In stead she plans on taking a university degree in the humanities.

## ADELE'S BELIEFS' TRANSPOSITION

*STABLE:*

Adele keeps talking nicely about her mathematics teacher and the mathematics teaching over the three years. She does her best to put it in a nice way, but she may not be that keen on the attitudes from those peers that reminds her that she feels challenged in the subject, and due to this maybe also less accepted among some peers.

*CHANGED:*

The main change in Adele's beliefs happened before the first year questionnaire; she realised that she would not get the grades for studying medicine. Originally this was an important reason for choosing this study programme. During upper secondary school she sees herself as just below the middle in mathematics, whereas she is successful in the humanities. On this basis, Adele deselects tertiary education in STEM-programmes – and medicine requiring A-level mathematics for admittance – and now opts for a university study in which she can be successful.

# *THE CASE OF GARY*

*Gary is a male student from Gamma Technical Upper Secondary School. Amongst the students keeping their third year rating of mathematics at a level similar to the first year level, Gary is the student rating mathematics lower; he gives mathematics a "5" on the 'Favourite Subject Scale' both years.*

*The case of Gary is in some sense an example of a development in which circumstances outside school has a lot to do with the decision to change plans from opting for a tertiary education in engineering to considering tertiary education programmes directing to professions such as social education or lower secondary school teaching in mathematics.*

| Gary | Date for Questionnaire | Date for interview |
|---|---|---|
| 1st Year | 24 November 2010<br>( + supplement<br>16 December 2010) | 6 April 2011 |
| 3rd Year | 17 December 2012 | 14 March 2013 |

*Table 83: Dates for Questionnaires and Interviews*

## GARY'S 1ST YEAR QUESTIONNAIRE

| Q1-A | TRANSITION | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [5] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [616] | [Working on your own] |
| **3** | Did you like mathematics when you went to lower secondary school? [617] | [Yes, it was fine] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | It has become harder, and the expectations to your level in different areas is high, and work methods[618] |
| **4b** | Is there anything you liked better before?' | No, I think it is nice with a change.[619] |
| **4c** | Is there anything you like better now? | Yes, more group work and so on[620] |

*Table 84: GARY'S 1st year questionnaire, part A – TRANSITION*

616 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

617 Options:: [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

618 Det er blevet sværere, og forventningen til niveau på forskellige områder er højt, og arbejdsmetoder.

619 Nej, jeg synes det er godt med forandring

620 Ja, mere gruppearbejde osv.

| Q1-B | FOR SCHOOL | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | Because it is important that you can compute, bills and so on. Denmark also needs engineers.[621] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **6** | What made you choose a study programme involving A-level mathematics? | It was the best match to the ideas I had about career[622] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | Physics, chemistry, technology is affected by it, I think[623] |

*Table 85: GARY'S 1st year questionnaire, part B - MATHEMATICS AND SCHOOL*

621 Fordi det er vigtigt at man kan regne, regninger etc. Danmark har også brug for ingeniører.

622 Det passede bedst sammen med de tanker jeg havde omkring karriere.

623 Fysik, kemi, teknologi synes jeg er berørt af det.

| Q1-C | BEYOND SCHOOL | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | Banks, shops, etc.[624] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? | [Mathematics is invented] |
| **10** | What do you think a professional mathematician at a university is doing? | I do not know.[625] |
| **11** | Would you have to be a genius in order to study mathematics in university? [626] | [I do not know] |

*Table 86: GARY'S 1st year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

624 Banker, forretninger, osv.

625 Aner det ikke.

626 Options:[Yes]; [No]; [I do not know]

| Q1-D | IMRPOVING | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | All the different rules/formulae, it is a little hard for me to keep track of it[627] |
| **12b** | What do you think is the best means for improving in mathematics? | Good teaching, and conversely, good intentions to receive it[628] |
| **12c** | What do you do to improve in mathematics? | I engage in it as much as possible[629] |
| **13a** | What do you do if you get stuck on a task at school? | Ask the person next to me, or the teacher[630] |
| **13b** | What do you do if you get stuck on your homework? | Ask a classmate[631] |

*Table 87: GARY'S 1st year Questionnaire, part D – STRATEGIES FOR IMPROVING*

627 Alle de forskellige regler/formler, jeg har lidt svært ved at holde styr på det.

628 God undervisning, og omvendt god vilje til at modtage den.

629 Jeg engagerer mig så meget som muligt i det.

630 Spørger personen ved siden af, eller læreren.

631 Spørger en klassekammerat.

| Q1-E | CHALLENGES & SUPPORT | | GARY |
|---|---|---|---|
| # | *Question* | | *Answer* [632] |
| **14** | What issues involve more challenges to you? | a) Remembering | [3] Moderate challenges |
| | | b) Computing | [4] Few challenges |
| | | c) Figuring out the purpose of a task | [3] Moderate challenges |
| | | d) Finding a way to solve a task | [5] The fewest challenges |
| | | e) Reading and understanding the textbook | [3] Moderate challenges |
| **15** | Where can you find support for mathematical activities? [633] | | [From classmates] |
| **16** | Did you parents take the Upper Secondary School Leaving Certificate? [634] | | [Yes, my mother did] |

*Table 88: GARY'S 1st year Questionnaire, part E – CHALLENGE AND SUPPORT*

632 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

633 [From parents]; [From siblings]; [From uncles or aunts]; [From cousins]; [From classmates]; [Other places] If other places, from where or from whom?

634 [Yes, my father did]; [Yes, my mother did]; [Both my parents did]; [None of them did]

| Q1-F | IN CLASS | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? | [1-3] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [I do not know] |
| **17c** | Are you content with that? | Yes |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? | [1-3] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | No |
| **19a** | In your class, is it okay to be good at mathematics? | Yes, it is a mathematics class, after all[635] |
| **19b** | In your class, is it okay to have difficulties in mathematics? | Yes, people are helpful. We help one another.[636] |

*Table 89: GARY'S 1st year Questionnaire, part F - MATHEMATICS IN CLASS*

635 Ja, det er trods alt mat A klasse.

636 Ja, folk er hjælpsomme, man hjælper hinanden.

| Q1-G | PLANS | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | Bachelor of Engineering at "GREENGATE University College of Engineering". Electrical Engineer[637] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [Yes] |
| **21b** | Comments: | I know that the Engineering Programmes contain a lot of Mathematics.[638] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [No] |
| **22b** | Comments | |

*Table 90: GARY'S 1st year Questionnaire, part G – PLANS*

637 Diplomingeniør på ["Greengate" University College of Engineering]. Elektroingeniør

638 Jeg ved at ingeniøruddannelser indeholder meget matematik

# GARY'S 1ST YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 6 April 2011 | 23:01 |

***1.A. TRANSITION***[639]

Q1: The first question is how you think mathematics has changed from when you went to lower secondary school, till now, in technical upper secondary school?

GA1: It is by all means a higher level, and that makes it more group work oriented, I would say, because some of the things here might not suit me, I think. Anyhow, there is a lot of geometry and such things. It was never me. Then it may be good that I can get help from somebody else. Otherwise I always just solved things myself in lower secondary school, so in that sense it has changed quite a lot.

639

Q1: Det første spørgsmål, det er, hvordan du synes, matematik har ændret sig fra du gik i folkeskolen og så nu her på HTX?

GA1: Det er i hvert fald et højere niveau, og det gør det sådan lidt mere gruppearbejde-orienteret, vil jeg sige, fordi der er nogle af tingene, der måske ikke lige passer til mig, synes jeg. I hvert fald er der meget sådan geometri og sådan noget. Det har aldrig været mig. Så der er det måske meget godt, man lige kan hente hjælp fra nogle andre. Ellers så har jeg altid bare selv lavet det i folkeskolen, så på det punkt har det ændret sig ret meget.

***1.B. CHALLENGES***[640]

Q2: But when you run into those things, when you think: "Okay, here I have some challenges I do not... then you find that it is your peers that are good to ask?

GA2: Yes, if I do not know how to work it out, I always try to solve the tasks myself. If I do not see any meaning in them, then I try to ask others for a hint or for getting some help with it.

Q3: But before you ask you do something yourself. What would you typically try to do, if you are to try to do something with a task?

GA3: Take a look at it. See if I can find some kind of system in it or something. What can I do to arrive at the result I am looking for? So it is a quite systematic approach I have for tasks and such. I like to put things into bullet points...

Q4: Does it give you an overview, so you can...

GA4: Sometimes. Then there are also just some things that I cannot work out. Then you get in touch with some friends or something or the teacher and ask.

640

Q2: Men når du støder ind i de her ting, hvor du tænker: "Okay, her er der nogle udfordringer, jeg slet ikke (...)" så er det kammeraterne, du synes, der er det godt at spørge?

GA2: Ja, hvis jeg ikke selv kan finde ud af det, jeg prøver altid at løse opgaverne selv. Hvis jeg ikke kan se nogen mening med det, så prøver jeg at spørge andre for sådan at få et hint eller at få lidt hjælp til det.

Q3: Men inden du spørger, så gør du lige nogle ting selv. Hvad kunne du typisk finde på at gøre, hvis du sådan skulle prøve at gøre et eller andet med en opgave?

GA3: Kigger på den. Ser om jeg kan finde et eller andet system i det eller et eller andet. Hvad kan jeg gøre for at komme frem til det resultat, jeg skal have? Så det er sådan meget systematisk, det er en ret systematisk tilgang, jeg har til opgaver sådan. Jeg kan godt lide at stille ting op i punkter. (utydeligt.)

Q4: Giver det dig så noget overblik, så du kan?

GA4: Somme tider. Så er der også bare nogle ting, jeg ikke kan finde ud af. Altså så er det der, man henvender sig til nogle venner eller et eller andet eller læreren og spørger.

*1.C. GETTING GOOD MARKS*[641]

Q5: Do you feel a difference in the support you get from your peers now, and the context around learning now, compared to lower secondary school?

GA5: Yes. I do. Because there was not really anyone interested in Mathematics in my old school. So it was, like, me people asked. Then it is a little different here, where I ask people and such. So yes, and in lower secondary school I never really made an effort. I could get good marks, if I felt like, but it is different now (unintelligible) have to fight for it.

Q6: Yes. How do you like that?

GA6: I think it is fine. Then you cannot just sit and relax. It is fine that you have to do something.

*1.D. HELP IN CLASS*[642]

Q7: What about how much time there is for helping, on the part of the teacher, I mean?

GA7: I do not ask that frequently, I would rather ask my peers, and if so, if we cannot find out ourselves, then, the teacher, because he is a little busy, since we are [many] students. Then it might be that he does not have the time. So you have to limit it as much as possible.

641

Q5: (…) Kan du mærke, at der er forskel på den støtte, du får fra kammeraterne nu, og det miljø, der er omkring at lære nu i forhold til i folkeskolen?

GA5: Ja. Det kan jeg godt. Fordi der var ikke rigtig nogen, der var interesseret i matematik i min gamle skole. Så var det sådan mig, folk kom og spurgte. Så det er lidt anderledes nu her, hvor jeg (utydeligt) spørger folk og sådan nogle ting... Så ja, og i folkeskolen har jeg egentlig aldrig rigtig gjort noget ved det... Jeg kunne få gode karakterer, hvis jeg havde lyst til det, så det er lidt anderledes nu. (utydeligt) nødt til at kæmpe for det.

Q6: Ja. Hvad synes du om det?

GA6: Det synes, jeg er fint nok. Så kan man ikke bare sidde og slappe af. Det er meget godt, at man skal lave noget.

642

Q7: Hvad med hvor meget tid, der så er til at hjælpe (…) Fra lærerens side, tænker jeg?

GA7: Jeg spørger ikke så tit, jeg vil hellere spørge mine klassekammerater, og hvis det så er, at vi ikke selv kan finde ud af det, så læreren, fordi han har jo også lidt travlt, når det er, at vi er [mange] elever. Så kan det være, at han ikke altid har tid nok. Så må man begrænse det så meget som muligt.

***1.E. WHO SHOULD LEARN MATHEMATICS?***[643]

Q8: Another thing is who do you think should learn mathematic;. if everybody should learn mathematics?

GA8: Everybody should learn mathematics, it is very important. At least the fundamental things. Things like bills, end-of-year account statements and so on. I think it is quite important, that you learn this in any case, this is like basic things in mathematics.

***1.F. WHY A-LEVEL MATHEMATICS?***[644]

Q9: What made you choose exactly this study programme?

GA9: Most of all that my study counsellor recommended it to me. And because of the study programmes I could choose, it was the most interesting. Yes, and because I was to study engineering [for tertiary education] so this was a good option.

***1.G. IMPROVING***[645]

Q10: Another thing that I ask: "*What one can do to improve in mathematics?*" I can see what you answered: You "*engage as much as possible*"?

GA10: Yes, to get a better understanding you have to acquaint yourself with it as much as possible. Then, I do not know,

643

Q8: En anden ting, (…) det er hvem du synes, der skal lære matematik, om alle skal lære matematik?

GA8: Alle skal lære matematik, det er jo meget vigtigt. I hvert fald grundtingene. Sådan noget med regninger og ja, årsopgørelser osv. Det, synes jeg, er ret vigtigt, at man lærer i hvert fald, det er sådan grundting i matematik.

644

Q9 (…) Hvad fik dig til at vælge lige præcis den her studieretning?

GA9: Mest af alt, fordi det var det, jeg blev anbefalet af min studievejleder. Og fordi af de studieretninger, jeg kunne vælge, så var det det, der var mest interessant. Ja, og så fordi jeg gerne vil læse til ingeniør, så var det en meget god idé at vælge.

645

Q10: En anden ting, jeg spørger til, det er, hvad man kan gøre for at blive endnu bedre til matematik (...) Jeg kan se, hvad du svarer der (…) Du engagerer dig så meget som muligt i det (…)?

GA10: Ja. For at få en større forståelse for det, så at sætte sig så meget ind i det som muligt i hvert fald. Så ved jeg ikke, så skal man måske også bare have nogle evner for sådan at forstå nogle sammenhænge og sådan noget. Det ved jeg ikke, om man ellers kan tilegne sig på andre måder.

Q11: Altså evner, det kan man jo tænke på på den måde, at det er noget, man enten har eller ikke har. Men det der med, hvad man kan gøre med de evner, man nu har, det?

GA11: Ja, man kan godt forbedre sine evner. Det kræver bare, man har dem.

one may also just have some flair for, like understanding relations and such things. That, I do not know, if you can learn by other means.

Q11: Well, "flair", you may think of is as something you either have or do not have. But what about what you can do with the abilities, you do have?

GA11: Yes, you can improve your ability. It just requires that you possess some.

***1.H. CHALLENGES***[646]

Q12: And then you say that some of the greatest hindrances for improving in mathematics are all the rules and formulae you have to be on top of.

GA12: It is not always I can remember the formulae. Then it is nice to have a collection of formulae in your bag. So in any case that is some of what I have had difficulties with, remembering formulae.

***1.I. STRENGTHS***[647]

Q13: But you say, well, you experience the least challenges in finding strategies for solving a task?

GA13: In general I am good at it, at least.

Q14: It sounds like an excellent thing to be good at?

GA14: Yes, I just think there are...At least in this part of the curriculum, I have had difficulties in understanding some of the things, so I have not really been able to connect them with the task, so.. Otherwise, in general, I am quite good at finding solutions to problems and such like.

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646

Q12: Og så siger du, noget af den største hindring for at blive bedre til matematik, det er alle de forskellige regler og formler, man skal holde styr på.

GA12: Det er ikke altid, jeg kan huske formlerne sådan. Så er det godt, man har en formelsamling nede i tasken. Så det i hvert fald noget af det, jeg har haft svært ved, det er at huske formler. Med mindre jeg har brugt dem ret mange gange.

647

Q13: Men du siger, altså du har færrest udfordringer med at finde en måde at løse en opgave på?

GA13: Jeg plejer at være ret god til det i hvert fald.

Q14: Det lyder da som en god ting at være god til.

