EMERGENCE AND DOWNWARD CAUSATION

by Donald T. Campbell, Mark H. Bickhard, and Peder V. Christiansen
Abstract

This preprint contains two articles from the collection "Downward Causation" (in preparation to be published by Aarhus University Press). The collection contains views from many different academic disciplines (literature, media science, history, social science, psychology, biology, and physics). The two papers presented here are mostly related to physics. The first article (M.B.H. & D.T.C.) treats the subject "Emergence" from a philosophical and field-theoretical point of view, whereas the second (P.V.C.) is more specific about the emergence of surfaces considered as the first step in the semiosis of inorganic nature.

Acknowledgements

The editor of this preprint (PVC) wishes to thank the other members of the editorial board, Peter Bøgh Andersen, Niels Ole Finnemann, and Claus Emmeche for good initiatives and cooperation, also thanks to the author Mark H. Bickhard for allowing the inclusion of his paper (with excuses for possible editorial errors). Finally, thanks are due to Jesper Hoffmeyer for inspiration and useful suggestions.

Roskilde, February 1999, Peder Voetmann Christiansen
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Mark H. Bickhard with Donald T. Campbell

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Foreword by Mark H. Bickhard

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Deepest thanks are due to the Henry R. Luce Foundation for support to Mark Bickhard during the preparation of this paper.* This paper was to have been written jointly with Don Campbell. His tragic death on May 6, 1996, occurred before we had been able to do much planning for the paper. As a result, this is undoubtably a very different paper than if Don and I had written it together, and, undoubtably, not as good a paper. Nevertheless, I believe it maintains at least the spirit of what we had discussed. Clearly, all errors are mine alone.

Draft: 10 Jul 96; 29 Sep 96; 11 Oct 96; 2 Nov 96; 20 Dec 96
Emergence
Mark H. Bickhard with Donald T. Campbell

Abstract
Accounting for emergence has proved to be extraordinarily difficult, so much so that whether or not genuine emergence exists seems still in doubt. I argue that this difficulty is primarily due to an assumption of a false and inappropriate metaphysics in analyses of emergence. In particular, common assumptions of various kinds of substance metaphysics make the notion of causally efficacious emergence seriously problematic, if not impossible. There are, however, many problems with substance metaphysics — arguably fatal problems — and an alternative process metaphysics makes causally efficacious emergence much more natural.
1. reality or epiphenomenon

Consider a kitchen table. A table appears to be an entity in its own right — large, with a particular shape, solid, capable of supporting smaller objects, and so on. — But we also assume that it is made of molecules, and, in turn, atoms, and, in further turn, various subatomic particles. Perhaps the only physical reality is the swarm of quarks, gluons, and electrons that make up the table, and all of the other properties, of solidity, shape, and so on, are no more than manifestations of the interactions among those particles. Perhaps the properties of the table, and even the existence of a distinct object that we call a table, are all just epiphenomenal to the fundamental particle interactions. This is epiphenomenality in the sense of an appearance being false about underlying reality, such as the apparent motion of objects when watching a movie, when all that is really happening is a rapid succession of still pictures that happen to be sufficiently similar to each other to give an impression, a strictly false impression, of objects and people and caused motion. Perhaps being solid, for example, is mere appearance, merely epiphenomenal, from the level of the fundamental particles. Most of us would prefer that our experiences of tables not be false, not be merely epiphenomenal. It would be a strange world in which virtually all of our experiences were in fact false to reality. The issues become even more focused and interesting, however, when we consider not just tables, but living things, and things with minds — animals and other people — and, most especially, our own mind. The supposed lessons from science are just as strong about plants, animals, and minds, as about tables. It would be a strange person indeed who would feel satisfaction in the conviction that his or her own mind did not really exist, but was merely an epiphenomenal manifestation of fundamental particle interactions. We would like for tables and their properties to be real, as well as life and mind. But our best science suggests strongly that the world is integrated, that there are not different sorts of substances or fluids for every new kind of phenomena. We have learned that fire is not a substance phlogiston, heat is not a substance caloric, life is not due to vital fluid, and very few philosophers or scientists today are substance dualists about mind compared to matter. Instead, these phenomena are understood as the result — the natural result — of processes involving atoms and molecules that are familiar from other kinds of phenomena. Fire, heat, life, and so on, and, presumably, mind, are integrated with the rest of the natural world. Naturalism about the world is clearly the best bet. But, so long as naturalism seems to suggest that the only real reality is basic particles, the apparent dilemma remains. Perhaps phenomena such as life and mind are somehow emergent out of lower level particles and processes. Perhaps they only exist insofar as those lower level particles and processes exist and occur, but they nevertheless have a reality of their own that
comes into being, that emerges, when certain patterns or quantities or some other threshold criterion is satisfied. And, furthermore, perhaps, the reality they have makes a difference. It is of little satisfaction if mind proves to be real in the sense of involving properties that genuinely exist, if those mental properties nevertheless have no causal power in the world, if they merely float along the basic particle interactions for the ride, but make no difference themselves. We all know in our own experience that mind, whatever it is, exists, but it would also be nice if our impressions of being able to make decisions and do things in the world are not themselves just epiphenomenal (Heil & Mele, 1993).

2. Downward causation

So, for emergence to do what we would want it to do, we need not only emergent instances of properties, but the emergence of properties or entities or processes that have genuine causal powers. It has proven remarkably difficult to make good on these intuitions of emergence. The inexorable reality of quantum particles keeps grabbing all of the causal powers, leaving nothing for purported emergents. Perhaps we must simply accept this apparent lesson of contemporary science — that we ourselves are mere epiphenomena. I will be arguing that genuine emergence does exist, and that the difficulties encountered in trying to make sense of it have been exacerbated by the presupposition of a false metaphysics — a metaphysics of substances (particles) and properties. There are good reasons to abandon such a metaphysical framework, and to substitute a process metaphysics. In this alternative process metaphysical framework, the possibility of emergence, including genuine causally efficacious emergence, is found to be trivial — the in-principle mystery of emergence is dissolved. Accounting for any particular emergence, however, such as that of mind, remains a deep, complex, and difficult problem. The intuition of emergence is that of novel causal powers coming into being at specific levels of ontology (Beckermann, Flohr, & Kim, 1992; Beckermann, 1992b; Hooker, 1979, 1981a, 1981b, 1981c). The causal powers of purported emergents are the focus of much concern (Campbell, D. T., 1974b, 1990; Kim, 1992a, 1993b), but the criteria of novelty and the notion of levels are also of importance and interest (Wimsatt, 1976a, 1976b). I will have a few things to say about each of them, and begin with novelty. Novelty The novelty of emergents, or potential emergents, can be construed with respect to time or with respect to ontology (Stephan, 1992). Emergents in time — in history or evolution or cosmology, for example — are simply the first occurrences of whatever the emergent is claimed to be. Emergence in ontology is the stronger concept, and refers to something new coming into being with each instance of some level or pattern of lower level constituents. The two construals are closely related in that, on naturalistic
accounts, temporal emergents would be the first instances of particular ontological emergents; conversely, an ontological emergent would be a temporal emergent the first time an instance appeared. The emergence of novelty per se, at least in the sense of novel properties, seems uninterestingly trivial. There was presumably a first time for the cosmological emergence of an instance of the shape rectangle or the configuration of one thing being above something else. Among other requirements, these had to await the emergence of entities out of the original superhot fields of the Big Bang, and, for the relationship of above presumably the aggregation of a mass with a significant gravitational field so that the directions of up and down would be determined. But the simplicity with which such a criterion of novel property emergence can be met seems to render it almost nugatory, and, correspondingly, novelty is generally considered to be a weak necessary criterion with little intrinsic interest. If we turn the novelty criterion around, however, and consider it not just a requirement to be able to account for something new — anything — coming into being, but, rather, consider that most everything we are scientifically interested in did not exist at the moment of the Big Bang, and, therefore, that most everything we are scientifically interested in had to emerge since that time, novel emergence can become a very powerful negative criterion. In particular, any purported model of $X$ — for any phenomena $X$ — that cannot account for the historical and ontological emergence of $X$ since the Big Bang is thereby at best incomplete. More importantly, any model of $X$ that makes the emergence of $X$ impossible is thereby refuted. This holds even if we ignore any issues regarding the causal status of $X$, though, of course, in most cases of scientific interest, $X$ presumably will have some causal status. Contemporary models of cognitive representation, for example, generally begin with some set of representational atoms, each with its own representational content, and attempt to account for all representation as various combinations of these atoms. But such models cannot, in principle, account for the emergence of the representational atoms themselves. The attempts to account for representation (combinations) already presupposes representations (atoms). There are rejoinders to such a claim, of course, and the issues are not trivial, but this characterization of the current scene is at least prima facie correct, and I argue that it is in fact deeply correct of symbol models, causal models, information models, current functional models, and connectionist models alike (Bickhard, 1993; Bickhard & Terveen, 1995). If so, this inability in-principle to account for the emergence of representation refutes these models of representation. In any case, this characterization of current models of representation well could be correct, and that is all that I need at this moment to illustrate the potential power of emergence, even of just novelty, as a principle by which theories and models can be evaluated. Any theory of $X$ must be at least consistent with the emergence of $X$ or else it commits a
non-naturalism of cosmology. If X cannot have historically emerged, then either it existed from the beginning or it was non-naturally introduced. Our best current science tells us that nothing familiar existed from the beginning, and that nothing was non-naturalistically introduced. Consistency with the possibility of emergence, then, is a scientifically necessary requirement — given contemporary science — as well as a powerful metaphysical requirement, for any model of any phenomena.