GA14: Ja. Jeg tror bare, der er nogle... I hvert fald denne her del af pensummet, der har jeg haft lidt svært ved at forstå nogle af tingene, så har jeg ikke rigtig kunne sætte det ind i en sammenhæng i opgaven, så... Ellers plejer jeg at være meget god til at finde løsninger på problemer og sådan noget.

***1.J. SUPPORT***[648]

Q15: We already talked about where you can find support for mathematical activities, and in this context you especially mention your peers.

GA15: Yes, I think so. I think at least that it is a good idea (unintelligible) peers at school, then they also said that it was a good idea to team up with peers, to get everything done. Not only in mathematics, but also in other subjects.

***1.K. SUPPORT FOR HOMEWORK***[649]

Q16: What if you are doing something at home, if you sit in your room and do your homework and...

GA16: Then we are so lucky that we live in a world in which there are many means for getting in touch with other people. Otherwise you could just call or something, and ask people. Or ask your parents, but I do not think my mother knows too much about high-level mathematics, so...It would rather be friends and (unintelligible) the class.

648

Q15: Og vi har sådan set allerede snakket om, hvor man kan få støtte og hjælp til matematik, og der nævner du især kammeraterne.

GA15: Ja. Det synes jeg. Jeg synes i hvert fald, det er en meget god idé (utydeligt) kammerater i skolen. Og da jeg var nede og snakke med uddannelsesvejlederen her på skolen, der sagde de også, at det var en god idé, at man arbejdede sammen, sådan så man kunne nå alle tingene. Ikke kun i matematik men også andre fag.

649

Q16: Hvad så hvis man skal lave noget derhjemme. Altså der... Hvis du sidder på dit værelse og skal lave lektier og?

GA16: Så er vi jo så heldige, at vi lever i en verden, hvor der er mange forskellige måder at komme i kontakt med hinanden på. Ellers så man jo bare ringe eller et eller andet, og så spørge folk. Eller henvende sig til sine forældre, men jeg tror ikke, min mor har så meget forstand på høj-niveaus-matematik, så... Det bliver nok mere venner og... (utydeligt) Klassen.

***1.L. FAVOURITE SUBJECT SCALE***[650]

Q17: I ask, among other things, well, on a scale from 1 to 10, with 10 being your favourite subject, where would Mathematics be. Your answer is somewhat in between. What subjects do you really like?

GA17: I think it has changed a little.

Q18: It has changed a little?

GA18: It depends on what topics we deal with.

Q19: How would it be now, then?

GA19: Mathematics and Physics would probably be quite high.

Q20: Mathematics and Physics would be quite high?

GA20: Especially Physics, now after I went to a Physics Camp. Then I think it has become quite exciting. I definitely like Physics. It is also, it relates to things in a more simple way than Mathematics does.

650

Q17: Ja. Jeg spørger bl.a. til, altså på en skala fra 1 til 10, hvor 10 er ens yndlingsfag, hvor matematik ligger. Der svarer du sådan lidt midt imellem. Hvad for nogle fag kan du rigtig godt lide?

GA17: Jeg tror, det har ændret sig lidt.

Q18: Det har ændret sig lidt?

GA18: Det kommer an på, hvad for nogle emner, vi kører.

Q19: Hvordan ligger det nu, synes du?

GA19: Matematik og fysik ligger nok ret højt.

Q20: Matematik og fysik ligger ret højt, ja.

GA20: Specielt fysik her efter jeg har været på sådan en fysik-camp. Så synes jeg, det er blevet ret spændende. Jeg kan i hvert fald godt lide fysik. Det er også, det forholder sig også lidt mere enkelt til nogle ting end matematik gør.

***1.M. PLANS***[651]

Q21: Your plans after upper secondary school? When I asked in the questionnaire, then it was Electrical Engineering, as far as I remember, and you also knew exactly where to study it?

GA21: It still is, and I still know where. At "GREENGATE", at "GREENGATE University College of Engineering. And the reason why is that it is more practically oriented compared to university. At least as far as I have understood. And they also have more traineeships than university studies do.

Q22: And electro, it has your interest?

GA22: Yes, I like electro. It always had my interest. And now we will start out on "Technical Subjects A[652]", and then I will choose electro. There are different streams you can choose between.

Q23: Did you know that you wanted these studies, before you chose which Upper secondary school and what study programme to take? Well, was it part of, did it influence which study programme you chose, or?

651

Q21: Dine planer efter gymnasiet? Da jeg spurgte i spørgeskemaet, så var det elektroingeniør, såvidt jeg husker, og du vidste også lige præcis, hvor du gerne ville læse det henne.

GA21: Det er det stadig, og jeg ved stadig væk hvor henne. På "GREENGATE", ude på GREENGATE UNIVERSITY COLLEGE OF ENGINEERING. (…) Og grunden til, at jeg vælger det, det er sådan også, fordi det er lidt mere praktisk orienteret end universitetet. I hvert fald så vidt som jeg har forstået på det. Og de har også en del mere praktik end universitetet.

Q22: Og elektro, det er det, der interesserer dig?

GA22: Ja, jeg kan godt lide elektro. Det har altid interesseret mig. Og nu skal vi have Teknik A [Teknikfag, der er en type valgfagsretninger], her 3. år, der tager jeg også elektro. Der er sådan nogle forskellige retninger, man kan vælge.

Q23: Vidste du også, at du gerne ville læse det her, inden du skulle vælge gymnasium og inden du skulle vælge linje, altså var det med, havde det betydning for, hvad for en linje, du valgte eller?

GA23: Ja, det havde det. Jeg vidste ikke helt, hvad for en slags ingeniør, jeg ville læse til. Til at starte med, men det har jeg sådan fundet ud af i løbet af 9. klasse. Der var jeg nemlig på praktik ude på GREENGATE. Så altså det trak lidt der. Men jeg har altid gerne villet være ingeniør. Det har jeg villet siden 6. klasse eller sådan noget, og jeg har altid vidst, det skulle være et eller andet med computere eller elektronik. Det har altid sådan interesseret mig (...)

652 "Technical Subjects" in the technical upper secondary school programme are not the same as "Technology". Technical Subjects are elective courses combining practical and theoretical aspects (e.g. design, machine or electronics). Technology in the technical upper secondary programme, is an interdisciplinary project oriented subject, oriented around different themes (e.g. recycling, transportation).

GA23: Yes, it did. I did not know exactly what kind of engineer I wanted to study for. At the beginning. But I found out during ninth grade. Then I was in an internship at GREENGATE. So it appealed to me somehow. But I always wanted to become an engineer. I have wanted it since sixth grade or so, and I always knew I wanted to do something related to computers and electronics. It always had my interest.

***1.N. INTERDISCIPLINARITY***[653]

Q24: Can you use the mathematics you learn here in your other subjects?

GA24: Yes, I suppose so. At least in interdisciplinary projects. And yes, now we just had a project about (something). It was both Mathematics, Physics and Chemistry and Technology, that were involved, so then I could use it, yes. So in interdisciplinary contexts, yes. And then I do not know, I suppose I can also use it in Physics and Chemistry, because there you also work with numbers and data and so on. So you can use it in different contexts, but it depends a little on what kind of, well, what kind of level, you delve into, and yes, how advanced it is, I would say.

***1.O. APPLICATIONS***[654]

Q25: Yes, then I also ask what mathematics is used for, like, outside the school here?

GA25: Everyday arithmetic. There are also work places and such where you apply it. If you are to be an architect or something, then you need to know some trigonometry at least. And craftsmen and engineers apply it. There are

653

Q24: Altså kan du bruge det, du lærer i matematik, i dine andre fag her?

GA24: Ja, det kan jeg vel godt. (Gary svarer tøvende.) I hvert fald i tværfaglige projekter. Og ja, nu har vi faktisk fx lige haft sådan et projekt om (noget). Det var jo både matematik, fysik og kemi og teknologi, der var inde over det, så der kunne jeg godt bruge det, ja. Så i tværfaglige sammenhænge, ja. Og så ved jeg ikke lige, jeg kan nok også godt bruge det i fysik og kemi, fordi der arbejder du også med nogle tal og data osv. Så du kan godt bruge det flere forskellige steder, men det kommer lidt an på, hvad for en, altså hvad for et niveau, du dykker ned i, og ja, hvor avanceret det er, vil jeg sige.

654

Q25: Ja. Så spørger jeg også til, altså hvad matematik bliver brugt til sådan uden for skolen her.

GA25: Hverdagsregning (...) Der er også nogle forskellige arbejdspladser og sådan noget, hvor du også bruger det. Hvis du skal være arkitekt eller et eller andet, så skal du kunne noget trigonometri i hvert fald. Og håndværkere og sådan ingeniører bruger det. Der er mange steder, hvor det bliver brugt, hvor man egentlig ikke rigtigt tænker over det, og så er der jo nærmest matematik over det hele.

many places where it is applied, where you do not really think about it, and then there is more or less mathematics everywhere.

***1.P. THE NATURE OF MATHEMATICS***[655]

Q26: I also ask a question concerning how mathematics emerges, if it is something invented by human beings or if it exists already, and then is discovered by human beings.

GA26: I would say that it exists already. It is human beings that makes it real, kind of, because there are always... (unintelligible) Now, I can see two nuts over there, then you can say "1 + 1". And then it is human kind that makes it happen. So it is everywhere. But we add some numbers to it and formulae and so on.

655

Q26: Jeg stiller også et (..) spørgsmål (...) Det handler om, hvordan matematik opstår, om det er noget, som mennesker opfinder, eller om det er noget, der findes i forvejen og så bliver det opdaget af mennesker.

GA26: Jeg vil sige, det findes i forvejen. Det er mennesket, der sådan gør det til noget virkeligt på en eller anden måde, fordi der er jo altid... (utydeligt) Nu kan jeg jo så se, der ligger to nødder(utydeligt?) derovre. Så kan du så sige: "1 + 1". Og det er mennesker, der får det til at ske. Så det er noget, der er over det hele. Men vi sætter nogle tal på det og formler osv.

## THEMES IN GARY'S 1ST YEAR

Gary chose a study programme encompassing A-level mathematics because of its applicability to his plans for further education, namely an education in engineering.

### MATHEMATICS AT SCHOOL

Gary answers that he prefers working on his own, but at the same time he mentions group work as something he appreciates in upper secondary school compared to lower secondary school (GARY: **Q1-A**).

Now, in upper secondary school, Gary finds that mathematics is the expectations to the students are much higher than in lower secondary school. Now Gary needs help from peers which he did not rely on earlier (GARY: 1.A.). He can get help from fellow students for mathematical challenges now. He has not been used to needing it earlier, but he seems to appreciate it. In lower secondary school never had to make an effort to get good grades, but he recognises that it is needed now (GARY: **Q1-A**), (GARY: 1.C.).

### MATHEMATICS AS A DISCIPLINE

In technical upper secondary school Gary has had experiences with mathematics in interdisciplinary project work together with physics, chemistry and technology [656] and, when asked directly, he could think of mathematics as a tool in physics and chemistry, since you work with numbers and data, but he makes clear that it may be on a different (lower) level[657] (GARY: 1.N.).

656 "[...]nu har vi faktisk fx lige haft sådan et projekt om [noget]. Det var jo både matematik, fysik og kemi og teknologi, der var inde over det, så der kunne jeg godt bruge det, ja. Så i tværfaglige sammenhænge, ja."

657 "Og så ved jeg ikke lige, jeg kan nok også godt bruge det i fysik og kemi, fordi der arbejder du også med nogle tal og data osv. Så du kan godt bruge det flere forskellige steder, men det kommer lidt an på, hvad for en, altså hvad for et niveau, du dykker ned i, og ja, hvor

Gary does not have any image of what a university mathematician would be doing. In his questionnaire he suggests that mathematics is invented, but when interviewed his comments suggest a static view of mathematics: "*I would say that it exists already. It is human beings that makes it real, kind of...*"(1.P.)

### MATHEMATICS IN SOCIETY

When lining up examples of the use of mathematics outside a school context, Gary arrives at the conclusion, that "It is actually used a lot of places, in which you do not think about it, and then you could almost say that there is mathematics everywhere"[658]. He mentions these places: "*...There are also work places and such where you apply it. If you are to be an architect or something, then you need to know some trigonometry at least. And craftsmen and engineers apply it*" (GARY: 1.O.)

### MATHEMATICS & ME

#### *RATING*

At the time of replying the questions sheet, Gary rated mathematics a "5" on the favourite subject scale on which 1 would be your favourite subject. When interviewed, he now would rate mathematics and physics "*quite high*" and he indicates the rating to relate to the current topics (GARY: 1.L.).

#### *MATHEMATICS LEARNING*

A combination of effort and flair is needed to improve in mathematics: "*...you can improve your ability. It just requires that you possess some*"(GARY: 1.G.).

In general, Gary indicates to have problems in remembering formulae (GARY: 1.H.), but his general strength, finding a way to solve a task, is also challenged now;

avanceret det er, vil jeg sige."

658 "Der er mange steder, hvor det bliver brugt, hvor man egentlig ikke rigtigt tænker over det, og så er der jo nærmest matematik over det hele."

"*[I]n this part of the curriculum, I have had difficulties in understanding some of the things, so I have not really been able to connect them with the task*" (GARY: 1.I.). Friends rather than parents are sources to support for mathematical activities for Gary, even when not at school (GARY: 1.K.).

#### *PLANS*

Gary seems to have a quite specific plan for his further studies; he wants to become an engineer and he has decided it should be within electronics. The plan apparently has been there for a long time, because he mentions having had it since he was in grade 6. Also he knows exactly where he wants to take his degree; from a specific college near the capital. These plans played a role when he decided to choose the study programme (with A-level mathematics and A-level physics) for upper secondary school, and also that he preferred the technical upper secondary school over the general upper secondary school programme. (GARY: 1.F., 1.M.)

### MATHEMATICS IN SOCIETY

Gary thinks that mathematics is something everybody should learn, since basic mathematics play a role in paying your tax and paying bills. Friends or classmates are the main source of support for mathematical activities for Gary: "*I do not think my mother knows too much about high-level mathematics, so...It would rather be friends*"(1.K.).

## GARY'S 1ST YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

Mathematics in upper secondary school is different since it is harder and you need more help from your friends. In that sense it initiates and facilitates more group work.

### *MATHEMATICS AS A DISCIPLINE*

Gary has no idea of the activities of a university mathematician. His idea of the nature of mathematics seems to change from the view of mathematics as invented in the questionnaire to thinking that mathematics has always been there, but that human beings makes it real, which was what he said in the interview.

### *MATHEMATICS IN SOCIETY*

Gary thinks that mathematics is everywhere; in workplaces and in everyday arithmetics. And that everybody should learn it.

### *MATHEMATICS & ME*

Mathematics plays a role in Gary's future plans, since he wants to become an electronics engineer. This has been a dream since lower secondary school.

## GARY'S THIRD YEAR QUESTIONNAIRE

| Q3-A | TRANSITION | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **1** | On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics? | [5] |
| **2** | Are there any forms of organisation you prefer in mathematics (teaching) [659] | [Project Work] |
| **3** | Did you like mathematics when you went to lower secondary school? [660] | [Yes, it was one of my favourite subjects] |
| **4a** | In what ways has mathematics changed from when you were in lower secondary school to now, when you are in upper secondary school? | I have fallen behind – the learning situation is not good for me.[661] |
| **New 4d** | In which ways has mathematics changed from first year to third year of upper secondary school? | It has become somewhat harder and more demanding. [662] |

*Table 91: GARY'S 3rd year questionnaire, part A – TRANSITION*

659 [On your own]; [In pairs]; [Group Work]; [The whole class together]; [Project Work]; [Other:]

660 [Yes, it was one of my favourite subjects]; [Yes, it was fine]; [It was okay]; [It was not really me]; [No, I did not like it at all]; [Other:]

661 Jeg er faldet bagud – læringssituationen er ikke god for mig

662 Det er blevet noget sværere og krævende

| Q3-B | FOR SCHOOL | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **5a** | Why do you think it has been decided that everybody in Denmark should learn mathematics? | Because mathematics is applied everywhere and at a higher level it can be transferred to many things[663] |
| **5b** | Is mathematics something you think everybody should learn? | [Yes] |
| **7a** | Is mathematics related to your other subjects? | [Yes] |
| **7b** | Please give reasons for your answer: | Physics is in many ways linked up with mathematics[664] |

*Table 92: GARY'S 3rd year questionnaire, part B - MATHEMATICS AND SCHOOL*

663 Fordi matematik bruges alle stedet og på et højere niveau kan det overføres til mange ting

664 Fysik hænger på mange måder sammen med matematik

| Q3-C | BEYOND SCHOOL | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **8** | Where is mathematics employed when not directly related to education? Can you give examples of where mathematics is employed? | Calculations, e.g. a capacitor when Ohm's Law is not sufficient any more and differential equations become effective[665] |
| **9** | How does mathematics develop?<br>- Is it invented by human beings?<br>- Or does it exist already, and then discovered by human beings? [666] | [Both] |
| **10** | What do you think a professional mathematician at a university is doing? | - |
| **11** | Would you have to be a genius in order to study mathematics in university? [667] | [I do not know] |

*Table 93: GARY'S 3rd year questionnaire, part C- MATHEMATICS BEYOND SCHOOL*

665 Beregninger på for eksempel en kapacitator hvor ohms lov ikke er nok længere – differentialligninger træder i kraft

666 Options: [Invented]; [Discovered]; [Both]; [None of these options]; [I do not know]

667 [Yes]; [No]; [I do not know]

| Q3-D | IMRPOVING | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **12a** | What do you think is the greatest obstacle for you to improve in mathematics? | A great effort is demanded[668] |
| **12b** | What do you think is the best means for improving in mathematics? | A good exposition of the tools available[669] |
| **12c** | What do you do to improve in mathematics? | Not really anything – primarily due to personal problems and a gradual lack of interest[670] |
| **13a** | What do you do if you get stuck on a task at school? | Stop – or ask a classmate[671] |
| **13b** | What do you do if you get stuck on your homework? | Stop – or ask a classmate |
| **13c** | What do you do if you get stuck on your written assignments? | Stop – or ask a classmate |

*Table 94: GARY'S 3rd year Questionnaire, part D – STRATEGIES FOR IMPROVING*

668 En stor indsats er krævet

669 En god gennemgang af de værktøjer der er tilgængelige

670 Ikke rigtig noget – skyldes primært personlige problemer og en efterhånden manglende interesse

671 Stopper eller spørger en kammerat

| Q3-E | CHALLENGES & SUPPORT | | GARY |
|---|---|---|---|
| # | *Question* | | *Answer* [672] |
| **14** | What issues involve more challenges to you? | a) Remembering | [3] Moderate |
| | | b) Computing | [4] Few challenges |
| | | c) Figuring out the purpose of a task | [1] The most challenges |
| | | d) Finding a way to solve a task | [5] The fewest challenges |
| | | e) Reading and understanding the textbook | [4] Few challenges |
| **15** | Where can you find support for mathematical activities? | | [From classmates] |
| **New 16a** | Do you find that you meet some special challenges related to succeeding in mathematics compared to your classmates? | | To understand etc.[673] |
| **New 16b** | Do you find that you have some some special strengths related to succeeding in mathematics compared to your classmates? | | No |

*Table 95: GARY'S 3rd year Questionnaire, part E – CHALLENGE & SUPPORT*

672 Options: [1]= The most; [2]=Several; [3]= Moderate; [4]= Few; [5]=The fewest

673 At forstå osv.

| Q3-F | IN CLASS | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **17a** | How often would you typically raise your hand to ask questions during a mathematics lesson? [674] | [0] |
| **17b** | Do you think that you ask questions more frequently than other students in class? | [No] |
| **17c** | Are you content with that?[675] | No |
| **18a** | How often would you typically raise your hand to answer questions during a mathematics lesson? [676] | [0] |
| **18b** | Do you think that you answer questions more frequently than other students in class? | [No] |
| **18c** | Are you content with that? | No |
| **19a** | In your class, is it okay to be good at mathematics? | Yes[677] |
| **19b** | In your class, is it okay to have difficulties in mathematics? | I am not sure[678] |

*Table 96: GARY'S 3rd year Questionnaire, part F - MATHEMATICS IN CLASS*

674 Options: [0]; [1-3]; [4-8]; [More than 8 times]

675 Er du tilfreds med det?

676 Options: [0]; [1-3]; [4-8]; [More than 8 times]

677 Er det, i din klasse, OK at være god til matematik? "Ja"

678 Er det, i din klasse, OK at have svært ved matematik? "ved ikke"

| Q3-XA | UNDERSTANDING | GARY |
|---|---|---|
| # | *Question* | *Answer*[679] |
| **X.1.** | Have you recently experienced to *understand* what you worked with in mathematics? | [No] |
| | If yes, on which occasion? | - |
| **X.3.** | Have you, during upper secondary school experienced that you *understood* what you worked with and then s*ubsequently learned it by heart*? | [Yes] |
| | If yes, on which occasion? | Some geometry |
| **X.5.** | Have you during upper secondary school experienced *understanding something* but *never learning it by heart?* | [No] |
| | If yes, on which occasion? | - |

| Q3-XB | LEARNING BY HEART | GARY |
|---|---|---|
| # | *Question* | *Answer*[680] |
| **X.2.** | Have you recently experiences having to *learn something by heart*? | [I do not know] |
| | If yes, on which occasion? | - |
| **X.4.** | Have you, during upper secondary school experienced that you had to *learn something by heart*, and then, *subsequently understood it*? | [Yes] |
| | If yes, on which occasion? | Because I do not learn until I work with it[681] |
| **X.6.** | Have you during upper secondary school experienced *learning something by heart without ever understanding it*? | [No] |
| | If yes, on which occasion? | - |
| **X.7.** | Additional comments on understanding or rote learning in mathematics | "Swear word"-questionnaire[682] |

679 Options: [Yes], [No] or [I do not know]

680 Options: [Yes], [No] or [I do not know]

681 Fordi jeg først lærer noget når jeg arbejder med det

682 "Bandeord"-skema

| Q3-XC | A-LEVEL EXAMINATION | GARY |
|---|---|---|
| **X.8.** | *Topic* | *Answer*[683][684] |
| **(a)** | Parabola | [Okay] |
| **(b)** | Exponential | [Okay] |
| **(c)** | Pythagoras | [Readily] |
| **(d)** | Sine and cosine relations | [Readily] |
| **(e)** | Definition of differentiability | [Rather not] |
| **(f)** | Sum and product of differential functions | [Rather not] |
| **(g)** | Indefinite integral | [Rather not] |
| **(h)** | Volume of solid of revolution | [Rather not] |
| **(i)** | Differential Equations and their solutions | [Rather not] |
| **(j)** | Vectors in the plane, including scalar product | [Okay] |
| **(k)** | Lines and planes | [Readily] |
| **X.9.a.** | Which topic is your favourite? - and why? | Geometry |
| **X.9.b.** | Which topic would you rather avoid? - and why? | Differential Calculus[685] |

*Table 97: GARY: Topics for oral and written examinations in A-level Mathematics for Upper Secondary School Programmes in Denmark 2013*[686]

683 Options: [Readily], [Okay], [Rather not], [I do not know]

684 Svar: [Meget gerne]; [OK], [Helst ikke], [Ved ikke]

685 Differentialregning og integrering

686 Topics common to both the Technical (htx) and the General (stx) upper secondary programme examinations are suggested.

| Q3-G | PLANS | GARY |
|---|---|---|
| # | *Question* | *Answer* |
| **20** | What are your educational plans so far after upper secondary school? | School of Engineering |
| **20b** | What kind of influence would your final marks in mathematics have on your plans (or hope for plans) after upper secondary school? | A good deal[687] |
| **20c** | What kind of influence would your experiences with mathematics in upper secondary school have on your plans after upper secondary school? | Negative[688] |
| **21a** | Could you imagine opting for an education involving a good deal of mathematics? | [I do not know] |
| **21b** | Comments: | I do not feel that I am strong enough, but I do believe that I could become so[689] |
| **22a** | Could you imagine trying to avoid an education involving a good deal of mathematics? | [Yes] |
| **22b** | Comments | - |

*Table 98: GARY'S 3rd year Questionnaire, part G – PLANS*

687 En del

688 Dårlige

689 Jeg føler ikke jeg er stærk nok, men tror også at jeg kan blive det

# GARY'S 3RD YEAR INTERVIEW

| Date | Duration (mm:ss) |
|---|---|
| 13 March 2013 | 20:38 |

### *3.A. NOT REALLY ME* [690]

Q1: How do you feel about Mathematics for the time being?

GA1: Not that good, actually. I found out that it was not really me anyway. So I am not interested in it any more.

Q2: No, but it does have quite an emphasis here in technical upper secondary school, does it not? In your study programme?

GA2: Yes, it...yes.

Q3: But you write that you want to become an engineer afterwards, or what?

GA3: I have actually also become uncertain about that. Right now I do not really know what I want to do. I have some personal problems and such, something which is also part of the reasons for it.

690

Q1: Hvordan har du det med matematik for tiden?

GA1: Øh, ikke så godt. Jeg har fundet ud af at det ikke var så meget mig alligevel. Så interesserer det mig bare ikke rigtig længere.

Q2: Nej. (…) Men det fylder en del, her på HTX, gør det ikke? På jeres studieretning?

GA2: Jo, det… Ja.

Q3: Men du skriver, du vil egentlig gerne være ingeniør bagefter, eller hvad?

GA3: Det er jeg faktisk også blevet lidt usikker på. Jeg ved ikke rigtig hvad jeg vil lige nu. Jeg har lidt personlige problemer og sådan noget, det er en af grundene til det.

***3.B. BEHIND***[691]

Q4: Do you know if you will try and take the exam this summer?

GA4: Yes, I think so. I would like to get the diploma.

Q5: You want to get this, and then take things from there?

GA5: I think I will take a break, unless I come up with something else relatively soon.

***3.C. BEHIND...***[692]

Q6: Yes, but you say you are lagging behind, in your...

GA6: I am lagging behind a lot.

Q7: You are lagging behind a lot?

GA7: I have been lagging behind since the second year.

Q8: And is there something you have to hand in to be allowed to attempt the exam, or is it rather lagging behind in terms of understanding?

GA8: Both.

Q9: What do you say, both?

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691

Q4: Ved du om du går op til eksamen til sommer?

GA4: Ja, det regner jeg med. Jeg vil gerne have afgangsbevis.

Q5: Du vil gerne have det i hus, og så tager du den derfra.

GA5: Jeg tror lige jeg holder en pause, ellers, medmindre jeg finder ud af et eller andet, sådan, her, indenfor et kort stykke tid.

692

Q6: Ja. Men du siger… Du er også kommet bagud, siger du, i dit…

GA6: Jeg er meget bagud.

Q7: Du er meget bagud?

GA7: Det har jeg været siden andet år.

Q8: Og er der noget du skal aflevere for at få lov til at gå til eksamen, er det mere det med at være bagud for at forstå tingene?

GA8: Begge dele.

Q9: Hvad siger du, begge dele?

GA9: Jeg har ikke rigtig haft mentalt overskud til at lave så meget. Det har været lidt svært for mig at komme i gang. Det gik jo sådan set relativt godt første år.

Q10: Men du siger, at det handler mere om noget udenfor skolen end det handler om det på skolen, eller spiller det sammen på en måde?

GA10: Begge dele, fordi at både presset hernede og så hjemme, så… Det er lidt begge dele.

GA9: I have not really had the mental capacity to do that much. It has been difficult for me to get started. It actually went quite well the first year.

Q10: But you say that to a large extent this concerns something outside school, rather than something in school, or is there an interrelation in some way?

GA10: Both, because it is both the pressure here and at home, so...It was kind of both.

### *3.D. PLANS*[693]

Q11: Will you leave mathematics completely behind after technical upper secondary school, or may there be a bit of it?

GA11: Well, I am considering social work or lower secondary school teacher or something, So it is not completely... It will always be there, I presume. So I could probably not skip it, but if I can skip it to such an extent that it is not as complicated as at A-level, then it would be fine with me. It is what...It takes a lot of dedication, I think. I have not really had the extra it takes to do that.

Q12: No. But...teacher education and social work, that is completely new, well, it is some quite new directions to consider. What do you like about them?

GA12: Well, I like communication things, and I always like, like, to be with children. And just, kind of, people younger than me. I think it is quite nice.

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Q11: Vil du så lægge matematik helt bag dig efter HTX, eller må der godt være lidt af det?

GA11: Altså, jeg tænker lidt at jeg måske kunne tænke pædagog eller sådan noget. Eller lærer eller et eller andet. Så det er jo ikke sådan helt… Det vil jo næsten altid være der, går jeg ud fra. Så jeg kan jo nok ikke helt slippe for det, men hvis jeg kan slippe for det i en sådan grad at det ikke er helt ligeså kompliceret som på A-niveau, så ville det være fint nok med mig. Det er sådan det der gør at… Det tager meget dedikation, synes jeg. Det har jeg ikke rigtig lige haft overskud til.

Q12: Nej. Men… Lærer og pædagog, det er nogle helt nye, altså, det er nogle helt nye retninger at kigge i. Hvad kan du godt lide ved det?

GA12: Altså, jeg kan godt lide at formidle ting, og jeg har altid været glad for at, sådan, være sammen med børn. Og bare, sådan, mennesker som er yngre end mig, egentlig. Det synes jeg er meget sjovt.

### *3.E. SUPPORT?*[694]

Q13: Who can you draw on, when you are behind with your things, both in terms of understanding and handing in things, who could you draw on? Where could you get help?

GA13: Some of my friends, maybe. I do not really have an interplay with GAMMAGAARD, I think. He is not my type of teacher. Then I can, maybe, talk to GAMMABO, but... we should have made some kind of arrangement or something, but it has not really come into being. I cannot really do it at home, because my father does not really have an education, and my mother is kind of the same.

### *3.F. LEARNING BY HEART*[695]

Q14: Some of the questions I ask may not make to much sense to you, and I do not know if they are relevant questions to ask at technical upper secondary school, because you do not take an exam without supporting materials. So I do not know if I hit the wrong goal compared to where you are.

GA14: There probably were some of them that did not make much sense to me.

Q15: Is it necessary at all to learn anything by heart for this programme?

GA15: Yes, I would say so. Because I never learned anything by heart, but I need that. I do not take notes and such. I never really got used to it, ever.

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Q13: Hvem kan du trække på når du nu er bagud med nogle ting, både med at forstå nogle ting og med at aflevere nogle ting, hvem kan du så trække på? Hvor kan du få hjælp henne?

GA13: Nogle af mine venner, altså. Måske min lærer også. Jeg har bare ikke lige samspil med GAMMAGAARD, synes jeg. Han er ikke lige min type lærer. Så kan jeg måske snakke med GAMMABO, men… vi skulle også have lavet en eller anden aftale eller sådan et eller andet, men det blev aldrig rigtig til noget. Jeg kan ikke rigtig gøre det derhjemme, fordi min far, han har ikke rigtig noget uddannelse, (...), og min mor er sådan lidt det samme. Så (...)

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Q14: Der er nogle af spørgsmålene som jeg stiller, som jeg ikke er sikker på giver særlig meget mening for jer (spørgsmål angående at *forstå* versus at *lære udenad*, red.) (…) men jeg ved heller ikke om det er nogle relevante spørgsmål at stille på HTX, for I skal jo ikke til eksamen uden hjælpemidler, så jeg ved ikke om det er fordi jeg rammer ved siden af der hvor I er.

GA14: Der var nok noget af det som ikke lige var så relevant for mig.

Q15: Er det overhovedet nødvendigt at lære noget som helst udenad på den hér uddannelse?

GA15: Ja, det vil jeg sige, fordi jeg har ikke rigtig lært noget udenad, og det har jeg brug for. Jeg tager ikke rigtig noter og sådan noget. Det er ikke rigtig noget jeg sådan lige har fået vænnet mig til, nogensinde.