Causality

But this is just a requirement to able to account for the novel emergence of X, because there was a time at which X did not exist. If X supposedly has any causal powers of its own, then accounting for X must account not only for its cosmological and ontological novelty, but also for those emergent causal powers. This has been the focus of most of the concern about what emergence is and whether it exists or not — can genuine, and genuinely novel, causal powers emerge? Emergence presupposes a notion of levels. The universe at its origin was a superhot flux of quantum fields; everything since then is the result of condensation, symmetry breaking, and organization out of that original flux, sometimes with clear hierarchical levels of organization. Quark excitations stabilize in combination with other such excitations into nucleons, which combine with electrons to form atoms, which combine chemically to form molecules, which combine gravitationally to form planets or in derivative chemical ways to form rocks, water, cats, humans, and, presumably, minds. This hierarchy of levels is one of the inspirations for the intuition of emergence: maybe everything has arisen in at least a generally similar way. Note that successively higher levels often require successively lower temperatures to emerge.

Downward Causation. If causal powers do emerge, then, within the framework of any reasonable naturalism, any causal consequences of those higher level emergent powers will themselves involve constituent levels of matter, or at least constituent levels of organizations of quantum processes. That is, any consequences of emergent causality will affect lower levels, constituent levels, of pattern and organization as well as the level at which the emergence occurs. More concisely, causal emergence implies downward causation (Campbell, D. T., 1974b, 1990; Hooker, 1979, 1981a, 1981b, 1981c; Kim, 1992a).

3. Hierarchy of levels

Since interesting emergence involves causal emergence, and causal emergence implies downward causation, downward causation becomes a strong criterion for genuine causal emergence and for interesting emergence more generally. Levels? Emergence involves higher levels, but what constitutes the difference between
higher and lower? What counts as a level? These questions lead in several directions, one of which I will focus on in particular. Note first, however, that the paradigmatic hierarchy of ever higher levels traces progressively lower temperatures of emergence and stability. Each level condenses out of lower levels with weaker forces, and, therefore, are stable and persistent in time only at lower temperatures. For at least some levels, such a differentiation of energy regimes in which stability is possible might seem to be definitive of the levels, though not necessarily of the particular kinds of emergents at those levels. This temperature differentiation of emergence levels, however, ultimately proves unsatisfactory. Higher levels might exhibit stability in the same temperature regime as constituent levels, such as for strictly mechanical machinery, or even manifest stability at higher energy levels. If, for example, an organism can protect itself against high temperatures, perhaps with perspiration and the production of heat shock molecules, the whole organism may remain viable at ambient temperatures at which isolated proteins would denature. The strong intuition about the nature of levels remains that of ontological inclusiveness: higher levels include lower levels as constituents — regardless of the energy realms for stability. Later I will argue that even this seemingly most basic sense of levels is flawed. A Logical Point. Emergence seems prima facie to be in conflict with naturalism. Higher levels of organization or constituency would seem to have whatever properties they have solely in virtue of those constituents and the relationships among them. If there were anything emergent beyond that, it could not be causally efficacious on pain of violating the completeness of the account of the physical world at those lower levels. One powerful way of putting this is to point out a problem: If the lower level includes everything that is physically — causally — relevant, then higher level emergence can be causally efficacious only at the cost of violating the causal closure of the physical world (Kim, 1993a, 1993b). Such a result seems wildly non-naturalistic and something to be resisted. But if causal emergence yields such a result, then perhaps causal emergence too should be resisted. On the other hand, there are certainly laws of regularity of causal efficacy that emerge at higher levels of pattern or organization — e.g., atomic stability and chemical valence (Hooker, 1981c) — that cannot be deduced from lower level laws alone. The pattern or organization of the constituents, minimally, is also required. One aspect of the issue of what counts as higher and what belongs to lower, then, focuses on such patterns and organizations. They constitute initial and boundary conditions with respect to lower level laws, and they are necessary to be able to account for higher level causal properties (Hooker, 1981c; Kfppers, 1992). Should they be included as part of the lower level, in which case we again face the consequence that any resultant causal properties will be counted as not emergent? Or should they be counted as constituting (part of?) the higher level, in which case novel causal
properties clearly do emerge (van Gulick, 1992)? In part this is a stipulative difference, and our preferential stipulation will depend on how strong or weak a notion of emergence we wish to consider (Beckermann, 1992a; Horgan, 1993a; Hoyningen-Huene, 1992, 1994; McLaughlin, 1992; O'Connor, 1994; Stephan, 1992; StÜckler, 1991). Within the perspective developed to this point, our choice of which seemingly arbitrary stipulation to make might depend most reasonably on what is at stake. Neither choice violates naturalism; countenancing emergence, however — counting pattern as higher — fits our naive intuitions and shields the causal efficacy of, for example, emergent mind, which most of us would probably appreciate. So, perhaps the best of all possibilities is to accept a conception of emergence that accepts causal-property resultants of organization as of higher level, and, therefore, emergent: we retain naturalism, emergence, and the causal reality of, among other considerations, mind. Ultimate Reality: Microcausation? But is the situation that simple? It seems reasonable within its own framework, but, even accepting emergence as the result, for example, of organizational boundary conditions on the manifestations of lower level laws, there nevertheless remains a strong seduction toward the conclusion that all real causality occurs only at the ultimate level of physical reality, presumably some class of fundamental particles (Kim, 1989, 1990, 1991, 1992b, 1993a, 1993b; Klee, 1984). In this view, the merely stipulative distinction between whether to count organization as part of higher or lower levels may usefully diagnose issues concerning relatively higher and lower levels where all levels under consideration are higher with respect to ultimate micro-levels, but it does not even address considerations that might privilege that ultimate micro-level itself above all other levels. It may be the case that particular consequences in the world depend on initial and boundary configurations, patterns, and organizations of fundamental particles, but, it might seem, all genuine causality occurs, and only occurs, at this ultimate level of particle mechanics. However much it may be the case that the outcome of causality depends on the patterns in which it works its causal consequences, nevertheless the only causal powers extant are those of these basic particles. So, all other lawful regularities, at whatever level of emergence, are really just supervenient on and epiphenomenal with respect to that basic level. Of course it is necessary to take into account the space-time configurations within which basic particle mechanics plays out its causal dance, but the only genuine causality is in the interactions among those particles. Causal consequences may depend on higher level patterns, but the only causal powers are those of fundamental particles. This is prima facie an extremely attractive picture. Its conceptual attractiveness is not diminished at all by the recognition that particular kinds of initial or boundary conditions can reliably yield particular kinds of regularities of consequences, and that these can look like emergents. All that follows from the view of ultimate reality being ultimate microcausation; it is not
it is not in contradiction to it. So, no matter the analysis of the distinction between relatively higher and lower levels, and no matter the semantic choices made about what counts as higher and what as lower, this view remains as a continual deflator of pretensions of emergence. What might appear to be emergence is really just basic, very micro-, particles interacting with each other.

4. Fields.

But, such particles are not all there is. There are also fields, and, in particular, quantum fields. Quantum field theory yields a very different picture than that of micro-particle mechanics. Quantum fields yield non-local interactions, such as result in the Pauli exclusion principle. Note in contrast that, in the particle picture, all causality is itself atomized to the very local points of particle to particle encounters. Quantum field theory yields a continuum of never ending activity, of process, even in a vacuum (Aitchison, 1985; Bickhard, in preparation-c; Brown & HarrÃ, 1988; Saunders & Brown, 1991). The background is not one of nothing happening except geodesic motion and local particle encounters — of an inert stage for particle mechanics — but, rather, a background of seething continuous creation and annihilation of quantum excitations of the field with various symmetries, therefore conservations, constraining the interrelationships within this activity. Ontology is not atomized to particles on a space and time stage, and cause is not atomized to points of particle encounters. In fact, there are no particles. Quantum field theory yields the conclusion that everything is quantum field processes (Brown & HarrÃ, 1988; Davies, 1984; Weinberg, 1977, 1995, 1996; Saunders & Brown, 1991). What appear to be particles are the consequences of the quantization of field excitatory activity, which is no more a particle than is the quantization of the number of waves in a vibrating guitar string. To illustrate the reality of this continuum of non-particle field processes, consider what is known as the Casimir effect. Two conducting plates held close together in a vacuum will inhibit the virtual excitations between the plates because the waves of those excitations will be constrained by the physical distance between the plates. There is no such inhibition of the foam of virtual creations outside of that gap. Therefore vacuum activity between the plates will be less than outside of the gap, and this results in a difference of pressure exerted on those plates. The net effect is a force pushing the plates toward each other, which has been experimentally verified (Aitchison, 1985; Sciama, 1991; Weinberg, 1995). Note that this force does not involve any particles; instead it is the result of that continuum of vacuum activity that is so unlike the atomization of substance and cause in the standard view. Quantum field theory eliminates the localization and atomization of substance into particles, the localization and atomization of cause into particle encounters, and the localization and atomization of levels of systems into objects. Everything is organizations of quantum processes (van Gulick, 1993); causality
is constraints on that quantum field activity, such as those that yield momentum or energy conservation (Aitchison & Hey, 1989; Bickhard, in preparation-c; Kaku, 1993; Ryder, 1985; Nahorara, 1992; Sudbery, 1986; Weinberg, 1995). In this view, everything is organization of process. There is no ultimate level of real particles on which everything else is supervenient, and with respect to which everything else is epiphenomenal. So that seduction is eliminated. The ultimate level of micro-particle micro-causeation does not exist. It might seem that the micro-causation argument against emergence could simply be recast with respect to quantum fields instead of particles: the only reality is quantum fields, and everything else is epiphenomenal to that. The first part of this point is correct: everything is quantum field processes. But the critical point is that quantum field processes have no existence independent of configuration of process: quantum fields are process and can only exist in various patterns. Those patterns will be of many different physical and temporal scales, but they are all equally patterns of quantum field process. Therefore, there is no bottoming out level in quantum field theory — it is patterns of process all the way down, and all the way up. Consequently, there is no rationale for delegitimating larger scale, hierarchical, patterns of process — such as will constitute living things, minds, and so on. That is, quantum field theory is an antidote to the seduction of including all patterns in the supervenience base, and, therefore, not counting properties that are dependent on perhaps complex patterns as constituting any kind of emergence. The point of quantum field theory in this discussion is to eliminate the temptation to devalue pattern so that pattern does not support emergence. In quantum field theory, pattern is everything because there is no level at which something unique and bottoming out, e.g., particles, can be found. It is, therefore, at best incomplete to say that everything is quantum fields: everything is organizations of quantum field processes — at many different scales and hierarchical complexities. Micro- and macro- alike are such organizations.- This resurrects the possibility of choosing to consider manifestations of organizational boundary conditions as of higher level, thereby resurrecting a naturalized emergence. More correctly, the recognition that everything is organization of process — just at differing scales and with differing hierarchical organizations — makes the choice to consider pattern and organization as of lower level, and thus to render properties of those patterns and organizations as epiphenomenal, a choice that renders everything epiphenomenal because there is no level at which anything is other than an organization of quantum field process, including even the smallest scale quantum fluctuations. The choice between countenancing organizational emergence and not countenancing it, then, is no longer arbitrary: to reject this form of emergence is to eliminate any level of non-epiphenomenality. That would seem to be a reductio ad absurdum of that position. In particular, in quantum field theory (or any process metaphysics),
there is no basis for excluding pattern from supporting emergence because everything is equally pattern, including higher level things such as minds. Minds cannot be merely epiphenomenal unless everything is taken to be epiphenomenal because there is nothing else that can be privileged in the metaphysics other than pattern, and there is no inherent reason to privilege any particular scale of such pattern over any other. But the consequences of shifting to a quantum field view ramify more densely and more distantly than emergence per se, and at least some of those further consequences need to be examined lest we implicitly presuppose a micro-atomization ontology even while explicitly rejecting it.