### 3.G. UNDERSTANDING[696]

Q16: What does it take to pass, Mathematics this summer? What does it take to get through?

GA16: I almost need to get it crammed in to understand it, I think. And I do not think anybody has the time to do that with me. If it gets too complicated, I have a problem in dealing with it.

Q17: I think some of the examination tasks I see for Technical Upper Secondary School, they look hard. It is not simple tasks, you are given. It is not something about just finding a formula.

GA17: No, I also find it hard. That is kind of my problem. Because I do not feel like doing it, if I do not understand it.

### 3.H. PASSING EXAMINATIONS?[697]

Q18: Is there something where you can say "*If I draw this question, then I might be lucky, and then things will work, and*

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Q16: Hvad skal der til for at bestå for eksempel matematik-faget her til sommer? Hvor meget skal der til for at komme igennem?

GA16: Jeg skal nærmest have tæsket det igennem hovedet for at forstå det, tror jeg. Og det tror jeg ikke der er nogen der har tid til at gøre med mig. Hvis det bliver for kompliceret, så har jeg et problem med lige at overskue det.

Q17: Jeg synes at de eksamensopgaver jeg ser på HTX, de ser sværere ud. Det er ikke simple opgaver man bliver stillet. Det er ikke noget med lige at finde én formel frem.

GA17: Nej, jeg synes også det er svært. Det er lidt det der er mit problem. Fordi så har jeg ikke rigtig lyst til at lave det, hvis ikke jeg kan forstå det.

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Q18: Er der noget du kan være heldig at: "*Hvis jeg trækker dét her, så kan det være jeg er heldig, og så går det, og hvis jeg trækker det her, så har jeg svært ved at se hvordan det kommer igennem*"?

GA18: Jeg tror, så længe jeg ikke trækker sådan noget som mundtlig matematik eller sådan noget, så tror jeg godt jeg kan klare det meste af det, sådan lige... Og måske lidt mere end bare det. (...)

Q19: Og matematik skriftlig?

GA19: Det synes jeg bare er lidt nemmere at klare, fordi du kan sidde og tænke lidt over det. Og så også prøve at løse nogle af opgaverne i den rækkefølge du nu vil. Og så er det også med hjælpemidler, så det…

Q20: Så der er en chance for at overleve uden at du skal lave om på noget du ikke kan lave om på? Jamen jeg tænker på, altså, med de muligheder for at få hjælp fra nogen til at forstå tingene og til at få ting afleveret. Er du nødt til at skrue op for noget? Er du nødt til at gøre mere af noget? Er du nødt til at have fat i nogle nye personer?

GA20: Jeg regner med at jeg skal forberede mig lidt mere op til eksamen. Det har jeg nok brug for. Jeg har nok også brug for at få kigget lidt på noget matematik der, og blive lidt bedre til det. Vi skal også have lavet nogle opgaver, jeg og nogle kammerater sammen.

*if I draw that question, then I can hardly see how to get through it?"*

GA18: I think that as long as I do not have to attempt on oral mathematics exam, then I think I will be able to work out most of it...and maybe even more than that.

Q19: And the written Mathematics exam?

GA19: I think this is a little easier to handle, because you can think it over. And also try to solve some of he tasks in the order you prefer. And then it is with supporting materials, so that...

Q20: So there is a chance that you might pass without changing something that you cannot change? I mean, with the present possibilities for help from someone to understand things and hand in things. Do you have to "turn up" something? Do you have to make more of an effort? Do you need to get in touch with new people?

GA20: I expect that I need to prepare a little more for the exams. I might need that. I may also need to have a look at some Mathematics, to improve. We also intend to do work on some tasks, some friends and I.

#### *3.I. NEW WAY OF THINKING*[698]

Q21: Will you be able to use what you got from here, from Mathematics?

GA21: Hmm. I do not know about Mathematics, but yes, some of it I will use. It also changed my way of thinking a little, being here. The way I perceive things and such.

Q22: Yes. Can you say some more about that?

GA22: I approach problems in a different way. A little, like, more analytically and such. It is easier for me to deal with things compared to how I felt earlier. And I have also improved in group work and in cooperating some more with other people. I have not been that good at this earlier.

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Q21: Kommer du til at bruge det, det du har med herfra af matematik?

GA21: Hm. Jeg ved ikke med matematikken, men jo, noget af det kommer jeg jo til at bruge. Det har også ændret min tankegang lidt at være herude. Hvordan jeg ser på tingene og sådan noget.

Q22: Ja. Kan du sige noget mere om det?

GA22: Jeg går til problemer på en anden måde. Lidt sådan, lidt mere analyserende og sådan noget. Jeg har nemmere ved at overskue tingene, også, end jeg havde før. Og så er jeg også bare blevet bedre til gruppearbejde, og at samarbejde noget mere med mennesker. Det har jeg ikke været så godt til før i tiden.

### *3.J. A-LEVEL DEMANDS*[699]

Q23: Did you get surprised by the demands in Mathematics, here at A-level Mathematics in Technical Upper Secondary School?

GA23: Well, the first year I thought it was fine. And then in second year, it suddenly began to go wrong. And then I felt this pressure from home, and such, and then it just went worse and worse. I think it is really tough right now. It is hard for me to see how I, like, should become really really good at it, before my exams. But given that this is A-level, I think the demands are fair.

### *3.K. DEDICATION FOR LEARNING*[700]

Q24: You write: [as an answer to] "*What are the best means for improving in mathematics?*" "*A good exposition of the tools available*" And who should make such an exposition?

GA24: Some teacher who knows about it.

Q25: So, it means that you are dependent on somebody who can help in that area. But you say [for] "*What is the greatest hindrance?*" "*That it demands a lot of dedication to become really good at it*"

GA25: As I said, it demands a lot of dedication to become really good at it...

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Q23: Blev du overrasket over kravene til matematik, her på HTX A-niveau?

GA23: Altså, første år syntes jeg det var fint nok. Og så på andet år begyndte det at gå dårligt lige pludselig. Og så følte jeg det dér pres hjemmefra og sådan noget, og så gik det bare dårligere og dårligere. Jeg synes det er rigtig svært nu. Jeg har svært ved at se hvordan jeg, sådan, skal blive rigtig god til det, inden jeg skal til eksamen. Men i forhold til at det er A-niveau, så synes jeg at kravene er rimelige nok.

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Q24: Du skriver, "*Hvad er det bedste middel til at blive bedre til matematik?*" "E*n god gennemgang af de værktøjer der er tilgængelige.*" Og hvem er det der skal lave den gennemgang?

GA24: Enten en lærer eller en anden en som har forstand på det.

Q25: Så det vil sige, der er man afhængig af at der er nogen der kan, der kan hjælpe en på det felt dér. Men du siger… Og "Hvad er den største hindring?" "At det kræver en stor indsats af kræfter, at gøre noget ved det."

GA25: Som jeg sagde, jeg synes det kræver meget dedikation at blive rigtig god til det.

### *3.L. NEW PROBLEMS & CALCULATIONS*[701]

Q26: I ask: "*What can mathematics be used for outside school?*" [You write:] "*Calculations, e.g. a capacitor when Ohm's Law is not sufficient any more and differential equations becomes effective*"

GA26: Yes, it was because I went to WESTPORT College of engineering, and then it was, it is just an example, this. It was something they talked about, that Ohm's Law was suddenly not sufficient. Then they would calculate by way of integration and differential equations and such like instead.

### *3.M. MATHEMATICS TEACHER IS A POSSIBILITY*[702]

Q27: If we imagine that you study social work or that you study to become a teacher. Would you choose mathematics as a specialisation at teacher college, for example?

GA27: I might specialise in it for lower secondary school, because I feel quite okay with that. And it would probably also be a lower secondary school teacher that I preferred to become. Now I might not want to become an upper secondary school teacher.

Q28: So even given the mathematics you meet now, and what it takes to deal with it, you have not been scared away from dealing with it in the future?

GA28: No, it is fine. I like mathematics. It is a good tool. It is just the things we deal with right now, they are not really me. It is too high a level, I think.

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Q26: Her, jeg spørger "*Hvad kan man bruge matematik til udenfor skolen?*" "*Beregninger på kapacitator, hvor Ohms lov ikke er nok længere, differentialligning træder i kraft.*"

GA26: Ja, det var fordi jeg var på "WESTPORT" Ingeniørhøjskole, og der var det, det er jo bare et eksempel, det dér. Det var noget de snakkede om, der var Ohms Lov ikke lige pludselig nok. Så skulle de regne med integrationer og differentialligninger og sådan noget for at løse det.

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Q27: Hvis vi nu forestiller os du læser pædagog eller du læser lærer. Ville du vælge matematik på linje på en læreruddannelse, for eksempel?

Q27: Jeg kunne godt finde på at tage det i folkeskolen, fordi det har jeg det sådan, rimelig okay med. Og det er nok også helst folkeskolelærer jeg vil være. Nu gider jeg nok ikke rigtig være gymnasielærer.

Q28: Så selvom at den matematik I møder nu, og det det kræver at have med den at gøre, så er det sådan set ikke fordi du er blevet skræmt væk fra at have med matematik at gøre?

GA28: Nej, det er sådan set fint nok. Jeg kan godt lide matematik. Det er et godt værktøj. Men det er bare de ting jeg arbejder med nu der ikke lige er mig. Det er lidt for højt plan, synes jeg.

### *3.N. THE NATURE OF MATHEMATICS*[703]

Q29: Mathematics, one can look at it as something created by human beings, or something that was already there, which we then discover. Is there any of those two ways of perceiving it, that you find makes more sense?

GA29: I think Mathematics exists. Well, as you just create it, you put up some systems, that makes it work for us, so we can kind of decode some of the things that happen in the world, and such things. If that makes sense.

Q30: Are we done with the process? Is mathematics complete, and then it is just to teach it, or are there ways to create new mathematics? Or discover?

GA30: Well, if one just takes a look at everything that is happening now, like with particle accelerators and such things, mathematics is also applied. But there has not been discovered any new mathematics since the 1960's or something like that.

Q31: New mathematics? Or new Physics?

GA31: Now I think of equations and such things, that have been set up. But there can always emerge something new.

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Q29: (…) Matematik, man kan se på det som noget som er menneskeskabt, eller som noget der findes i forvejen, og så er det noget vi opdager der findes (…) Er der en af de måder du synes giver mere mening end den anden?

GA29: Jeg tror matematik findes. Altså, som man lige skaber det, man lige sætter nogle systemer op, som gør det kan fungere for os, så vi ligesom kan afkode nogle af de ting som sker i verden, og sådan noget. Hvis det giver mening.

Q30: (…) Er vi færdige med processen? Er matematik færdig og så er det bare at undervise i det, eller er der måder at lave ny matematik på? Eller opdage?

GA30: Altså, hvis man bare tager og kigger på alt det der sker nu, sådan med partikelaccellerator og sådan noget, der bliver der jo også anvendt matematik. Men der er jo ikke blevet opdaget ny matematik siden tresserne eller sådan noget.

Q31: Ny matematik? Eller ny fysik?

GA31: Nu tænker jeg på ligninger og sådan noget, der er blevet opstillet. Men der kan altid komme noget nyt.

## THEMES IN GARY'S 3RD YEAR

Now in third year, Gary is not longer too fund of mathematics, mainly due to personal problems. Mathematics is rated with a [5] on the favourite subject scale, which was also the case in the first year questionnaire. Nevertheless, it had a peak at the end of first year; at the time of the first year interview, he would have rated mathematics "quite high".

### *CHALLENGES & NEW PLANS*

In the third year questionnaire, Gary now indicates that he is in doubt if he is strong enough in mathematics to go for an education containing a good deal of mathematics, but at this point he mentions that he might be able to change this and to gain the strength. In the period from when the questionnaire was answered and until the interview was given, Gary has now given up the plan of becoming an engineer. He is behind in handing in tasks and also behind in understanding the topics. A-level mathematics is too demanding for him in his situation, but it is not as if he would be deselecting Mathematics as a Discipline. He mentions teacher education or social education as possible paths for him, and it would involve mathematics to some extend. He also mentions teacher education specialising in mathematics teaching as a possibility, since he is not 'scared' from dealing with mathematics in general, but rather he distinguishes between the demands of mathematics at A-level and the general usefulness of mathematics in general.

### *RELATIONAL RATIONALE FOR LEARNING*

Gary shows an interesting perspective on whether he can use the mathematics he has learned in upper secondary school in his life outside school; he mentions that it has changed his way of thinking and that he now is better equipped in dealing with problems because he now has a more analytical way of approaching them.

Giving an example from physics, he also mentions problems that are solvable only by means of the mathematics he has now studied in the third year of A-level mathematics in upper secondary school; "differential equations and integration" . All in all he seems to be left with the impression of mathematics being applicable as such - even the more complicated topics.

### MATHEMATICS & ME

All in all Gary has kept an idea of the usefulness of the mathematics he has been taught. He clearly distinguishes between the challenges relating to his own situation, and his abilities as a mathematics learner. However, his experiences and his interpretation of his situation results in giving up the idea of becoming an engineer. Even though his experiences with mathematics in upper secondary school have a negative influence on his plans for further education it seems fair to presume that his situation outside school has a greater deal of the responsibility for this result than the experiences with A-level Mathematics at School, even if is a question of some kind of threshold between the resources for engaging in learning and the demands from an ambitious study programme.

### MATHEMATICS AS A DISCIPLINE

To Gary, the nature of mathematics seems to be static, whereas the applications of it may develop: "*I think Mathematics exists. Well, as you just create it, you put up some systems, that makes it work for us, so we can kind of decode some of the things that happen in the world, and such things*". In the questionnaire, Gary suggests that mathematics is both invented and discovered.

### *WHO IS GARY AND WHAT DRIVES HIM?*

Somehow we see a strong connection between Gary's role as a mathematics doer and learner and the demands from A-level mathematics. What is worth

noticing is that his beliefs about himself as a mathematics learner is distinguishing between his possible abilities and the circumstances he is in. That being said, the result of the situation is that he in the third year indicates that he is no longer doing anything to improve in mathematics due to his personal problems. This was not the case in first year, when he indicated to engage as much as possible in improving in mathematics.

The lack of engagement in improving in mathematics has not had a negative influence on the ability to see the usefulness of mathematics outside the subject itself. He perceives even advanced mathematics as highly useful - maybe a derivative belief from his knowledge of the applicability of differential equations or similar experiences from technical upper secondary school of from his interest in engineering. Taking a further look at his beliefs about mathematics shows that he may nor perceive mathematics a developing, but rather society and science - leaving new areas for application of mathematics as the element of development, rather than mathematics itself.

This leaves us with indications of a person with existing beliefs about Mathematics in Society and about mathematics as a tool for other sciences, but his beliefs about mathematics as a science on its own right asking and answering questions in its own domain, still developing, and not, as suggested, left unchanged since the 1960's has not been developed.

## GARY'S 3RD YEAR BELIEFS

### *MATHEMATICS AT SCHOOL*

A-level Mathematics has become much harder.

### *MATHEMATICS AS A DISCIPLINE*

Mathematics is both discovered and invented. There has been no new mathematics developed since the 1960's.

### *MATHEMATICS IN SOCIETY*

Mathematics is everywhere. You cannot avoid having to deal with it.

### *MATHEMATICS AND ME*

Gary has more or less given up as a learner and doer of A-level mathematics. Gary is behind in handing in tasks and in understanding the topic. He is unsure of he can pass the final exam. He is considering new plans outside STEM-related tertiary education.

## GARY'S BELIEFS TRANSPOSITION

From the first to the third year of upper secondary school, these trends seems to be of major importance for Gary's plans for tertiary education:

### *MAJOR CHANGES*

In the first year, Gary chose an A-level study programme due to his plans of becoming an engineer. In third year, A-level mathematics has become too hard for him and this has made him change plans from a STEM-subject – engineering - to a job as social worker or lower secondary school teacher with mathematics. In the sense, the influence changes direction from going from "*Mathematics & Me*" to "*Mathematics at School*" to the opposite direction.