5. Supervenience

Notions of supervenience are attempts to distill the intuition that higher level properties depend on lower level properties. No change at the higher level without a concomitant change at the lower is the motto. There are importantly different varieties of attempts at explication of this intuition, but the issues that I want to focus on seem to be in common at least to both weak and strong supervenience (Kim, 1990). The lower level of a supervenience dependency, the supervenience base, must include both lower level constituents and relationships between them. Sphere is not supervenient on two hemispheres that are physically distant from each other, but would be supervenient on precisely the same constituents if they were in the proper physical relationship with each other (Baker, 1993). A supervenience base, however, does not include any relations external to the unit or system being considered. The property of being the longest pencil in the box, for example, is not supervenient on the molecules and internal relations that make up that pencil (Teller, 1992). By adding a new longer pencil to the box, the original pencil ceases to have that property, yet nothing of the supervenience base has changed. The property of being the longest pencil in a box is not of great independent interest, but there are other properties that are of deep importance that are similarly externally relational. Global quantum field constraints, such as the exclusion principle or a conservation constraint applying across spatially separated parts of a quantum system, are externally relational — they are not local. The property of being in thermodynamic equilibrium is relational to the environment, and so, consequently, is the property of being a far-from-equilibrium system. Necessarily open systems are those that are inherently far-from-equilibrium, and, therefore, require constant or at least intermittent interaction with an environment to be able to exist over time — otherwise they move to equilibrium and the far-from-equilibrium system ceases to exist. This implies that far-from-equilibrium systems, and all of the properties that they have qua far-from-equilibrium systems, are externally relational and, therefore, cannot be supervenient in the standard sense. A flame,
for example, is not supervenient: its existence is dependent on its environment (adequate oxygen, not too low a temperature, and so on) as well as on its own constituents _per se_. Furthermore, its supposed supervenience base is constantly changing, and any supposed micro-particle base is similarly in constant flux. The only persistence that constitutes the persistence of the flame is a persistence of an organization of process, not of the constituents that undergo that process. That organization of process, in turn, can be persistent only if appropriate transactions with the environment are possible and do in fact continue, such as inflows of oxygen and fuel vapor and outflows of combustion products. Conversely, if the constituents of a flame at a particular point in time were frozen — literally — then the supervenience base would remain the same, but there would no longer be a flame. Other even more important examples of far-from-equilibrium systems, and, therefore, of the limitations of the supervenience explications, are living things and minds. The supervenience intuition seems strong: higher levels depend on lower levels. But far-from-equilibrium systems constitute counterexamples to any presumed general applicability of supervenience as currently explicated. What is the source of the problem? Supervenience is explicated in terms of entities — particles — and properties (Kim, 1989, 1990, 1993b). This is basically an Aristotelian metaphysics, and is an inadequate metaphysics for relationships and process, most especially open process. _Entities_ that are organizations of underlying far-from-equilibrium process are not supervenient so long as supervenience discounts external relations, and so long as it counts lower level constituents as part of the supervenience base. Flames, waves, vortices — none are supervenient on underlying constituents. They are more like knots or twists in an underlying flow — nothing remains persistent other than the organization of the knot itself. They are topological entities, not substantive entities. _Living_ cells may contain structures that are in equilibrium stability, at least on relatively short time scales, but remaining alive requires continuous maintenance of far-from-equilibrium conditions, and, therefore, continuous flow and exchange with the environment. _Living_, then, is not a supervenient property: it is externally relational, and it requires a continuous flow of constituents. I argue that normativity, from functional normativity (functional - dysfunctional) to representational normativity (true - false) (Bickhard, 1993) and on up through rationality (Bickhard, forthcoming) and ethics (Bickhard, in preparation-a), is dependent on far-from-equilibrium systems properties. If this is so, or even if it is plausible, then the stakes involved in overlooking the inability of constituent and property based explications of supervenience to apply to far-from-equilibrium systems are quite serious. The sense in which everything is organization of quantum process, then, is even deeper than might at first appear. A first temptation in understanding _organization of process_ is a constancy of constituents — particles — engaged in some motions and interac-
tions; perhaps particles running around each other to form an atom. But far more important are organizations of process that have no constituents, or certainly no unchanging constituents. The organization is everything; the constituents either do not exist or are not part of the supervenience base. Quantum field theory suggests that there are no constituents in the classical sense at any level. There are only certain wave properties that are maintained in the flux of quantum vacuum activity, like a soliton wave in water, but for which the vacuum takes the place of water. What we normally consider as constituents, as particles or entities, are persistences of instances of organizations of underlying quantum process: they are topological. If those persistences are due to equilibrium stabilities, then we have classical paradigm cases such as atoms for which it is easy to overlook that quantum field nature, thus process nature, of even the electrons and quarks. If those persistences are far-from-equilibrium system persistences, then we must look elsewhere than equilibrium to understand such persistence, and the relevance of external relations is directly manifest; the basic reality of the organization of process, relatively independent of whatever engages in that process, is more likely to be forced on us. The dependence of higher on lower, then, remains. But the explication of supervenience as attempts to capture that dependence must relinquish the conception of the supervenience base as involving particular constituents and their internal relations. The types of the instances of lower level process patterns involved may be important — e.g., oxygen rather than nitrogen for a flame — but the dependence on the identities cannot remain. Furthermore, dependence cannot be simply mereological even with that modification: among other reasons, the necessity of external relationships must be accommodated. A vortex in a flow cannot exist if the flow itself does not exist. Note that this view not only eliminates the localization and atomization of substance (substance disappears) and causality (point-localized particle encounters), but also of entities. Waves do not have definite boundaries; neither do flames, vortexes, and so on. A thorough and deep de-localization and de-atomization is required. We do not have an acceptable and well understood metaphysics of this sort. In this view, the possibility of emergence, even causally efficacious emergence, is — at least in principle — trivial. There is no mystery, no non-naturalism. Everything is process organization, and, therefore, every causal property is a property of process organization. Higher levels and lower levels alike are levels of the organization of process. There cannot be the temptation, therefore, to privilege the constituents at the lower level, or even at some ultimate level, because there are no particles, and even lower level instances of process organizations may be in constant flux. It's pattern and organization all the way down. So a higher level causal emergent is just as legitimate as a lower level causal emergent. Accounting for the emergence and causal efficacy of any particular kind of phenomena, of course, can still be of
causal efficacy of any particular kind of phenomena, of course, can still be of enormous difficulty and complexity, but the impossibility in principle of any such emergence that a substance metaphysics yields (no new substances can emerge within a substance metaphysics, only combinations or blends of the basic substances can occur) is eliminated. At least in principle, in this view, the possibility of causally efficacious emergence is trivial, though the specifics of any particular emergence may well not be. Reduction and Anti-reduction. A particle and property metaphysics tempts us to think that the only real causality is found at the micro-particle level. If so, then anything that is a resultant of those particle interactions working their way within some initial or boundary condition constraints is most fundamentally due to those particle causal powers and particle interactions. Everything else is epiphenomenal to that, and can be eliminatively reduced to it — perhaps with the caveat of the cognitive limitations of human beings to handle the complexities required. In this view, higher levels are necessary considerations only because of their relative cognitive simplicity for humans, not for any metaphysical or even physical reasons. Common sorts of rejections of such eliminative reductionist conclusions include the claim of multiple realizability of the higher level in the lower level and of cross-cutting kinds from higher to lower. The central point in such objections to eliminative reduction is that higher properties (or kinds) cannot always be eliminated in favor of lower properties (or kinds) because there can be multiple ways — perhaps unbounded or infinite numbers of ways — in which the higher level can be realized in the lower. The necessary correspondences between higher properties (kinds) and lower, then, do not hold. There are an unbounded number of ways to physically construct a computer, and therefore being a computer cannot be defined in terms of any of them. The disputes in this area turn on what counts as a property or kind, in particular whether or not disjunctions of properties or kinds are themselves legitimate properties or kinds, on the nature of laws, and the relationship among laws, properties, and kinds, and so on (Burge, 1989, 1993; Fodor, 1981; Kim, 1989, 1990, 1992b, 1993b; van Gulick, 1989). If, for example, potentially unbounded disjunctions of kinds are legitimate kinds, then what it is to be a computer can be defined in terms of the disjunction of all of the physically possible ways that one could be realized. So long, however, as the temptation remains to grant ultimate reality only to an ultimate micro-particle level of reality, it seems that the issue regarding reduction is foregone. Metaphysically everything is either at the micro-particle level, or else it is epiphenomenal and reducible to that level. Human cognitive limitations may require consideration of higher level epiphenomena because they are simpler, but they have no more metaphysical reality than that. In the quantum process view, however, issues of multiple realizability and cross-cutting kinds still exist, but they exist as issues of what sorts of organizations of what sorts of
process organization instances will yield particular emergent properties. Computers can be silicon, vacuum tubes, fluidic, even mechanical (though they tend to be rather slow), so long as certain organizational relationships are realized. This is the same point as is made within a particle view, except that there is no temptation to eliminate everything above the level of fundamental particles — there aren’t any. The organizational properties that constitute something as a computer are just as legitimate as those that constitute something as an atom or cell or brain. The special properties that emerge with each of these need to be accounted for — a decidedly non-trivial task — but there is no need to fend off possible eliminative reduction to fundamental particles. Even within a particle view, the organizational properties cannot be ignored. But in a process view, such organizational properties (perhaps richly hierarchical) are all that there is. There is no more basic or fundamental reality. The Emergence of Properties and Entities Because everything is organization of process, every causally efficacious property is a property of organization of process. The possibility of causally efficacious property emergence, therefore, is assured. But what about entities? Particles have been eliminated, so entities cannot simply be combinations of particles. But how do we get to entities from properties and process organizations? Paradigm entities are stable instances of organizations of underlying process, such as atoms or animals. There are two kinds of such stability: 1) equilibrium or energy well stability, and 2) open process, far-from-equilibrium, stability. Energy well stabilities are those process patterns that would require energy input to destabilize them. They exist, or would exist, at thermodynamic equilibrium. So long as the ambient energy is not sufficient to destabilize them, to disrupt their cohesion (Collier, 1988, 1995), they will tend to persist. Atoms are a paradigm example. Necessarily open system stability, in contrast, cannot exist at equilibrium. Necessarily open systems are inherently far from equilibrium and cease to exist if they approach equilibrium. But approach equilibrium they inexorably will unless there are continuous exchanges with the environment that maintain the critical far-from-equilibrium conditions. The stability of far-from-equilibrium systems, then, depends on the stability of those conditions in the environment and relations to the environment that maintain the necessary far-from-equilibrium conditions. In some cases, all such conditions of stability are in the environment per se, and the system stability is completely dependent on that environment. A far-from-equilibrium system in which chemical solutions continuously flow into a container, for example, can exhibit fascinating properties (such as self-organization), but the stability of any such system is captured in the reservoirs and pumps for the chemical solutions, not the open system per se. A flame, in contrast, contributes to its own stability. It generates above-combustion-threshold temperatures, and, in an atmosphere and gravitational field, that yields convective inflow of oxygen and outflow of
combustion products. The heat also releases fuel vapor from the substrate, such as a piece of wood. The flame makes no contribution to the general availability of oxygen or fuel (though that might be disputed in the case of a fire storm), but it does contribute to the temperature requirement and to the local availability of oxygen and fuel and the dispersal of waste. I call such systems self-maintenant systems — they contribute to their own maintenance. Consider now a far-from-equilibrium system with the following general property: it has more than one way of being self-maintenant, and it can shift between or among available ways with at least some degree of appropriateness to what environmental conditions require. A bacterium, for example, might keep swimming if things are getting better, and tumble for a moment if they are not (Campbell, D. T., 1990). In conditions of getting better, keep swimming; in conditions of getting worse, randomize direction. Note that the switching between forms of contribution to self-maintenance requires some signal from the environment that can be used as an indication of which form is currently appropriate. I call such systems recursively self-maintenant — they tend to maintain (with respect to variations in the environment) their own condition of being self-maintenant (in those environments). I now want to offer some extremely inspissated outlines of how this framework might be able to account for some normative emergences. Note that a self-maintenant system either succeeds in maintaining system stability or it does not. If it does, the system remains stable in the world, and its causal consequences continue. If it does not, then the system ceases to exist, and its causal consequences qua that system cease. If the match flame has gone out, then the paper will not burn. The flame, then, serves a function (actually several) relative to the maintenance of the flame itself. And it makes a causal difference, an asymmetric difference, in the world whether or not that function is well served or not served. The difference between the flame existing or not existing is obvious; the asymmetry derives from the persistence of the relevant emergent properties if it continues, and the cessation of those emergent properties if it ceases. The asymmetry, then, derives from the asymmetry between the existence of open system emergents and the non-existence of those emergents — from the basic asymmetry between far-from-equilibrium and equilibrium. I claim that this is the general form in which function, and dysfunction, emerge. Function is contribution to self-maintenance, and is relative to the far-from-equilibrium system whose maintenance is in focus (Bickhard, 1993, in preparation-a). Note also that a recursively self-maintenant system could be wrong in its switching from one manner of self-maintenance to another. In particular, such a shift of process involves an implicit anticipation of subsequent self-maintenant interactions with the environment, but the environment may or may not cooperate. If the environment misbehaves, if things are actually getting worse for the bacterium in spite of continued swimming that is
supposed to make things better, then that implicit anticipation has been falsified. Furthermore, the system may be able to detect such a falsification: tumbling may be triggered yet again. In a more complicated system, perhaps a higher level signal (perhaps generated internally to the bacterium) indicates falsification even while the signal to switch from swimming to tumbling remains with the swimming. Any such higher level error signal would have to be a surrogate or vicariant for overall system stability in order for the error to be functionally genuine error for the system (Campbell, D. T., 1974a). But even the existence of such an error detector would do the bacterium no good unless that signal could in turn control or trigger some further self-maintaining process. It might, for example, shift to an entirely different set of interactive strategies for self-maintenance, or, in a much more complex system, such error signals may guide learning, not just subsequent behavior. My basic point, however, is that such implicit anticipations, and their potential falsification in and of and by the system itself, constitutes an emergence of truth value in the system itself. Truth value is one of the criteria, and a crucial and very difficult criterion to meet, for the emergence of representation. I argue, in fact, that such truth-valued anticipations constitute the most primitive form of emergent representation, out of which all other representation is differentiated and derived (Bickhard, 1993, in preparation-b). I have barely outlined these two claims of normative emergence, of function and of representation; I have not offered anything like an adequate argument for these particular emergents here. My point, however, is illustrative, not conclusive. My point is to illustrate a prima facie not-implausible possibility. Note that, in these models, function and representation emerge as properties of certain kinds of open, far-from-equilibrium, systems. That is, they emerge in certain kinds of organization of process. The possibility of their emergence, therefore, and of their causally efficacious emergence, is not precluded. Not precluded, of course, is not the same as accounted for. That requires the full arguments not presented here. But, for them to be not metaphysically precluded is already a large step beyond the intricate impossibilities yielded by standard particle and property metaphysics. As mentioned at the beginning of this paper, requiring that a model of X not preclude the emergence of X already rejects every model of representation and function; (Bickhard, in preparation-b) available in the contemporary literature.

6. Conclusion

The intuition that genuine causally efficacious emergence occurs — of mind, for example, especially yours or mine — is very strong. But serious difficulties have been encountered in trying to account for the mere possibility of any such emergence. I suggest that these difficulties are due to an inadequate and,
according to our best current science, false metaphysical framework that is presupposed in attempting those accounts. Within a more acceptable process metaphysics, the mere possibility of emergence is trivial, and the hard work of creating good models of actual emergents can proceed.

7. References


8. Footnotes

British emergentists had a kind of organizational conception of what counted as lower, and still wanted to claim that something else could be emergent at the higher level (Beckermann, 1992a, 1992b; McLaughlin, 1992; Stephan, 1992; StÜckler, 1991). The emergent property supposedly came into being with particular organizations of constituents, but it was in-principle not derivable from lower level considerations. Such emergence was itself presumed to be part of the physical laws of the universe: under such and such organizational or patterns conditions, this new causal property comes into being. This position may constitute a physicalism, but it violates the non-ad-hoc-ness of naturalism.

There is an epistemological view of emergence that depends on higher level properties not being derivable from lower level considerations, as a distinct issue from that of whether or not the higher level properties are determined by lower level properties and relations (Hoyningen-Huene, 1992). In such a view, chaotic systems provide a clear kind of (epistemic) emergence in that their course over time is not calculable in-principle, even though it is completely determined. Among other consequences, this implies that it may not be determinable which of two or more different attractors a given system is or will be in because the attractors themselves or (inclusive) their basins of attraction may be chaotically mixed and not separable in any physically realistic sense (e.g., Newman, 1996). I find this to be an interesting conception of emergence, but it is not the one at issue in this paper. I am concerned with issues of ontological and physical emergence, not only epistemological unpredictability (Hooker, 1979, 1981a, 1981b, 1981c).

This would likely be considered to be too weak a notion of emergence by some — the British emergentists, for example. But the point of the concept of emergence is to differentiate novel causal powers. Causal powers that are in principle not derivable from lower causality and initial and boundary conditions would certainly be a kind of emergence — though likely an empty kind, and certainly an ad-hoc kind — but it is difficult to find a reasonable argument that this should be held as the only notion of emergence. Conversely, the point of reduction, at least in the sciences, is to reduce the number of ontological kinds necessary to understand the world, without necessarily prejudicing, and certainly without necessarily rejecting, the reality of at least some aggregations of instances of those kinds. Hooker, for example, distinguishes between ontological reality, which is a reality of ontological kinds, and physical reality, which can include aggregations of instances of those kinds. Ontological reduction can, in
ontological kinds are of sub-atomic particles (Hooker, 1979, 1981a, 1981b, 1981c). That is, ontological reduction of X does not necessarily carry the implication of the elimination of the reality of X. The key point would seem to be that of the existence of genuine emergent causal powers. If it were held that higher level physical systems might exist, but that their causal consequences were strictly a result of the working out of the causal powers of the fundamental particles that constituted them, then that physical existence might seem unacceptably pale and unsatisfying as a notion of emergence. This stance depends on a strong distinction between causal consequences and causal powers, because it is clear that differing organizations of particles will have, in general, differing causal consequences. So the issue is whether or not there are emergent causal powers, whatever those might be. The assumption that this distinction between consequences and powers makes sense, in turn, depends on the assumption that there exists something that bears those genuine causal powers — distinct from mere causal consequences. Fundamental particles are the obvious candidate for these bearers of ultimate causality. It is to this set of issues regarding causal powers that I now turn in the main text. Assuming that minds can be understood naturalistically as organizations of particular kinds of processes. It is arguable, incidentally, that the basic particle reduction picture is not just factually false, but it is also logically incoherent. For example, if the particles have no extension, then a field view is forced in order to account for particle interactions, since the probability of such particles ever actually hitting each other is zero. If particles have finite extension, however, then they pose problems of compressibility, velocity of transmission of force through their diameter, extreme difficulty in explaining differing kinds of interactions (gravity, electricity, etc.), and so on. If a move is made to a combination of particles and fields (the typical contemporary semi-sophisticated view), then all of the basic issues are already granted anyway in the granting of fields at all. Any field view destroys the reduction into a micro-particle reduction because configurational and organizational properties make differences in causal power, not just in the working out of lower order causal power. There are no particles, but, even if there were, so long as fields are granted at all, the microreduction motivation fails — and a strict particle view is not only factually false but conceptually incoherent as well. (It is worth pointing out that Special Relativity forces a field physics, and, thus, a field metaphysics.) Though it is not clear what is supposed to bear those internal relations. The syntactic assimilation of relations to properties as all being just N-adic predicates for varying Ns seems to have obscured the ontological problems that relations pose to any substance-property metaphysics (Olson, 1987). It is already clear that causally relevant properties are not necessarily local, and, therefore, not necessarily supervenient (Burge, 1989, 1993; LePore & Loewer, 1987, 1989; van Gulick, 1989). The point here
is an extension of that to the existence of certain kinds of systems — in particular, of far-from-equilibrium systems. For other discussions of inadequacies of the concept of supervenience, see Collier (1988) and Horgan (1993a, 1993b). And quantum field theory requires that all entities are topological entities, not substance entities. Topological entities are defined in terms of what classes of shapes can and cannot be continuously deformed into each other without breaking or tearing anything. A surface with one hole in it, for example, can be smoothly deformed into a teacup, but a surface with one hole in it cannot be smoothly deformed into a surface with two holes in it — something has to tear. Similarly, a sphere cannot be smoothly deformed into a torus (doughnut), and a simple loop cannot be smoothly deformed into a simple overhand knot (with the ends joined). Such considerations at the level of vacuum processes have proven to be central to quantum field theory (Atiyah, 1987, 1991; Dijkgraaf & Witten, 1990; L. Kaufmann, 1991; Weinberg, 1996; Witten, 1988, 1989). Clearly they are important at a macro-level: a flow with a vortex in it is causally different from a flow with no vortex. There exist, of course, questions about the nature of the vacuum processes which are (hierarchically) organized at so many different scales. That nature is largely unknown (Atiyah, 1991; Bickhard, in preparation-c; Brown & HarrÃ, 1988; Misner, Thorne, Wheeler, 1973; Saunders & Brown, 1991). But continuity, non-locality, and virtual excitations, for example, compel that that nature is not particle-like. The British emergentists not-withstanding, the scientific use of the concept of emergence fits quite well with this notion of emergence in organization, rather than some sort of emergence beyond anything non-ad-hoc attributable to organization (e.g., Anderson & Stein, 1984; Bechtel & Richardson, 1992; Broschart, 1996; Careri, 1984; Chapman & Agre, 1986; Cherian & Troxell, 1995; Maes, 1992). There is also a form of persistence of types of process organization that is the result of instances of that organizational type causing, or at least increasing the probability of, the creation of more instances of that organizational type, such as in auto-catalysis or reproduction. I will not address these here (Bickhard, 1993; Bickhard & Campbell, D. T., in preparation). The illustration leaves the realm of biological reality here. I haven’t bothered to find out if any actual bacterium is capable of this. My point is more general, and this is illustration. and of all other forms of normativity as well.
Downward Causation
from macro- to micro-levels in physics

Abstract

Downward Causation in a physics-context is viewed as the influence of macroscopic boundary conditions on the microscopic dynamics of a thermodynamic system. Three cases are considered, corresponding to the three phenomenological categories of C.S. Peirce: 1: The irreversible approach to the maximum entropy equilibrium state of a homogeneous phase. 2: A symmetry-breaking phase transition (emergence) forming a separating boundary between two phases, like the surface of a liquid. 3: adaptive behaviour associated with the surface-modes such as self-organized criticality.
1. Wholes and parts

The metaphorical use of the words "upward" and "downward" in connection with "causation" is generally understood as involving wholes and parts of a system. Thus, the system is a whole that is distinguished from its surroundings by certain boundary conditions, and inside the system we may find interacting parts. In general systems theory words like "inside" and "boundary" also have a metaphorical character: the system is not necessarily like a container in ordinary space; for example we may speak of the system of electrons in a metal as something separated from the system of elastic vibrations in the same metal, although the "boundary" separating these two systems does not have the character of the wall of a container but is a sort of energetical constraint that connect the two systems weakly throughout the three-dimensional space of the metal.

We shall, however, in this chapter mostly be concerned with systems that really are containers, e.g. a gas that is separated from its surroundings by a solid wall. The gas as a whole has certain properties, like volume, pressure, and temperature that are conditioned partly by the wall and partly by properties of its constituent molecules. Thus, if the wall is heat conducting (diathermic) we may assume that the temperature has a fixed value, determined by the temperature of the surroundings, and the pressure and volume have a reciprocal relation to each other, whereas, if the wall is heat-insulating (adiabatic) both pressure and temperature will change, when the volume of the container is changed. The boundary conditions in this way determine the laws on the macroscopic level of the whole, i.e. the thermodynamic relations that are appropriate to the system, and they restrict the motion of the microscopic parts. We can therefore say that the boundary conditions exert a "horizontal" and a "downward" causation. Also, it is clear that there is an "upward" causation in the system, because macroscopic properties, like the heat capacity of the system depends on microscopic features, like the shape and rigidity of the molecules.

One may say that the restricting influence of the walls on the motion of the molecules is not genuine downward causality, because the laws of molecular motion, like Newton's law of action and reaction, are unchanged by the walls. This, however, is a limited truth, because the boundary conditions determine how these laws are to be applied. We may state the law of action and reaction by saying that the force molecule A exerts on molecule B is equal in magnitude (but with opposite direction) to the force molecule B exerts on molecule A, but this law then presupposes that molecule A and B have individual existence, so that
they do not react chemically with each other and form a compound or split into other parts that are not identical with the original molecules, and whether such reactions take place or not is determined by the boundary conditions, e.g. whether the walls are rapidly changing their positions or whether they are able to conduct heat from surroundings with a sufficiently high temperature.

The temperature is the most important macroscopic property that determines what type of laws describes the dynamics on the microlevel. One may say that temperature determines what type of parts we may consider as having individual existence. An examination of the concept of an ideal gas will illustrate that.

At room temperature we may consider a quantity of atmospheric gas as consisting of rigid diatomic molecules that are able to move freely in the three dimensional space within the confinement of the walls and perform free rotations around their center of mass. Thus, each molecule has 5 degrees of freedom in their motion, namely three translational motions and two rotational, and each of these degrees of freedom contributes on the average with a fixed amount $\frac{1}{2}kT$ (where $k$ is Boltzmann's constant and $T$ the absolute temperature) to the total energy of the system (assuming that the interaction between molecules is weak). The heat capacity of the system is therefore $(5/2)k$ times the number of molecules.