### *STABLE DEVELOPMENT*

Mathematics is still perceived as useful in society and in disciplines from the natural sciences.

# AN IDEAL TYPICAL STUDENT'S BELIEFS

*In this Section, I will present an "IDEAL TYPICAL A-LEVEL STUDENT'S TYPICAL BELIEFS ABOUT MATHEMATICS". The description is based partly on statistics from the whole population and partly on material from the CASE informants; both interviews and selected items from the questionnaires. This Ideal Typical account summarises what has been accumulated in the minds of upper secondary school students in study programmes involving A-level Mathematics.*

The exposition of a set of typical beliefs in a typical upper secondary A-level Mathematics student is organised in the four categories of aspects of beliefs:

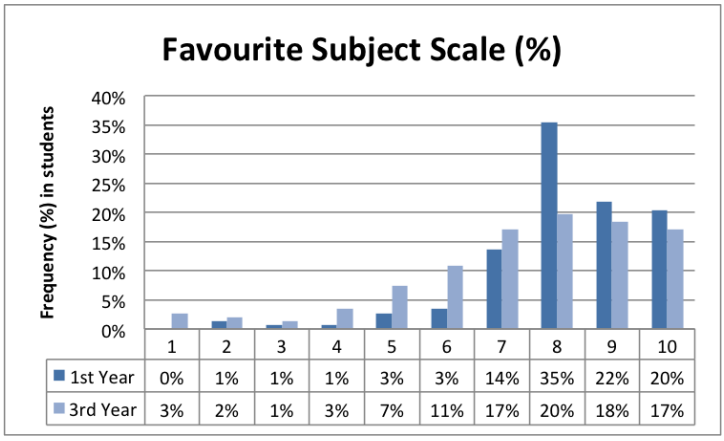
- Mathematics at School
- Mathematics as a Discipline
- Mathematics in Society
- Mathematics and Me

For each aspect, a summary of typical beliefs is presented, followed by selected quotes from the case interviews and graphs from the quantitative analysis supporting the formation of this ideal type.

## TYPICAL BELIEFS ABOUT MATHEMATICS AND ME

*A typical A-level student beliefs that Mathematics is fun, when he or she understands it, and feels less well about Mathematics if he or she is lagging behind. Even though the rating may change frequently due to ups and downs in terms of understanding the topics, Mathematics may well be found amongst the top-3 of school subjects for the typical A-level student.*

One important instrument for assessing how the respondents feels about mathematics, has been the favourite Subject Scale (#1 in both the first year and the third year questionnaire: Q1-A and Q3-A, respectively). In the large population there is a trend toward a general decline in the rating from the fist to the third year.




| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1st Year | 0% | 1% | 1% | 1% | 3% | 3% | 14% | 35% | 22% | 20% |
| 3rd Year | 3% | 2% | 1% | 3% | 7% | 11% | 17% | 20% | 18% | 17% |

*Illustration 1: Item #1: On a scale from 1 to 10, on which 10 stand for your favourite subject, how would you rate mathematics?*

Despite this general decline, the most typical response in both the first and the third year, is to give Mathematics the rating [8], which is 35% of the students in the first year and 20% of the students in the third year.

Suggestions of which experiences that might lead to this sort of development can be found in the interview excerpts. For Donna, the experience of beginning to

understand Mathematics is of paramount importance. Also to Dylan, the organisation of the teaching and how this supports his understanding is also connected to his rating of Mathematics on the favourite subject scale. Grace relates her rating to whether "it" [Mathematics] makes sense, whether you understand in or not and whether you are in control of it or not. Adele is fine with mathematics, but it bothers her that she is not as fast as her peers to understand new things. Instead Adele prefers the disciplines of the humanities, in which she finds herself to be more successful. Brandon loves Mathematics as a school subject, because it is very easy to him. And Gary does not feel very well about mathematics. He is behind in both understanding and handing in mandatory tasks and he can hardly see his way out of that situation due to personal problems in his home.

| | **FAVOURITE SUBJECT SCALE** |
|---|---|
| **Name** | **Quotes from the 3rd Year Interviews** |
| **DONNA (2 → 7)** | 3A.<br>Q1: How do you feel about mathematics now?<br>D1: Much better. I actually think that mathematics has started to become fun again, so much better.<br>Q2: Why is it about to become fun?<br>D2: Because I have started to understand the topics. And when I understand the topics, then I see the logic of it and then everything begins to become much more fun than it were in the 1st year and year 2, when I did not understand anything of anything, then it just was a pain to have mathematics. It is not like that any more. |
| **DYLAN (8 → 6)** | 3.A.<br>DY1: Now, today, I am fine, but lately (…) February – March I think it has been quite (…) I am being challenged more now, than I was during the last two years.<br>3.Q.<br>DY29: Well, in a week, then I think I would feel better about the project we work on now. Right now, just today, it is about preparing for the end of term exam (…) one can take some theory, and then one can work with it, and you can do some tasks and see how much one can manage, and then … work a little differently with the tasks, and thereby learn more. |
| **GRACE** | 3A. |

| | FAVOURITE SUBJECT SCALE | |
|---|---|---|
| **(8 → 4)**<br>*(→ 8)* | G1: | I feel good. I think I am better than I were in first and second year. So that must be positive. |
| | G3: | How I fell now? I think it is on [8] right now. |
| | G4: | I just think it makes more and more sense. And yes, if you really hang on, because I thought in the beginning, that something was really hard, then maybe you did not make that much of an effort for learning it, because you thought: "I*t will come, eventually*". And that just made it really hard, in stead of hanging on and keep asking the teacher, if there is something you do not understand, keep asking your peers. Eventually, one is just so much in control of it, that one ends up helping the others. |
| **ADELE**<br>**(7 → 7)** | 3.M. | |
| | A14: | Well, but I am fine with mathematics, I am glad that I chose it, because, as I said, it is a good thing to have and to know. And then, because you can almost enjoy sitting and doing little tasks at home and have a good time with that. And this thing about, that one can describe, well, all kinds of things in everyday life by some calculations. And then sometimes it can be a little difficult, but that is life. I think a lot of people feel like that about mathematics. Yes. |
| **BRANDON**<br>**(10 → 10)** | 3.A. | |
| | B1: | Mathematics? It is probably the best subject in school, I would say. It is going fine, also in terms of marks, and I think the tasks, they are rather easy, for example compared to Physics and Chemistry. So it is going really well. |
| **GARY**<br>**(5 → 5)** | 3.A. | |
| | Q1: | How do you feel about Mathematics for the time being? |
| | GA1: | Not that good, actually. I found out that it was not really me anyway. So I am not interested in it any more. |
| | Q2: | No, but it does have quite an emphasis here in technical upper secondary school, does it not? In your study programme? |
| | GA2: | Yes, it...yes. |
| | Q3: | But you write that you want to become an engineer afterwards, or what? |
| | GA3: | I have actually also become uncertain about that. Right now I do not really know what I want to do. I have some personal problems and such, something which is also part of the reasons for it. |

### *TYPICAL BELIEFS ABOUT PLANS*

*The typical A-level student had an idea for tertiary education requiring A-level*

*Mathematics before choosing a study programme involving Mathematics at this level. However, two main obstacles may influence on these plans; firstly, if the average marks does not converge toward the requirements for that education, and secondly, if struggling with keeping up in Mathematics makes study programmes relying heavily on mathematics less appealing. Nevertheless, the ideal typical student will be likely keep up spirit and stay tolerant towards a tertiary study programme involving a good deal of Mathematics, even if the enthusiasm of some of his or her peers is more subdued now in third year.*

One of the most important questions for this study is whether the student could imagine positively choosing an education involving a good deal of Mathematics (Item #21a in Q1-G and in Q3-G) and whether one could imagine trying to avoid an education involving a good deal of mathematics (Item #22a in Q1-G and Q3-G respectively).

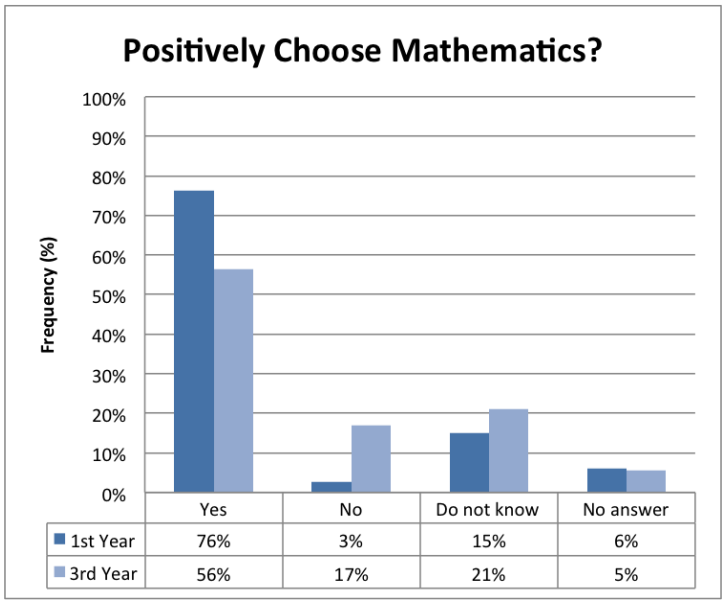



*Illustration 2: Item #21a: Could you imagine opting for an education involving a good deal of mathematics? (N=147)*

From the first to the first to the third year, there is a decline in the number of students answering [Yes] to the question of whether they would positively choose an education involving a good deal of mathematics from 76% in the first year to 56% in the third year.

Accordingly, there is a decline in students answering [No] to the question of whether they would try to avoid an education involving a good deal of mathematics from 80% in the first year to 60% in the third year.

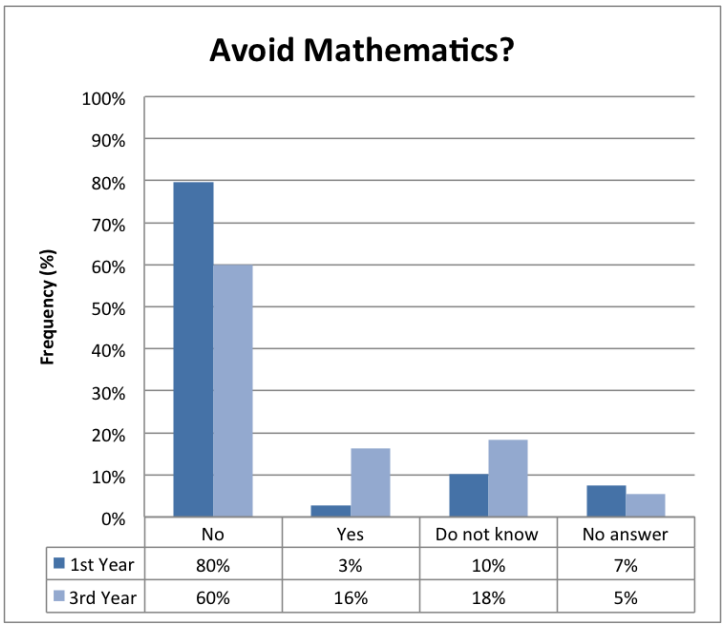



*Illustration 3: Item #22a: Could you imagine trying to avoid an education involving a good deal of mathematics?(N=147)*

In the interview transcripts, we get to learn how experiences in school influence on the students' plans for tertiary education and thereby their future life in society.

**Donna** knew already before choosing her programme for upper secondary

education which branches of tertiary education she would head for. Even when she was fighting the most to keep up in mathematics, she never allowed her struggle to prevent her from fulfilling her plans. In that sense she is able to show resistance towards hindrances.

3J.

D27: (…) I just think I have a propensity for choosing what I find exciting without really considering what I think I am good at. For example, here in upper secondary school, I am much better at languages that at anything else, but then again I chose science. That is because I find it more exciting. My abilities are not as remarkable in mathematics as they are in the history of ideas, for example, but even so I chose mathematics as the major subject for my Study Programme Project. So I choose what I find more exciting, and then I have to work a little harder for it. So yes, that is probably also one of the reasons that I dare choosing one involving mathematics, but I will make it if I have set my mind on it, you know.

To **Dylan**, his experiences with Mathematics in upper secondary school has resulted in change of plans due to two things: He does not have the average of marks for studying Medicine, and he does not have the confidence to tertiary education involving a good deal of mathematics:

3.J.

DY16: As it looks right now, my tertiary education only requires C-level Mathematics. And this relates mainly to the fact that my average of grades in general does not permit admission to a further education programmes which demand it on B-level or A-level. But I do hope that the mathematics I have learned will prove useful to me in my tertiary education, so it is not just a wasted effort. Yes.

3.M.

DY20: (...) I would not chose a tertiary education which required A-level mathematics, because it is not within Mathematics I would unfold myself

DY21: (...) I do not have the required average of grades, for studying medicine. So right now, I steer towards applying for the education programme as a laboratory technician, and to take a bachelor's degree.

Even though **Grace** had her ups and downs with Mathematics at school, it ended up not having any negative influence on her future plan. She ended up sticking to a branch of Chemistry Engineering, which was in line with some of her initial plans.

**Adele** realises that she will not get the average of grade get get admittance to

medical school. She also experiences felling less able in Mathematics than the average in her class. In the disciplines of the Humanities, however, she fells successful, and in third year she is sure that her plan for tertiary education will involve Humanistic subjects and no mathematics.

**Brandon** enjoys Mathematics in school and experiences being successful at it. I does not influence on his ideas for tertiary education. His first priority is Medical School, but even if he might not get the average marks for that plan, he does not consider anything related to Mathematics, since he cannot imagine any good jobs in it.

**Gary** wanted to become an Electrical Engineer since sixth grade in lower secondary school, but due to problems at home and problems in keeping up, he realises that these plans are not realistic. He does not mind Mathematics to play some part in his future education, since he considers lower secondary school mathematics teacher as a future plan, but he concludes that he is not able to make the effort that will make him succeed in A-level Mathematics.

| | **PLANS** |
|---|---|
| **Name** | **Reference to Data** |
| **DONNA** | Donna's plans for study after upper secondary school involves going to university to study Molecular- and Biomedicine (Donna: 3E). It depends on her final grades from examinations and evaluations whether it will work. (...) The plan seems to have changes a little since first year, when she intended to study Chemistry. |

| | PLANS |
|---|---|
| **Name** | **Reference to Data** |
| **DYLAN** | Dylan's average grades does not permit for him to apply for medical school. Instead, he thinks of taking a bachelor of professions as a laboratory technician. If so, he will not need the high-level mathematics in order to be admitted, but he hopes it will prove useful anyway (DYLAN: 3.J., 3.K.). |
| **GRACE** | Grace ended up deciding to study Food and Nutrition Engineering (3.J), which will involve some mathematical tools (3.K). It is a branch of chemistry engineering, which, among other quite different ideas, has been among her ideas since first year (GRACE: 3.K., 1.L). |
| **ADELE** | Adele is not likely to need A-level mathematics to be admitted to the study programmes she now finds appealing. (…) [S]he considers combining Languages with some kind of Business perspective in her future career (ADELE: 3.B.).<br>Even though Adele is rather keen on studying Languages or Philosophy, the idea of a career with these subjects does not appeal to her to the same extent as a career with point of departure in the Natural Sciences. Instead of deciding on a specific tertiary education immediately, she considers taking a year off before she decides (ADELE: 3.L.). |
| **BRANDON** | Brandon is quite fine with quitting mathematics after upper secondary school, and he does not seem to expect to keep up with it. He cannot really see any jobs in it. Otherwise, mathematics is what interests him the most. But if there had been some good jobs in mathematics, he would have been fine in carrying on with mathematics. Jobs concerning helping people appeals the most to Brandon, so jobs related to health and medicine appeals the most to him, So if he gets the grades for studying medicine, that is what he wants (BRANDON: 3.L.). |

| | PLANS |
|---|---|
| **Name** | **Reference to Data** |
| **GARY** | In the third year questionnaire, Gary now indicates that he is in doubt if he is strong enough in mathematics to go for an education containing a good deal of mathematics, but at this point he mentions that he might be able to change this and to gain the strength (GARY: Q3-G) In the period from when the questionnaire was answered and until the interview was given, Gary has now given up the plan of becoming an engineer (GARY: 3.A.). He mentions teacher education or social education as possible paths for him, and it would involve mathematics to some extend (GARY: 3.D.). |

## TYPICAL BELIEFS ABOUT MATHEMATICS AT SCHOOL

*All in all, it seems that the typical A-level mathematics student holds the beliefs that Mathematics as School relates very much to computing things, finding, remembering and applying formulae in order to solve tasks. Furthermore, a typical A-level student may believe that there is only one solution in Mathematics, and it is either right or wrong. In line with this, a typical A-level student believes that being good at Mathematics at school involves being good at finding and remembering formulae as well as entering numbers and performing calculations. What is learned in A-level Mathematics is not typically believed to be useful outside an educational setting by a Typical A-level student.*

To the typical student, the modelling aspect of mathematics stays in the background, but applications of ready-made formulae also in an extra-mathematical context is more common. To Adele, solving tasks is something she finds enjoyable, but even though she describes the power of mathematics for describing things, her examples of these features mainly relates to formulae already there, such as the equation of the circle.