When the temperature is raised the heat capacity begins to increase, because the molecules cease to be rigid. When the two atoms in a diatomic molecule are able to oscillate relative to each other there will be 6 degrees of freedom per molecule, and this is also the case when the temperature induced oscillations become so violent that the molecules split into two atoms each having three translational motions. A further increase in heat capacity due to additional degrees of freedom for the microscopic motion becomes evident when the atoms begin to loose their electrons and the gas becomes a plasma of charged ions and free electrons.

We may understand the increase of heat capacity as due to the occurrence of new degrees of freedom, but once we have understood that molecules consist of atoms that consist of electrons and nuclei that consist of protons and neutrons that consist of — — we are faced with a big problem: These additional degrees of freedom exist all the time in the molecules. How come that we do not "feel" them at ordinary temperatures? How is it possible at all to speak of a well defined micro level of a macroscopic system when the parts themselves are wholes consisting of smaller parts that perhaps again may be subdivided in even smaller parts? It looks as if there is no "bottom" for the physical description but
rather an indefinitely descending hierarchy of microscopic levels. Where do we find the bedrock of microscopic dynamic from which the upward reaching causality extends to the macro-surface of thermodynamic systems?

This is one of the paradoxes that haunted classical physics around the turn of the century and led to the invention of quantum mechanics. The answer to the problem is that sub-microscopic degrees of freedom are "locked" by quantization of energy, and the smaller parts we consider the larger is the separation between their energy levels. When the level differences are much larger than the average energy of thermic motion it is impossible to transfer heat to these parts and therefore they do not contribute to the heat capacity. Therefore we are allowed to consider the gas at room temperature as a collection of classically moving rigid bodies for the purpose of dynamics, although we know full well that they consist of atoms, electrons and nucleons. The laws of quantum mechanics come into play for higher temperatures to describe the gradual loosening of the motions of these smaller particles and also for lower temperatures to describe the locking in of motions that are free at room temperature.

We see, thus, that the downward causative influence of the macroscopic boundary conditions on the microscopic dynamics is far more profound than just to delimit a certain part of the state space as available (see the paper by Mark H. Bickhard). The very notion of a microscopic state depends crucially on our ability to heat and isolate systems, and this ability is not reducible to microscopic laws but depends on technology and intention. The physicists do not just isolate a natural system for closer study, but with their methods of preparation create the system, including the notion of microscopic parts and the laws that govern them.

The Nobel prize in physics for the year 1996 was given for the discovery of superfluidity of the Helium isotope $^3$He.\textsuperscript{1} However, this property only exist below a millidegree above the absolute zero of temperature, and, as the background temperature of the universe is between 2 and 3 degrees, more than a thousand times higher, we can be pretty sure, that superfluid$^3$He only exists where there are physicists to study it.
2. Irreversibility and noise

All microscopic dynamical laws in physics are reversible, or invariant under time reversal. This means, that there is a certain mathematical operation that changes time $t$ to $-t$ in connection with changes of other variables such that the same law applies to these transformed quantities. In classical mechanics we have to reverse all velocities when we reverse time. If we look at a motion picture of a lot of billiard balls in motion and compare a certain situation with the same situation in the same motion picture run backwards, then we see the same positions of the balls but the opposite velocities, but we cannot by watching of the two versions of the film decide which is run the wrong way, unless there is a situation that points to the setup or preparation of the scene. If, for example, we see ten balls lying still in a cluster and one rapid ball moving into the cluster scattering the others in all directions, then we would guess that we see the events in the correct order of time. The time reversed show of a lot of balls coming together in a multiple collision and transferring all their motion to a single ball would seem too improbable to be natural. We would know that no billiard player, however skillful, would be able to create such a sequence of events, except by sheer luck.

There is nothing in the laws of motion that forbid improbable occurrences, for the very notion of probability is totally alien to the laws, like the notions of skill and intention. When we introduce such considerations we are jumping from the microscopic, reversible world to the macroscopic world, where the laws are irreversible. A film showing an egg being dropped to the floor and splashed all over it displays this macroscopic type of behaviour, and nobody would be in doubt whether it is shown with the right or wrong direction of time.

Macroscopic irreversibility was first formulated in laws like Fourier’s law of heat conduction and Ohm’s law of electrical conduction. Later it was generalized by Clausius about 1860 in the law of the increasing entropy. This strange state function of thermodynamic systems has the peculiar property that it can only increase when it changes, and it does so whenever some spontaneous event takes place in an isolated system. We all know what such an event could be, e.g. self-ignition of burnable material, but the notion of spontaneity is just as alien to the microscopic dynamics as entropy and irreversibility.

In classical mechanics or quantum mechanics every change of the state of an isolated system is totally deterministic, being determined alone by the force-law and the present state. But in thermodynamics we cannot be sure that a quiet state
of equilibrium will remain so. It may be a metastable state, and a transition to a more favorable equilibrium (with higher entropy) may be triggered by unforeseeable fluctuations in an explosive way. There is a profound connection between irreversible behaviour and indeterminacy. If a system is able to reach a state of equilibrium in an irreversible way then there must be unpredictable fluctuations or noise in the system. Normally the noise will be sub-liminal, and it is neglected in laws like Fourier’s and Ohm’s. But there may occur situations where the future development may take several directions depending on marginal differences, and in such cases the presence of noise is crucial for the realization of macroscopic indeterminacy.

The intrinsic connection between irreversibility and noise is due to the statistical or probabilistic character of both. This was illustrated with the example of the billiard balls, and in general we can use statistical models involving a moderately large number of particles to mediate between the seemingly irreconcilable paradigms of reversible micro-dynamics and irreversible thermodynamics.

The first attempt to reconcile these two physical disciplines was made by L. Boltzmann with the H-theorem from 1872. Boltzmann set up an equation to show how an arbitrary initial distribution of velocities of the molecules in a gas would be changed by collisions and finally stabilize itself in a statistical equilibrium. This was done by introduction of the H-functional that exhibited irreversible properties and could be used as a definition of entropy in statistical terms. Boltzmann was convinced of the correctness of thermodynamics, but his H-theorem was met with severe criticisms from mechanists, Loschmidt, Zermelo, and Poincaré. The simplest objection was the Umkehr-Einwand by Loschmidt who simply pointed to the time-reversal symmetry of the mechanical laws and correctly concluded that no mechanical proof of the entropy law could be possible. The objection would not be so serious if it hadn’t been put forward in a philosophical ground of mechanical reductionism. Everybody seemed to believe that Newton’s laws of mechanics ought to explain everything, and the best arguments against this view and in support of Boltzmann were formulated by the physicist W. Gibbs and the philosopher C. S. Peirce, both in America far outside the European main stream of science at that time.

In 1911, five years after the death of Boltzmann, Paul and Tatjana Ehrenfest published a thorough discussion of Boltzmann’s theory and the objections against it. The Umkehr-Einwand was taken into consideration with a simple model of diffusion, that we shall briefly consider.
In the Ehrenfest diffusion model a collection of N numbered particles are distributed in two urns, or in the separate two halves of a container. Every second a number is chosen randomly between 1 and N, and the corresponding particle is transferred to opposite half-container. The figure below shows a simulation (or rather two simulations) with 200 particles (ragged curve). At time zero in the middle there are 180 particles in the right half of the container. Time proceeds from zero to 400 from the middle to the right boundary of the figure and from zero to -400 going to the left.

\[\text{Figure 1} \quad \text{The Ehrenfest diffusion model. Vertical axis (0-200) shows the number of particles in the right half container. Horizontal axis: time from -400 to 400. Ragged curve: simulation. Smooth curve: average relaxation.}\]

In this model time is just a counting number of a random draw and it makes no difference whether it is counted backwards or forwards. The leftward running simulation (0 to -400) of course looks slightly different from the rightward running (0 to 400) but that is just a statistical difference to be expected between two different simulations. The reversibility of the model is manifested by the approximate left-right symmetry of the figure.

In principle we could regard the whole run as one single simulation from -400 to 400. The resulting curve is a fractal with small fluctuations within larger ones
and one especially large fluctuation right in the middle. It is not impossible that such a simulation result could occur but one gets suspicious that the large deviation in the middle is prepared, because the most probable distribution of particles is 100 in each half of the container and large deviations from that number are extremely rare. If a single simulation produced such a result we would be tempted to to discard it because it is "untypical" just as if a shuffling of cards had produced a deck with all the diamonds in a single cluster with no other suits mixed in between. In fact, the probability of a random occurrence of 180 of the 200 particles in the right half-container is about $10^{-33}$, so if we draw one number per second we would have to wait about $3 \cdot 10^{25}$ years before such a combination could be expected to occur once if there were no "cheating". Considering that the universe is only about $10^{10}$ years old we are almost allowed to say that such a large fluctuation is impossible.

Knowing, however, that the situation at time zero is prepared by the experimenter and that in reality there are two simulations, one counting forwards and one counting backwards in "time" there is nothing strange in the picture. If we make a lot of simulations from the same initial condition at calculate the average number of particles in the right container for each step of time the result is the smooth curves in figure 1 showing exponential relaxation of the initial large deviation in both directions of time. The reversibility of the model is exact for the two relaxation curves taken together, although the phenomenon of exponential relaxation in physics is always connected with irreversible phenomena. The forward relaxation curve looks exactly like the discharge of a capacitor through an ohmic resistance. Ohm's law alone will give the smooth exponential, and the deviations from it shown by the simulation correspond to the Nyqvist-Johnson noise from the resistor as filtered by the capacitor.

How can the reversible Ehrenfest model then account for the irreversibility described by Boltzmann? By showing that irreversibility is a result of the experimenters ability to prepare an improbable initial state and letting the dynamical situation proceed forwards in time. It is only the right half of figure 1 that can be regarded as a physical model of diffusion. The experimenter can have the 180 particles put into the right half of the container at time zero and then let the system run its course by itself, but he cannot choose an initial state like the one at time -400 that will evolve by itself to the very improbable state at time zero. If the experimenter had a "memory of the future" he could perhaps do the trick, but he only knows the past and he therefore cannot select among all the many similarly looking states near equilibrium one of the few initial states that will develop into a conspicuously deviating state.
The question of how irreversibility arises is thus transferred from the domain of microscopis dynamics to the irreversible behaviour of the experimenter. How can it be that we only have a memory of the past and that our sense of time always proceeds in the same direction? This question cannot be answered reductionistically be considering a human being like a collection of molecules that act together according to the laws of mechanics, for, as we have seen, these laws are all reversible and have no sense of "time's arrow". But the human body works as it should only if it is inserted in an ecological system with available food and clean air and water, a thermodynamic system far from equilibrium. Such a system has a tendency to relax towards equilibrium producing entropy and it is this tendency that nourishes the organism and provides it with a sense of time. If the ecological system were isolated in the universe it would run down to equilibrium and the organisms would die. But it is maintained in the non-equilibrium state by a flow of low-entropic energy from the sun that can be converted into high-entropy heat radiation and scattered out into the background radiation of the universe. The question of the origin of irreversibility is thus pushed upwards as far as "up" goes in physics: to the irreversible evolution of the whole universe.

The recognition of this multi-level downward causation from the ecosystem through the experimenter's ability to select improbable initial states for a thermodynamic system changes the status of the sentence "the entropy increases" from a paradox to a tautology. For the prerequisite of being able to say anything is that the entropy of the universe is higher after the saying than it was before. The same entropic condition applies to any significant event, to every difference that makes a difference, i.e. rises appreciably above the noise level of fluctuations.

3. The emergence of boundaries

In the early universe matter is uniformly distributed in a gaseous state of internal thermodynamic equilibrium. In such a state there are no boundaries, it is impossible to separate a system from its surroundings, and no signs or significant actual events can exist. Nothing marks the space, and gravitation is cancelling itself out. However, the increasing scale or expansion sets up an external control parameter that gradually forces internal symmetry breaking choices that set up ordering fields acting as internal control parameters for the creation of significant
boundaries, limitations, and constraints. These constraints, in turn, lead to greater semiotic freedom, or liberation of the semiosphere, as pointed out by Jesper Hoffmeyer

A specific type of order, created by spontaneous symmetry-breaking may generalize itself by the action of the ordering field it makes. For example, a larger concentration of matter in a volume creates a gravitational attraction towards its center such that surrounding material gets sucked in making the gravitational pull even stronger. The resulting local inhomogeneity of matter creates a spreading tendency to form nucleation centers for matter in space. Gravitation, previously lying dormant, in this way becomes generalized to a habit of the universe, becomes significant.

According to Peirce this is semiosis at work. A slumbering affinity or similarity is an icon, an actual difference is an index, and a habit or general rule is a symbol. Symbols are general ideas that spread and loose intensity but become associated with other ideas whereby new symbols are created. This is Peirce’s law of mind.

The phenomenology and metaphysics of C. S. Peirce distinguishes between three ontological modes or categories:

1. Firstness: This is the mode of potentiality and being.

2. Secondness: Actuality and individual existence.

3. Thirdness: Generality and reality.

The categories follow each other such that Secondness presupposes and contains Firstness, and Thirdness presupposes and contains Firstness and Secondness.

The emergence of a boundary separating between spatially extended qualities is a Secondness arising as an actual distinction between Firstnesses. If we think of something like a water surface we can imagine how the constrained space of the surface evolves it own laws by downward causation, and indexical signs like drops of water become generalized and symbolized to rain and rivers and oceans (with birds, boats, and fishes).

Internally there is no qualitative difference between a gas and a liquid. There is no long range order and the molecules wander erratically around. So, if it is at all possible to distinguish between gas and liquid it must be due to the existense
of a surface that separates the denser liquid from the rarefied gas. Secondness enters the picture through the surface that distinguishes between the internal Firstnesses of the two phases, but while the gaseous phase keeps its unconstrained Firstness, the liquid is contaminated with Secondness, for the surface belongs to the liquid it confines. The surface introduces a tension that keeps drops of liquid together.

The qualitative features of the gas-liquid transition was first described mathematically with the Van der Waals equation of state (1872). This equation explains the existence of a critical point \((P_c, T_c)\) in the pressure-temperature plane such that the distinction between gas and liquid only exists for certain pressures when the temperature is below the critical temperature \(T_c\). Van der Waals’ equation (see appendix) has become paradigmatic, not because of its quantitative agreement with measurements for real gases (which is not impressive) but because it gives a simple conceptual scheme for the discussion of order-disorder transitions (or second order phase transitions). The hypostatic abstraction of this concept was perfected by L. D. Landau in the mean field theory of second order phase transitions\(^6\) (1950)\(^6\) and by R. Thom in the so-called catastrophe-theory (1978).\(^7\) The gas-liquid transition exemplifies the cusp-catastrophe of Thom, and this is the simplest model for describing how a type of order nucleates spontaneously and is able to induce similar ordering in its surroundings. By means of Van der Waals’ equation one is able to formulate a law of corresponding states for different gases. For example, the reduced pressure \(P/P_c\) of saturated vapour is a universal function of the reduced temperature \(T/T_c\) as shown in figure 2

\(^{6}\) Other examples of such transitions are the ferromagnetic and the superconductive transitions, and order-disorder transitions in alloys.
Figure 2 Reduced pressure of saturated vapour as function of reduced temperature according to van der Waals.
As shown in figure 3 the cusp catastrophe requires two control parameters, $a$ and $b$, where $a$ is the "external" control (temperature) and $b$ the "internal" control (ordering field). For the case of the gas-liquid transition $b$ is roughly proportional to the deviation of the pressure from the critical pressure. (see, however, the discussion of the control-parameters in the appendix).

The $a$- and $b$-axes in figure 2 are made to cross in the critical point. Above this point (for higher values of $a$) no ordering is possible (no surface), but below there is an interval of $b$ values where the two phases may coexist.

![Cusp catastrophe diagram]

**Figure 3** The cusp catastrophe. Potential as function of order parameter shown for marked points below the cusp.

The ordered phase is described by an *order parameter* which for the gas-liquid transition is the difference in density between the two phases separated by the surface. The equilibrium value of the order parameter is one that minimizes the thermodynamic potential (Gibbs' free energy). The *catastrophe set* in the control plane is a curve that exhibits a cusp in the critical point. This curve separates a
region where this potential has two minima below the cusp from another region where it has only one minimum.

The left minimum corresponds to the gaseous phase and the right to the liquid phase. Close to the critical point, where the saturated vapour pressure is equal to the critical pressure, the two phases may coexist in equilibrium only on the line \( b = 0 \). Normally in thermodynamics one assumes that the lowest minimum is the stable one, such that the two phases may coexist only when the two minima have the same height, i.e. on the \( a \)-axis below the cusp. This assumption corresponds to the so-called Maxwell convention. In reality, however, there may be a region with "superheated liquid" to the left of the \( a \)-axis and a region with "supercooled vapour" to the right and these regions of metastability may extend to the catastrophe curve, but not beyond, which is the convention of "maximum delay". Where the transition actually takes place is determined, among other things, by characteristics of the surface. Very small bubbles of liquid have a high surface tension which increases the internal pressure such that the bubble may be superheated.

The emergence of the phase separating boundary to a liquid phase in a gaseous region is a complicated cooperative phenomenon. A mist of small droplets appears, and gradually these droplets coalesce whereby the pressure is regulated through the action of surface tension (and perhaps gravitation). When the external control parameter is lowered (\( a \), the temperature) large density fluctuations will begin to appear, and these will adjust the internal control (\( b \), the pressure deviation) so as to pass through the critical point. Below criticality the fusion of droplets will tend to keep the system in the close neighbourhood of the \( a \)-axis, \( b = 0 \), the line of saturated vapour pressure.

The emergence of boundaries like the liquid surface is the first step in the semiosis of natural pre-biological evolution. It is the transition from the slumbering Firstness of icons to the specificity and actuality of indices. But the law of mind comes to play by the downward causative influence of habit formation. A habit is an emerging generality, a Peircean Thirdness that presupposes the significant difference of Secondness. An occurrence governed by habit is facilitated by its own previous occurrence. In this way the habit implies a self-reference that makes it a suitable third factor or interpretant of a symbolic sign relation.

We have seen that the surface tension is a feature that arises by downward causality. But liquid surfaces and other types of emergent boundaries tend to develop specific habits that do not belong to the world of microscopic dynamics.
The significance of singular shapes in the control space, like the cusp and the line of coexistence in figure 2 is due to a tendency of boundaries to proliferate themselves, and this is done most efficiently in the neighbourhood of critical regions. The phenomenon of self organized criticality (SOC) that has been described by Per Bak et al \(^8\) seems to be a most important fact for the understanding of semiosis in evolution. The simplest example is that of the sand dune that maintains a critical slope because just this slope has the maximal ability to respond by avalanches of all sizes to every disturbance.