The reasoning aspect of mathematics stays in the background. Even though Brandon emphasises proofs as something central and valuable, he still does not perceive Mathematics as a subject in which you should argue for your result. There is only one answer and this is actually also what he likes about the subject. Dylan explains that the theoretical aspects of mathematics, which he has not met until the third year of upper secondary school, is quite deviant from what he thought Mathematics was about. To him, the problem is that it has no role in terms of applying mathematics.

In stead, Dylan shows a considerable emphasis on finding and using formulae; both in terms of the activities of a mathematician, the features of his textbook and in

relation to his description of good teaching. Also to Grace, the features for providing formulae and examples, is what she cherishes from her textbook.

| Name | 3rd Year Interview Quotes |
|---|---|
| **DONNA** | "I was not fast enough to understand what I was supposed to do, so I managed to solve the tasks, partly at least, by just computing things. But to acquire the fundamental understanding of why, well there was not much of that" (DONNA: 3.H.D18:).<br>*"You need to work in an educational setting to be able to employ it [A-level Mathematics] in your job; for it to be useful. Because, it is too special, for being employed in everyday life...." (DONNA: 3.G.D16)* |
| **DYLAN** | Mathematicians may make formulae longer or shorter or make them contain more factors (DYLAN: 3.D.).<br>The best feature of Dylan's textbook seems to be its usefulness for providing ready made formulae in which you can enter values and compute (DYLAN: 3.H.).<br>"Yes, it relates to, that, yes, now we got these formulae, and then we just apply them in the task and so on, where here, in the third year, then it is something related to getting a lot of theory which fundamentally (sic.!) - as such - does not relate to what you should be able to apply. And, well, we have been assigned some tasks that I think have been, well, somewhat complex, and rather far from what I thought the subject was about" (DYLAN: 3.N. DY23) |
| **GRACE** | *Grace finds the textbook quite easy to understand, since it provides her with formulae and examples (Grace: 3.C. G8). In her programme, learning by heart is not really necessary, but the fundamental formulae might as well be learned by heart (GRACE: 3.D. G10).* |

| Name | 3rd Year Interview Quotes |
|---|---|
| **ADELE** | *For Adele, working with mathematical tasks seems rather enjoyable, and she repeatedly emphasises the power of describing things which Mathematics provides (ADELE: 3.M.).* |
| **BRANDON** | To Brandon, Mathematics is about solving tasks giving one specific result at the end. He appreciates that, because then you do not have to argue for it and such. There is only one answer. And that goes for physics and chemistry as well. In other subjects, you have to argue for your answer, which does not appeal to Brandon (BRANDON: 3.G.). |
| **GARY** | - |

## TYPICAL BELIEFS ABOUT MATHEMATICS AS A DISCIPLINE

*A typical A-level student beliefs that mathematicians in university teach. And maybe they do a project on a specific topic, or try to reformulate some formulae in an easier or faster way, shorter or longer form. The nature of Mathematics as a scientific discipline is hidden.*

First of all, a typical A-level student beliefs that it is not necessary to be a genius to study Mathematics in university. Nevertheless, it is widely recognised that it may take a considerable effort if this is what you want.

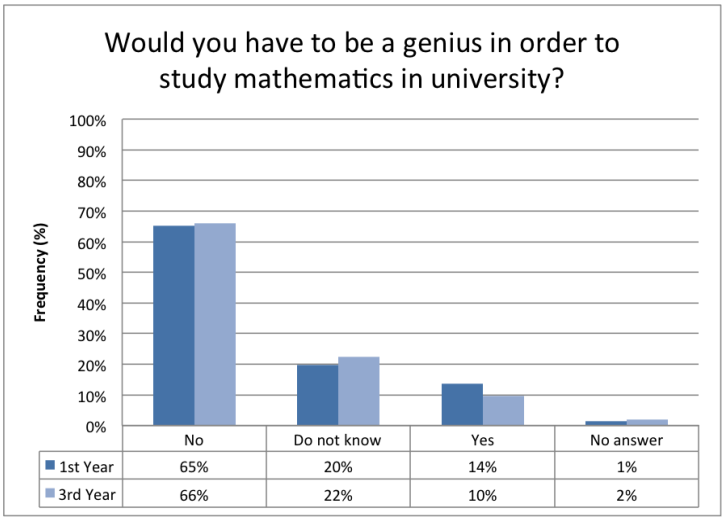



*Illustration 4: Item #11: N=147*

Besides that, most students do not have much of an idea of what a mathematician in a university is doing which in some sense might reflect what kinds of activities dealing with mathematics as a scientific discipline would involve. Even students with family members studying it, refer mainly to the teaching and learning aspects of mathematics, which is what they know from their own experiences in school.

Donna compares university Mathematics with "*a nice hobby*", whereas Dylan

suggests some kind of revising of formulae. Grace, on the other hand, does not seem to stick to a ready-made image of Mathematics; she indicates a view involving development of new mathematics: "*come up with more*", "*elaborate on the Mathematics*" and suggests that "*there must be someone who keeps inventing*". But also some aspects of revision is detectable in her quote; "*find, maybe, easier solutions, methods*" and "*finding easier ways to do things*". Adele knows something from her brother, who started studying Mathematics in University just that study year. Her impression involves that he reads, attends lectures and tutorials and solves tasks. All in all her impression of Mathematics from her brother is still dominated by the aspect of a teaching subject to a greater extend than that of a research subject. Brandon admits that he does not really know what mathematicians are doing, but suggests relating mathematics to society. Still, this would not be contradictory to an idea of mathematics as something that is already there, done, leaving nothing new to do, except applying it. To Gary, the applications of Mathematics seems to be his main impression of the Discipline of Mathematics, which, in his idea, stopped developing fifty years ago. So to Gary, the aspect of mathematics as a science in itself, developing new mathematics, is invisible.

| DISCIPLINE | Quote from the 3rd Year Interviews |
|---|---|
| **DONNA** | 3N.<br>D48: [Mathematicians in university] Yes, but...eh..it is because I actually do not really know what they are doing. I could imagine that one would try to explain how things are related. It may not be the only thing they do, but eh... That is at least what I could imagine. It would be a nice hobby, one might say. |
| **DYLAN** | 3.D.<br>DY9: ...for example the many long formulae we have today, they can probably be reduced, likewise they can be extended and include more factors and so on. |

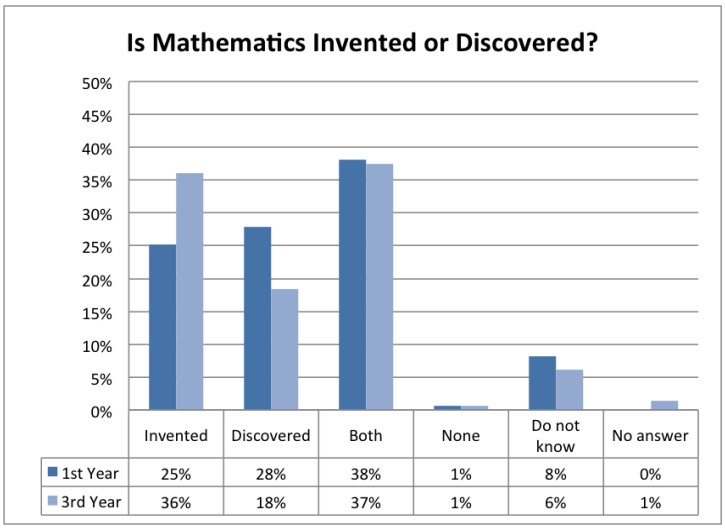
| DISCIPLINE | Quote from the 3rd Year Interviews |
|---|---|
| **GRACE** | 3.H.<br>G18: [Mathematicians in university...] to conduct research in the subject, well, to come up with more, to elaborate on the Mathematics, to find, maybe, easier solutions, methods, if it is a subject, as I think it is, Mathematics, that we human beings have come up with, then there must be someone who keeps inventing, and finding easier ways to do things. |
| **ADELE** | 3.J.<br>A10: Well, a Mathematician in university, yes, what is he doing? [My brother] he does assignments. Well, he reads really, really, really, really a lot, I know. And then he solves tasks. (…) that is the conception, I have. A lot of reading and a lot of lectures and tutorials in small groups in university.<br>3.K.<br>A11: When they are not lecturing, what are they doing then? It could be their own little project, they practice, or their...something they write a nice thesis about or...maybe they are sent out to upper secondary schools as external examinors or something, what do I know? |
| **BRANDON** | 3.J.<br>Q23: What about the mathematicians, what are they doing with this mathematics?<br>B23: I do not know, really. I do not know, well, it cannot just be tough tasks and such...they may see it compared to society, what exactly it is, I do not know. |

| DISCIPLINE | Quote from the 3rd Year Interviews |
|---|---|
| **GARY** | 3.N.<br>Q30: Are we done with the process? Is mathematics complete, and then it is just to teach it, or are there ways to create new mathematics? Or discover?<br>GA30: Well, if one just takes a look at everything that is happening now, like with particle accelerators and such things, mathematics is also applied. But there has not been discovered any new mathematics since the 1960's or something like that.<br>Q31: New mathematics? Or new Physics?<br>GA31: Now I think of equations and such things, that have been set up. But there can always emerge something new. |

## TYPICAL BELIEFS ON THE NATURE OF *MATHEMATICS*

*A typical A-level student beliefs that Mathematics is both invented and discovered. However, this is not a central belief, which means that it may easily change.*

The aspect of Mathematics in terms of its nature and ontology, is addressed by means of Questionnaire item #9 (addressed in Q1-C and Q3-C respectively):




| | Invented | Discovered | Both | None | Do not know | No answer |
|---|---|---|---|---|---|---|
| 1st Year | 25% | 28% | 38% | 1% | 8% | 0% |
| 3rd Year | 36% | 18% | 37% | 1% | 6% | 1% |

*Illustration 5: How does mathematics develop?- Is it invented by human beings? - Or does it exist already, and then is discovered by human beings? N=147*

The overall impression from the answers from the whole population (N=147; those answers that are comparable in a longitudinal setting) is not very clear. Most students (38% the first year and 37% the third year) answer that Mathematics is both discovered and invented. Besides this most common answer, there is an increase in students considering Mathematics to be invented from the first to the third year (25% the first year and 36% the third year) and accordingly, the is a decline in students considering mathematics to be discovered (from 28% the first year to 18% the third year).

The most common development for the questionnaire response is represented by Grace and Brandon, who both answer [Both] in both the first and the third year questionnaire. The interview answers, however, do not contribute to any convergence in the answers.

| | **Q1-C** | **1st year interview** | **Q3-C** | **3rd year interview** |
|---|---|---|---|---|
| **Grace** | [Both] | Both | [Both] | Created |
| **Brandon** | [Both] | Discovered (NOT invented) | [Both] | Discovered |

Also in the other interviewees, there is divergence to find; Donna arrives at Mathematics to be discovered by human being both in her third year questionnaire and in the following interview, which is new compared to the first year, when she found that Mathematics would be invented. Dylan arrives at Mathematics as invented in both the third year questionnaire and follow up interview, which is new compared to his first year questionnaire, in which he suggested Mathematics to be discovered. After refusing both options in her first year questionnaire and suggesting a mix the follow up interview, Adele tends towards perceiving mathematics as invented by human beings in her third year questionnaire, which is not contradicted (but neither confirmed substantially) in the follow up interview.

| **The Nature of Mathematics** | **Questionnaire Answers and Quotes from the 3rd Year Interviews** |
|---|---|
| **DONNA** | Q1-C: [Mathematics is invented]<br>Q3-C [Mathematics is discovered]<br>3N.<br>D50: (…) Mathematics has *always been there*. It is not like, we are just sitting and then, all of a sudden, then Mathematics is here. Mathematics has always been used in some way or another. We may not have known it, but Mathematics is also the relations between things, and relations between things have *always been there*. So... |

| The Nature of Mathematics | Questionnaire Answers and Quotes from the 3rd Year Interviews |
|---|---|
| **DYLAN** | Q1-C: [Mathematics is discovered]<br>Q3-C: [Mathematics is invented]<br>3.C.<br>DY8: [Mathematics:] It is created by human beings. Like the wheel. And if one person had not done it, I think someone else would. |
| **GRACE** | Q1-C: [Both]<br>1.M.<br>G21: I think *both*, actually.<br>G22: I think the human beings find out different ways to calculate things. So, somehow it is invented by us human beings, I suppose. But it is also, it is also. But they also *discovered how to do these things*. But, yes, we have found out how to calculate different things. So I think it must be inv... no, I think it must be a mix. It is a mixed answer.<br>Q3-C: [Both]<br>3.H.<br>G17: [Mathematics]Yes, it is something we *created*. It is *something we have arrived at*, and kind of *determined that it is like this and like that*, and if we do like this, it must give that. I think some of it is nature, *it is something that has always been logical*, but it is something that we have *created*, human beings have *created* in the last instance. |
| **ADELE** | Q1-C: [None of these options]<br>1.T.<br>A27: I would say it is a *mix*, because it is something you work out how to, yes, you find some formulae, for example; *circles have always been there*, you have been able to draw a circle in the sand. But, and then suddenly *you find out, that if we use this formula*, then we can find its circumference or we can find its centre and so on, right? So, in a way, it has, then it is just some numbers we enter, but in some way they have *always been there*.<br>Q3-C: [Mathematics is invented]<br>3.C.<br>A3: What is mathematics? Mathematics is a way of describing the world. It is a place where you can, well, define a movement, a shape (…) all kinds of things. (...) |

| The Nature of Mathematics | Questionnaire Answers and Quotes from the 3rd Year Interviews |
|---|---|
| **BRANDON** | Q1-C: [Both]<br>1.S.<br>B33: They did not invent it, they did not invent it. Well, *it has been there for a long, long time*, for example. There has always been something about trade. So there it is used., for example. And then, over time, some more difficult things, which I cannot think of right now, like for example cosine and sine, it may not always have been there, because...well, something which is hard, that you deal with in third year, it may not have been there one thousand years ago, for example. So one has *discovered* new things, some more things over time, but I just think it always has been there.<br>Q3-C: [Both]<br>3.J.<br>B24: They probably *discover* it, because it is something which has *always been there*. It is just to *discover* it, and then... you cannot invent it really, well you can invent some formulae and such, but otherwise, it is there, and then it just is there...<br>Q25: Are there any more formulae to invent at all? Have they not been found, all of them?<br>B25: I do not know, there may be more. It may be that they have already been found. I do not know. Well, *there is probably not a limit, is there*? So it is just...yes, there are more, I would say. |
| **GARY** | Q1-C: [Mathematics is invented]<br>1.P.<br>GA26: I would say that it *exists* already. It is *human beings that makes it real*, kind of, because there are always... (unintelligible) Now, I can see two nuts over there, then you can say "1 + 1". And then it is *human kind that makes it happen*. So it is everywhere. But *we add some numbers to it and formulae* and so on.<br>Q3-C: [Both]<br>3.N.<br>GA29: I think *Mathematics exists*. Well, as you just create it, you put up some systems, that makes it work for us, so we can kind of decode some of the things that happen in the world, and such things. If that makes sense. |

## TYPICAL BELIEFS ABOUT MATHEMATICS IN SOCIETY

*A typical A-level student beliefs that mathematics is everywhere and that everybody should learn it, at least the fundamental things, because you will need it for managing your life as a citizen in society.*

*The A-level student also has the conviction that Mathematics is useful for applications and for education.*

There is considerable agreement amongst A-level upper secondary school students, that Mathematics is a subject that everybody should learn. In the Questionnaires 94% of the student in the first year and 95% of the students in the third year, agrees on that.

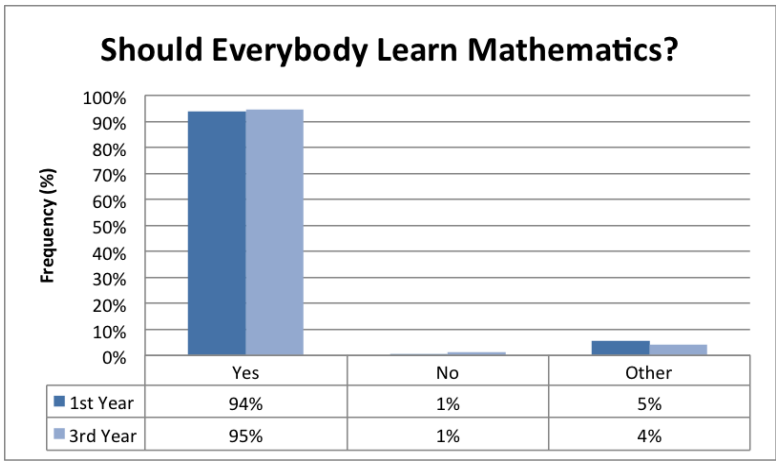



*Illustration 6: (Q3-B-#5b) Is mathematics something you think everybody should learn?*

The reasons for this type of answer, as they emerge in the interviewees, concerns the general education as a citizen in society.

| "Why do you think it has been decided that everybody in Denmark should learn mathematics?" | |
|---|---|
| **Name** | **#5a in Q3-B in the informants' 3rd year questionnaires** |
| DONNA | *Because it teaches us a lot about economy and other stuff you should be responsible for* |
| DYLAN | *It is a matter of course in order to manage oneself* |
| GRACE | *It is an important subject to master* |
| ADELE | *It is good to know, a good basis* |
| BRANDON | *A part of everyday life – simple mathematics in terms of trade and the like* |
| GARY | *Because mathematics is applied everywhere and at a higher level it can be transferred to many things* |

Nevertheless, not everybody should necessarily learn A-level Mathematics, according to the interviewees. Adele refers that not all her friends finds it equally interesting to be able to calculate what she learn to calculate:

1.I. A14: Well, sometimes I talk to some of my friends who chose e.g. business college or something. And then I find it really exciting to tell them: "*I found out how one can, e.g. this with the equation of the circle, or I now know how to find out the energy consumption of an electric kettle*". But they do not find that interesting. So I would say, I do not think it is something everybody should learn, only those interested in it. But I do think that one should know some mathematics for household use. Well, be able to calculate simple things. And if you have children, for example, then one should also be able to help them. But there are many who struggle with it.

And Dylan suggests that even though you choose an education which is not related immediately to mathematics, you may need in in situations in your life, or if you need to change direction in your career later on:

1.B. DY5: (…) maybe you choose a language oriented education; then everybody should have the chance to bring up mathematics, and they should also, one can say that they may not need it in their everyday life, but if they are in a situation, in which they needed some equations or formulae and so on, then they would need mathematics, if they did not know it already.

As suggested by Adele and Dylan, it seems that even though Mathematics is important to learn for everybody, A-level Mathematics may well be reserved for those interested in it. Whether the reason for it is the Discipline itself or its value for admission to or applications in tertiary education.

# SUMMARY OF IDEAL TYPICAL BELIEFS

### *MATHEMATICS AND ME*

- *Mathematics is on the top-3 of favourite subjects at school, even though this measure is highly sensitive to success and failure in understanding the topics on the agenda in school at the actual moment.*
- *An idea of tertiary education was the basis for choosing a study programme involving A-level mathematics*

### *MATHEMATICS AT SCHOOL*

- *Mathematics is about remembering and applying formulae in order to solve tasks*
- *There is only one solution in mathematics*
- *The A-level mathematics curriculum is not useful outside a school setting*

### *MATHEMATICS AS A DISCIPLINE*

- *Mathematics is a teaching subject rather than a developing research discipline*
- *Mathematics is both invented and discovered*

### *MATHEMATICS IN SOCIETY*

- *Mathematics is everywhere*
- *Everybody should learn fundamental mathematics*

# *INTERMEZZO*

## THEMES WITH VARIATIONS

PLANS are one of the main focus points in this study, and especially experiences - and interpretations of these - that lead to change in the student's plans. The interaction between elements from the four aspects of students' mathematics-related beliefs has shown some repertoires of interrelationships having an impact on plans. I will now deal with a selection of these interrelationships.

### *CONSEQUENCES OF AN INSTRUMENTAL RATIONALE FOR LEARNING*

Donna and Dylan have in common that they both chose a study programme involving A-level mathematics due to their initial plans for tertiary education. Both of them had professions in mind that did not directly seem to involve mathematics as a substantial part, but A-level mathematics was a necessary prerequisite for being admitted together with a general average of marks closer to A (12) than B (10).They both face a teaching subject much harder and much more demanding than they had imagined. In this sense, the interrelationship between mathematics and me and mathematics at school is similar:

| **Mathematics & Me** | | **Mathematics at school** |
|---|---|---|
| *PLANS* | → | *A-LEVEL STUDY PROGRAMME* |

*Table 1: DONNA & DYLAN: Similar goals. Mathematics & Me influence on Mathematics at School.*

During the years in upper secondary school, Donna and Dylan adopt different strategies; Donna asks, asks and asks again until she understands and she works diligently by taking notes and attending the Homework Café. As a side effect she ends up understanding mathematics and her plan is still realistic to her.

| **Mathematics at school** | | **Mathematics & Me** |
|---|---|---|
| *Mathematics has become harder* | *DONNA →* | *If this is what A-level mathematics is about, then I am really challenged.*<br>*(And so are my plans)*<br>*I see myself as a smart person*<br>*I have to overcome this* |
| | *DYLAN →* | This is not what I thought mathematics would be<br>I cannot relate to it<br>I do not engage that much in it<br>I have to change my plans |

*Table 2: DONNA & DYLAN: Different strategies*

Dylan describes that he does not always feel motivated for working with the projects and that his level of engagement influences his understanding and his appreciation of mathematics. Further, the theoretical aspect of mathematics that he meets in the third year has nothing to do with what he though mathematics was all about. He suggests that his experiences in upper secondary school mathematics has made him realise that he should not go for a tertiary study programme requiring A-level mathematics.

One difference between Donna and Dylan's plans may be that Donna has actually done internships twice and fallen for the professions she strives to get. In this way her rationale for the actual plans is not purely instrumental. She wants that career also because she would enjoy it. Dylan's career plans tend to have a more

instrumental focus; he wants to study medicine as a means for getting a high position in society, not, as far as it is apparent in the data, because the profession in itself is something he strives for.

These rationales may seem to be far from related to mathematics, but in some sense, they end up influencing the students' willingness or sense of meaning related to fighting for mastering mathematics.

### *CONSEQUENCES OF A SOCIAL RATIONALE FOR LEARNING*

Grace and Brandon did have some ideas of plans for tertiary education before choosing A-level mathematics. But in some sense these plans do not seem to be their only reason for choosing their respective study programmes. Beside working just fine for their initial plans, mathematics is also something they just like, and then it makes sense to pursue it further.

The subject as they encounter it in school, however, seems to give rise to quite different experiences. To Brandon, mathematics does not seem to change that much and it keeps being easy to him. Nevertheless, at the end of the 3rd year of upper secondary school his ideas of what mathematics could be and the role of mathematics in the surrounding world, does not seem to have the same coverage as we see in Grace.

Grace, on the other hand, meets a mathematics teaching quite different from what she used to know and appreciate. It appears as if she gains a lot of new terrain in mathematics; she seems able to reason and her ideas of the role of mathematics in different contexts is quite good. But her self esteem is challenged a lot and it influences her appreciation of the teaching subject mathematics. Nevertheless, it does not scare her away from having something to do with mathematics in her tertiary education; she goes for a STEM study programme relying on some

mathematics, but not with mathematics as its main component.

Brandon seems to have little ideas of the role of mathematics in professions. The teaching he has met has not enabled him to acquire new terrain in this field. He really likes mathematics, but he cannot see the point in pursuing it any further after upper secondary school.

### *CONSEQUENCES OF PREFERENCE FOR SUCCESS*

Adele and Donna have in common that they both have a brother in the same study programme as themselves and that their brothers end up studying mathematics at university, when Adele and Donna are in their third year. Both of them appreciated the help from their brothers when they were in the 1st year of upper secondary school.

Both Adele and Donna strongly appreciate feeling competent at what they are doing. For Donna this results in a strategy ensuring her the success she wants in mathematics, but for Adele it means that she ends up deselecting science related study programmes and choosing humanities instead, because success is easier for her to acquire in that domain.

### *CODA OF INTERMEZZO*

*More examples could be given to indicate the same point; the exposition of these themes with variations all indicate that phenomenon which on the surface may appear alike show deep differences behind this shell.*

# CHAPTER V: FINDINGS

At this point the reader has now had a chance to follow the study from the formulation of research questions in Chapter I, the decisions of how to approach the study methodologically in Chapter II, the choice of conceptual framework in chapter III and an account of the collection of and analysis of empirical material in Chapter IV. Now it is in its place to formulate what findings these attempts have given rise to.

## FINDINGS CONCERNING THE DEVELOPMENT OF BELIEFS:

The findings below are responses to the first research question:

**Research Question #1:** *How do the mathematics-related beliefs of students' develop during the three years in upper secondary school?*

The answers to this research question are organised according to the four aspects of beliefs; *Mathematics at School*, *Mathematics as a Discipline*, *Mathematics in Society* and *Mathematics and Me*:

### *MATHEMATICS AT SCHOOL*

Remembering is seen as an important aspect of mathematics learning; in the quantitative part, this is apparent concerning item #14 from the questionnaire concerning what issues involve more challenges to the students, but it also comes up in the interviews in the case analysis, and when students comment on the skills relevant for studying mathematics (item # 11). This facet of mathematics teaching

seems to indicate the lack of possibilities for understanding, since learning by heart is the first possible substitute when understanding does not occur. The emphasis on the importance of remembering is not out of line with the frequent references to the task discourse that comes up in the case analyses.

Many students do not seem to notice too big a difference between lower secondary school mathematics and upper secondary school mathematics, at least this is not detectable in the 1st year questionnaires. The comments mainly suggest that mathematics has become harder, but rarely in what ways. In the third year, however, mathematics becomes even harder according to the students, and also now the theoretical sides of the discipline are revealed to the students to a greater extent. This change may come as a surprise to many students and may imply that mathematics is not be recognisable as the subject they thought it would be.

Even though the students see mathematics as useful in general, A-level mathematics is not seen as useful outside a school setting, neither to the STX student nor to the HTX student.

The activities of mathematics at school involve finding and applying formulae for solving tasks – tasks that may relate to an extra-mathematical context in some way, but not in a way that makes A-level mathematics seem useful outside a school setting. The emphasis on formulae reflect the task discourse as suggested by Mellin-Olsen (1990).

### *MATHEMATICS AS A DISCIPLINE*

The discipline of mathematics lives a somewhat hidden life in students' beliefs; only the role of mathematics in other disciplines seems visible. Mathematics as a project of adding to the body of theory already existing does not appear to be visible to the students. We see that the vast majority of A-level mathematics students

recognise the role of mathematics in other subjects at school. Also students are keen on mentioning facets of mathematics as an educational subject when addressing possible activities of mathematicians at university. Very few students expect mathematicians to deal with theory building and processes related to this. Nevertheless, many students recognise the combination of discovering and inventing in relation to mathematics. Besides, A-level students are influenced in a way that leads to a decline in students perceiving mathematics as being discovered by human beings and an increase in the proportion of students seeing mathematics as invented. The facets of mathematics as a dynamic discipline under continuous development might be protected from other clusters of students' beliefs; the prevalent task discourse and the emphasis on learning by heart prevalent amongst the students does not correspond very well to these facets.

What is positive, from the point of view of recruiting students for mathematics-related studies, is that mathematics is not generally perceived as being restricted to a certain kind of people, and this feature is stable over the years of upper secondary school. Anyone who is sufficiently interested would be able to study mathematics at university – but he or she may need to work hard and be good at remembering formulae. The point of studying mathematics, however, may not be appealing to the students; in general, the purpose of doing so seems to relate to the educational facets of the discipline only; learning more mathematics yourself and passing it on to others, rather than theory building. The role as an aid to other disciplines, however, seems to be highly acknowledged by the students.

### *MATHEMATICS IN SOCIETY*

A-level students agree almost unequivocally that mathematics is important, and a subject everybody should learn in order to manage oneself as a citizen in society. This view is stable over the three years in upper secondary school, and no

major difference is seen on this perspective between the STX and the HTX groups of students.

How, exactly, mathematics is applied outside a school setting, is rarely recognised apart from everyday arithmetic and simple procedures relating to trade. Only in relation to the professions of architect and civil engineer students are able to realise the need for mathematics, but many other professions drawing heavily on mathematics are invisible to them; the possibilities for making a career from a mathematics-related tertiary education is widely ignored or discarded, if it is not precisely engineering.

There seems to be an under-explored potential for making visible the extent to which we rely on mathematics in professions and in societal institutions, but even more for making visible the possibilities of making a living out of a mathematics-laden educational background.

### *MATHEMATICS AND ME*

The transition from lower to upper secondary school involves a change in the role as a mathematics learner at school for A-level mathematics students. It is challenging to experience that you are not being the best in class any more; now you have to work much harder to keep up.

Students do not find that mathematics is a waste of time. Mathematics, in general, is seen as in important subject. However, there is a decline in the willingness to have to do with mathematics, but it is not out of a lack of respect for the subject or its importance.

Learning by heart, be it formulae, or procedures, may work in the beginning, but as the level of complexity increases in the third year of upper secondary school, a

year which is busy in many ways, the student may not have the chance of following any more, and might end up becoming detached.

We see a link between students experiencing to understand the mathematics they deal with in school and their appreciation of the subject; Mathematics is fun, when you understand it and lagging behind is not fun.

If the student has a plan beforehand, and this plan is very strong in her or his mind, he or she may be able to endure not understanding and keep fighting anyway, but otherwise, enjoyment of mathematics and understanding it are closely connected.

## FINDINGS CONCERNING INFLUENCE ON PLANS:

**Research Question #2:** *How are the students' ideas for choice of mathematics-related future study programmes influenced by their mathematics-related beliefs?*

These findings concern types of impact conceptualised by means of beliefs that relate to changes in plans for tertiary education, especially if the change involves de-selecting mathematics or deselecting the STEM disciplines.

Experiences in upper secondary school lead to a decline in students positively choosing mathematics-related tertiary education. This trend is even more prevalent in the HXT students compared to the STX students.

One major issue that has an impact on the students' plans is experienced lack of understanding of the mathematics dealt with in school. Lack of understanding influences negatively students' willingness to accept studies involving a good deal of mathematics. However, if A-level mathematics merely works as a means for getting admitted to a certain tertiary study programme, the students seem to be less sensitive to the lack of understanding. In line with this, it seems that undecided

students may be more prone to deselect STEM-studies than the general population; these students are more sensitive to their experiences in upper secondary school.

Also meeting an aspect of mathematics that is in contradiction to one's own idea of what it is all about may lead to deselecting mathematics; this can be interpreted as an influence from beliefs about *Mathematics as a Discipline* to *Mathematics and Me* (plans) via *Mathematics at School*.

## SERENDIPITY FINDINGS

Some findings that this study provided, did not come as an answer to one of the research questions:

The fact that more students loose enthusiasm than the opposite has come as unexpected – and this seems to relate to lack of understanding, which is less surprising. This finding follows from the development in the rating of mathematics on the favourite subject scale. This could relate to a steep development in the demands on the students, especially at the end, as many students indicate. I may relate to a very steep development from lower to upper secondary school, but it may also relate to a very steep development from the 1st year to the 3rd year demands on the students. Also the fact that understanding and rating on the "Favourite Subject Scale" were closely connected, was not fully expected beforehand, but it is seen clearly in the analysis of Donna, Dylan, Grace, Adele, Brandon and Gary. All in all, this study has shown that for many students A-level mathematics ends up being "too much for me".

Entailed in the decrease in enthusiasm towards mathematics, there is also traced a decrease in tolerance towards mathematics in tertiary education for the A-level students. For this finding, there are clear and surprising differences between the group of STX and HTX students; the HTX students being less inclined to deal

with mathematics in tertiary education compared to the STX students.

This study also showed that the A-level students rely mainly on each other for support for mathematical activities – and this trend becomes even more predominant in the 3rd year compared to the 1st year.

In relation to the belief aspect Mathematics in Society, the clear stable trend in students beliefs concerning mathematics as a subject everybody should learn, was not expected beforehand.

## FINDINGS THAT THIS STUDY DID NOT PROVIDE

Before turning to the discussion of the findings, I will point out some findings that this study did not provide, even though they might, in retrospect be seen as desirable:

In the later years the role of CAS tools has become more and more dominant in the mathematics teaching in upper secondary school. This study has not provided any new insight into this area. However, it did not seem to be predominant in the empirical material of this study either, but this might be due to the design of the study. It would have been possible to extend the study by questions concerning the role of CAS, but the extension would have been rather substantial.

The study did not relate the students' interpretations to the actual teaching in the classrooms in upper secondary school; neither in its planning, nor in its execution or evaluation. Neither did the study bring light to the actual development of conceptual understanding in A-level students and how this development specifically relate to the students' interpretations of experiences. However, that would have constituted an independent project.

The contribution from the group of students exiting the population before the 3rd year, could have shed light on types of development on the favourite subject scale before school drop out. Since school drop-out was not part of the research aim, but

rather understanding the processes leading qualified students to deselect mathematics-related tertiary education, this has been a conscious decision, but in another context dropping out of school would be of major importance to the schools and to society. This group might not have added to the proportion of students increasing their FSS rating, but since they are not included in the analysis, this issue remains unresolved.

# CHAPTER VI: DISCUSSION

*In this chapter I will discuss the findings representing my attempt to answer the research questions based on the research design, the conceptual framework, the empirical investigation and the analysis in this study. I have chosen to structure parts of the discussion by way of three main categories suggested by Schoenfeld (2007); trustworthiness, generality and importance. Trustworthiness and generality will be dealt with first, then, additionally, I will compare the means for answering the research question with other means, and also I will discuss possible contributions to the study by means of alternative research questions. Then, finally I will address the issue of importance which will lead to a statement of five major concerns for Danish Upper Secondary School, based on this study*

## TRUSTWORTHINESS OF FINDINGS

The issue of trustworthiness concerns a set of aspects; *descriptive and explanatory power*, *prediction and falsification*, *rigour and specificity*, *replicability* and finally *triangulation*. Beside the aspect of replicability, these above mentioned aspects will be discussed separately in the following sections. The aspect of replicability will not be dealt with separately as part of the trustworthiness discussion, but rather implicitly as part of the second *main issue* of *generality*.

### *DESCRIPTIVE AND EXPLANATORY POWER*

"[*T*]*he descriptive power denotes the capacity of theories or models to represent what counts*"(Schoenfeld, 2007, p. 83).

I wish to address this issue by discussing the appropriateness of the conceptual framework for addressing the research aim and thereby providing answers relevant to that.

*Research aim* → *Conceptual framework* → *findings relevant to the research aim*

The aspect of explanatory power concerns "*the degree to which a characterisation of some phenomenon explains how and why the phenomenon functions the way it does*"(ibid., p. 83). So, the explanatory power will be dealt with by discussing to which degree my study attempts to say how and why changes in beliefs correspond to the change of plans in the students.

In this study I do detect development in the beliefs of A-level mathematics students in upper secondary school over the three years of the study programme. The beliefs are conceptualised by means of the four aspects of beliefs defined in Chapter III. For each aspect of beliefs I have detected the development in both very broad belief objects, such as the nature of mathematics, as well as in less broad belief objects such as the role of formulae in mathematics education. Very narrow types of belief objects, such as single mathematical concepts, have not been in focus in this study.

By way of this conceptual framework this study has also provided answers to the question of detecting experiences leading to the change of beliefs. However, the characteristics of the teaching as planned and implemented, which may lead to these types of development in beliefs are not fully covered in this study.

Another question is which changes in the mathematics-related beliefs of the students relate to the fact that they develop from being big children to being young

adults. Becoming someone, is also on the agenda these years. When you study the development of mathematics-related beliefs, it is difficult to disregard other influences. In principle, this is impossible, and so I should be careful not to put everything on the mathematics-related beliefs-account, when assessing the answers to the research questions; as a big child, dealing with something in which you are not an expert may not be of major concern, but when you start thinking of your plans for managing yourself in society in your future life, you may evaluate your own success or failure in a sharper light.

Nevertheless, I do find mechanisms that relate interpretations of experiences leading to selecting or deselecting mathematics related tertiary educational options. Again, the circumstances around the activities of teaching and learning mathematics as they were carried out in concrete teaching cannot be characterised by way of this study.

### *PREDICTION AND FALSIFICATION*

The aim of the study has not as such been to predict certain phenomena, but rather to explore and provide descriptions with the aim of gaining better understanding of the *problématique*. However, the study points to certain indications of explanations for the relatedness of students' sensitivity towards mathematics-related study programmes in their plans for tertiary education and their experiences of understanding mathematics in their A-level study programme in upper secondary school. To be able to make predications, however, it would be necessary to design the study specifically for that aim, in terms of methods and sampling procedures.

### *RIGOUR AND SPECIFICITY*

In this study every attempt has been made in order to define precisely the concepts applied (Chapter III). This has been followed up by an account of the

appropriateness of the instruments for data generation – the questionnaires and the interview guides, with specific reference to the aspects of beliefs addressed by the items in the questionnaire. Also the criteria for selection of empirical material for further analysis have been explained clearly and the full range of empirical data for each of the six case informants has been displayed. Thus, it has been the aim of the reporting of this study to provide transparency of the process of analysis: Rich material from interviews and questionnaires allows the reader to follow the steps of analysis leading to the findings and thus to asses the process of analysis.

*TRIANGULATION*

In the process of interpreting the empirical material, I have also presented colleagues for interview transcript in order to check if other researchers using the definition of the four aspects of beliefs for the same empirical material would result in the same categorisation of the material. This effort provides feedback both for the results of my own categorisation of the material, but also for the quality of the definitions of the categories.

Moreover, since the same students were interviewed twice, I also had the chance to return to issues from the first year interview and invite the interviewee to deepen their interpretations of these.

Nevertheless, in this study, everything happens by means of informants: to interpret what people do, I have to presume that what the informants say and do is consistent – but it depends on their ability to express clearly what they mean. It is necessary to presume consistency between what they say and what they do, but it is a strong presumption.

## GENERALITY OF FINDINGS

For assessing the aspect of generality, one may, following Schoenfeld, wish to distinguish between the *warranted* generality of the study as opposed to either the *claimed*, the *implied* or the *potential* generality.

For studies relying on questionnaires, the response rate is of great importance to the possibility of obtaining warranted generality.

The carrying out of the quantitative part of the longitudinal study, worked considerably better with those school for which I had a close cooperation with the teacher by way of other activities such as the interviews. Thus, the process of learning what kind of material I was after and my attempts to clarify this in my instructions to the teachers, made the administration of the questionnaires easier and better and helped me to improve the response rate in the special project classes. I also improved in how, and by which channels, to address schools outside the special project and how to get better help from these schools for my study. Having known then what I know now, could have improved the response rates of the questionnaires considerably, I am convinced.

One of my main findings, the drop in enthusiasm towards mathematics as it is measured by way of the 'favourite subject scale' and other findings, is based on the questionnaire answers by the 147 students in the quantitative part of the longitudinal study. However, these 147 students represent only those students who did not drop out during the three years. This means that they all represents students that are recruitable for tertiary education, but those students dropping out of school during the three years may have given an even lower rating of mathematics on the FSS, and the other items, even though their development was not traceable, since they exited the population.

The possibility of generalising the findings obtained to the groups of STX and HTX students at large is very restricted due to the procedure for sampling and the response rate. Also some of the items, especially those relating to 'understanding versus learning by heart' in the 3rd year questionnaire, suffered from carelessness in students' responses. However, some of the differences are so clear that they constitute an indication which deserves to be studied further.

## ALTERNATIVES

*What would have been nice also to have had in the study/findings? Would other means of investigation have provided greater certainty or better coverage of the research aim?*

What I say in my answers to the research questions, is due to a set composed of of

- Research design
- Conceptual framework
- Methods for empirical investigation
- Process of analysis

Beside these means for answering the research questions, one might consider what kinds of answers alternative research questions could have provided. However, I will first comment on possible alternatives for the means for answering the actual research questions, and then proceed to discussing the possible consequences of applying other *research questions*.

### METHODS:

*The answers I have provided have been obtained by means of certain methods. What might other methods have given of answers and strength?*

Randomly selected schools for the questionnaire part of the longitudinal study would have entailed little less work and would have enabled a better foundation for generalising from the selected sample to the population at large. However, other criteria would be necessary as well; a predefined conceptualisation, which was not

possible due to the exploratory nature of the study. Also, for achieving these features, a high response rate would be necessary.

The study was carried out by paper based questionnaires. Many students commented on this, claiming that they would have commented more thoroughly on the open-ended questions, if the questionnaire had been web-based. Also online questionnaires would have facilitated an easier handling of the questionnaire answers and errors in the form of mis-typing answers could be avoided. On the other hand, I also received the feedback from at least one school, that they would not have accepted participating if the questionnaire had not been paper based, since they had experienced several problems with web-based questionnaires recently. Also web-based questionnaires require access to computers for all students. However, this may be more and more easy to obtain, at least in some schools.

As it may have been mentioned earlier, the actual teaching the students experienced, is not included in the study; neither the planning or the implementation of teaching is taken into consideration, only the impression the teaching left with the students, which is perfectly relevant for my research questions. The impression left with the students may be one-sided regarding what actually took place – but at the same time, it does form the point of departure for the students' decision-making.

Other means, as teaching experiments or ethnography, would in each their way have enabled the inclusion of actual experiences in mathematics teaching in a classroom context. These approaches could be candidates for investigating further the characteristics of teaching leading to different types of changes in beliefs.

**ALTERNATIVE RESEARCH DESIGN:**

This study has made use of inquiries into students beliefs by means of questionnaires and interviews in the 1st and in the 3rd year of upper secondary

school. However, inquiries could have been conducted more frequently and the time between the questionnaires and the interviews could have been shortened. On the other hand, given the resources for carrying out the research, this would have implied a smaller group of interviewees and thus a design less robust towards a decline in the population.

**ALTERNATIVE RESEARCH QUESTIONS:**

*This is an attempt to unfold the perspective of what I could have done and what it could have covered. What kinds of alternative research questions could have been relevant: What answers could they have provided?*

Having now recognised the disparity between the STX and the HTX group of students, the idea of linking technical versus general upper secondary mathematics teaching to the general trends in the development of upper secondary school students evolves. This would require a larger group of students in the quantitative part of the longitudinal study, preferably randomly selected and with a better control of the response rate. In addition to this, means of characterising the teaching in these two main types of upper secondary A-level mathematics education may enable means for comparing main strands of teaching approaches, say a problem oriented interdisciplinary project based approach with a considerable emphasis on problem solving and modelling, with an approach relating more to a "task discourse" millieu and an emphasis more closely related to the symbols and formalism competency (Niss & Jensen, 2011) and the passive part of the reasoning competency.

Also the question of how, precisely, strategies of learning by heart and imitative reasoning strategies interact with relational understanding and creative reasoning appears promising, and the question of whether it would be possible to trace influences from these student strategies on their enthusiasm for mathematics

emerges.

Finally, one could have approached the problem field by means of an approach oriented towards students' socio-economic and cultural background variables. There might be an overlap between sensitivity in the rating and other variables. However, background variables are not be easy to change, neither are beliefs – even though more so - but the ideas of addressing the foreground of students rather than their backgrounds (Skovsmose, 2011) adopted in this study seemed more promising from a teacher's perspective.

## IMPORTANCE OF FINDINGS

In the Introduction (p. 13) I held out prospects of contributions, not only to my own understanding of the problem field, but also to these two areas:

First, the research field of mathematics education, because I saw the potential of contributing to the understanding of the development of students' beliefs about mathematics in a longitudinal perspective. This study has provided empirical documentation of the development of four aspects of beliefs, as they were defined in this study. Hopefully, these findings will contribute to the field by way of this.

Secondly, I expected to be able to provide ground for reflection for upper secondary school teachers in (A-level) mathematics by way of describing mechanisms of interpretations in students in a millieu in which teachers also are part.

Beside these two areas, I am convinced that decision-makers involved in mathematics education at the upper secondary level as well as mathematics-related studies in tertiary education, will find the results of this study important.

*As a final word, I will call the attention to five major concerns for A-level mathematics:*

## *MAJOR CONCERNS FOR A-LEVEL MATHEMATICS*

Based on my study I call for attention towards these *problematiques* in Danish upper secondary school:

1. *Understanding and joy of mathematics are closely related*

2. *The complexity in the third year curriculum is surmountable, and managing it by means of learning by heart is neither viable, nor fair to the students. They should be given a chance to understand what they learn. The students need to have a chance to understand the mathematics they work with, if teaching to a greater extent should enable students to keep mathematics related studies as part of their future plans.*

3. *The role of mathematics in professions is hardly visible to the students; besides architecture and engineering, most students relate mathematics solely to everyday arithmetics.*

4. *Resources for support for mathematical activities are unevenly distributed; only half of the students can get help from their parents in the first year of A-level mathematics, and less than 30% in the third year.*

5. *Finally, the role of and impact from the widely implemented use of CAS and its influence on students' development of understanding needs to be examined*

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# *APPENDIX*

*INTERVIEWGUIDE OPFØLGENDE INTERVIEWS 1.G.*

1. **Hvordan har matematik ændret sig siden folkeskolen?**
    a. Indhold
    b. Arbejdsformer
    c. Lærer-elevforhold
    d. Yndlingsfag
2. **Hvem skal lære matematik?**
    a. Hvorfor på A-niveau
    b. Hvorfor stx/htx?
    c. Skal alle lære mat?
    d. Hvor meget?
    e. Er der noget, du lærer, som alle kan bruge? Ingen andre kan bruge?
3. **Hvor bedrives matematik?**
    a. Har mat noget med dine andre fag at gøre?
    b. Hvor mat, uden for skolen?
    c. Er mat opdaget/opfundet?
    d. Hvad laver matematiker på uni?
    e. Skal man være geni?
4. **Hvordan bliver man bedre til matematik?**
    a. Hvad kan man selv gøre
    b. Hvor har man brug for andre
    c. Anderledes nu i forhold til folkeskolen?
5. **Hvor kan man få støtte/hjælp?**
    a. Hvor opleves udfordringer?
    b. Hvor kan man få støtte?
6. **Eget engagement i matematik:**
    a. Hvor aktiv i timerne?
    b. Hvor aktiv i grupperne?
7. **Hvordan bliver man opfattet?**
    a. Nemt/svært ved mat?
    b. Aktiv eller passiv i forhold til at lære matematik?
8. **Planer efter gymnasiet:**
    a. Konkrete idéer?
    b. Vigtige kriterier ved uddannelse