Phase separating boundaries create a confined space for special types of disturbances, like ripples on the water. These modes have a dominating downward causative influence near the critical point, as described in the slaving principle by H. Haken:

"Haken’s slaving principle states that in the neighborhood of critical points, the behavior of a complex system is completely governed by few collective modes, the order parameters, that slave all the other modes."\(^9\)

\hspace{1cm}

**Figure 4** rippled surface by M.C. Escher. Lino-cut 1950
These surface modes have dynamic properties determined by overall metrical properties of the surface, like its fractal dimension. On the other hand, these modes have the function of maintaining the overall characteristics of the surface that maximizes the diversity of internal motion which is close to the critical region of marginal instability. The working of the pre-biotic law of mind may thus be described as a complex interplay of upwards and downwards causality.

4. Notes and references


2. The most general formulation of this connection is the fluctuation-dissipation theorem of Callen and Welton (1951).

3. J. W. Gibbs and C. S. Peirce were both born in 1839 and both graduated in chemistry and had some correspondence with each other. Gibbs developed his Statistical Mechanics about the same time as Boltzmann. Peirce wrote about the philosophical aspects of chance and necessity (The Doctrine of Necessity Examined, 1892), defended Boltzmann’s views against mechanists and pointed to the need for a new mechanical theory of atoms.


Appendix:
van der Waals' equation of state

1. Virials and Van der Waals.

In the article "Man's Glassy Essence" Peirce attempted to put up a materialistic theory for the metabolism and self-reproduction of living cells. The physical foundation for this theory was the virial theorem and van der Waals' equation of state. Even though Peirce's article takes an idealistic turn towards the end it is still important as an expression for Peirce’s semiotic realism, and as the theory also in the present article is basic for the discussion of emergence of surfaces, we shall briefly consider it in this appendix.

In classical statistical mechanics the so-called law of equipartition states that the average kinetic energy per degree of freedom is ½kT. In contradistinction, the average potential energy is not so easy to calculate. In stead we can express the deviation from ideality of gases by the virial, that is the average sum of attractive force times distance, that affects one molecule from all the others. The Virial theorem of Clausius can be stated in the form

$$kT = P\nu + \frac{1}{3}\sum F_r$$

where P is the pressure and ν the volume per molecule, i.e. the total volume V divided the number of molecules. The last term is the molecular virial. Here, the situation is viewed from the place of one, arbitrarily chosen, molecule, and we sum for all other molecules the attractive force $F$ (i.e. -F, if the force is repulsive) times the distance $r$ from the chosen molecule. Finally, this quantity is averaged over all molecules (denoted by the bar over $F_r$). If we can disregard the interaction between molecules, the virial disappears, and equation (1) degenerates to the state equation of ideal gases.

---


**) See e.g. D. Ter Haar: Elements of Statistical Mechanics. Holt, Rinehart, and Winston (1954)
We may get a simple expression for the $v$-dependence of the virial if we assume that the molecules are uniformly distributed in space. If we place a sphere of radius $R$ and center in a molecule, then the number of other molecules in this sphere will be the sphere's volume divided by $v$. We shall further assume that the attraction $F$ decreases with distance faster than $r^{-3}$ \(^\dagger\). We can then choose $R$ sufficiently big, so that the molecules outside the sphere don't significantly contribute to the virial. It then follows that the virial must be proportional to the number of molecules in the sphere, i.e. inversely proportional to $v$. Equation (1) can, therefore, be written in the form

$$kT = Pv + \frac{a}{v} = (P + \frac{a}{v^2}) \cdot v \tag{2}$$

where we have included the factor $1/3$ from eq. (1) in the constant $a$. The introduction of this constant instead of the individual virials makes the following theory belong to the family of mean field theories of second order phase transitions that is comprised by Thom's cusp-catastrophe. In this model we have disregarded the repulsive core of the intermolecular forces, but we can take it into account, roughly, by ascribing to each molecule a proper volume $b$. I van der Waals' equation (2) this is done by replacing $v$ with "the free volume" $v$-$b$. If, simultaneously, we introduce molar quantities in stead of molecular Eq (2) is changed to the traditional form of van der Waals' equation:

$$(P + \frac{a}{v^2})(v - b) = RT \tag{3}$$

where $R$ is the gas-constant, i.e. Boltzmann's constant $k$ times Avogadro's number. In the general thermodynamics of real gases one assumes that the virial can be series-expanded in the quantity $1/v$. The term $a/v$ in eq. (2) is the first term in the virial expansion:

$$Pv = RT \left[1 + a_1(T) \cdot v^{-1} + a_2(T) \cdot v^{-2} + \cdots \right] \tag{4}$$

For van der Waals' equation (3) we have:

\(^\dagger\) The attractive tail of the van der Waals-forces, due to mutually induced molecular dipoles goes as $r^6$.\n
\[ P = \frac{RT}{v-b} - \frac{a}{v^2} \quad (5) \]

by series expansion of the first term on the left side of (5) and comparison with (4) we then find, that the first two of the virial coefficients according to van der Waals are given by:

\[ a_1 = b - \frac{a}{RT} ; \quad a_2 = b^2 \quad (6) \]

Specifically, that the second virial coefficient is temperature-independent

2. Reduced variables and corresponding states.

Van der Waals’ equation (2) can be written as a cubic equation in \( v \):

\[ Pv^3 - (Pb + RT)v^2 + av - ab = 0 \quad (7) \]

One finds, that there is a critical temperature \( T_c \), such that there for \( T < T_c \) exists a \( P \)-interval with three solutions for \( v \). For \( T > T_c \) there will for each value of \( P \) only be one value of \( v \), that satisfies eq. (7). For \( T = T_c \) there will be one point, \( P = P_c, v = v_c \), where the polynomial \( Q(v) \) on the left side of (7) is zero, while, simultaneously, both its first and its second derivative vanish. In order to determine the critical values \( v_c, P_c, \) and \( T_c \) we have to solve (7) together with the equations:

\[ Q'(v) = 3Pv^2 - 2(Pb + RT)v + a = 0 \quad (8) \]

\[ Q''(v) = 6Pv - 2(Pb + RT) = 0 \quad (9) \]

By the solution, e.g. one may first find from (9), that

\[ Pb + RT = 3Pv \quad (9a) \]

Dette is then inserted in (8) and one gets:
\[ v^2 = \frac{a}{3P} \]  

(8a)

By insertion of (9a) og (8a) i (7) we find:

\[ 0 = v[Pv^2 - (P+RT)v + a] - ab = v[-2Pv^2 + a] - ab = a(v/3 - b). \]

Det critical volume is thus

\[ v_c = 3b \]  

(10)

og by insertion of this value in (8a) og (9a) we find:

\[ P_c = \frac{a}{27b^2} \]  

(11)

\[ T_c = \frac{8a}{27bR} \]  

(12)

we may then introduce dimensions-less, or reduced variables, viz. the reduced volume \( v \), the reduced pressure \( \pi \), and the reduced temperature \( \tau \) by the definitions:

\[ v = \frac{v}{v_c} ; P = \frac{\pi P_c}{P} ; T = \frac{T_c}{T} \]  

(13)

Inserting these expressions in van der Waals’ equation (3), it attains the dimension-less form:

\[ (\pi + \frac{3}{v^2})(3v - 1) = 8\tau \]  

(14)

In this way every reference to the specific properties of the gases vanishes. We say that two gases with the same samme values of the reduced variables are in corresponding states.
3. the pressure of saturated vapour.

We start by writing (14) as a cubic equation in \( u \):

\[
\begin{align*}
    u^3 + A\,u^2 + B\,u + C &= 0 \\
    A &= -\frac{1}{3}(1 + \frac{8\tau}{\pi}) ; \\
    B &= \frac{3}{\pi} ; \\
    C &= -\frac{1}{\pi}
\end{align*}
\]  \hspace{1cm} (15)

Even though it is easy to determine the isotherms \( \pi \) as function of \( u \) for fixed \( \tau \) by using af (14):

\[
\pi = \frac{8\tau}{3u-1} - \frac{3}{u^2}
\]  \hspace{1cm} (16)

it may in some situation be necessary to go the other way and find \( u \) as a function af \( \pi \) (for fixed \( \tau \)) by solving the cubic equation (15).

For \( \tau < 1 \) there exists a \( \pi \)-interval, where (15) has three solutions for \( u \). By placing a line of constant \( \pi \) in this interval, the areas between the isothermal--curve and the line be found analytically by integration of (16), when the points of intersection have been found by solving (15). The pressure of saturated vapour is then, according to Maxwell, that value of \( \pi \), that makes the areas over and below the linen of equal size.(see figure A2). This so-called Maxwell-convention corresponds to the thermodynamic condition of equilibrium that the chemical potentials of the two phases shall be equal. The isotherm can, according to (16) have negative values of \( \pi \) for \( \tau < 27/32 \), men that doesn’t matter, because the part of the isotherms, that lies under the Maxwell-line, is unphysical, anyway. The following table and curve (figure A1) shows the results for the pressure of saturated vapours reduced e rede tryk as function of the reduced temperature, determined by numerical solution of the Maxwell-condition.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96</td>
<td>0.8476</td>
</tr>
<tr>
<td>0.9</td>
<td>0.6470</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3834</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2005</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0869</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0278</td>
</tr>
</tbody>
</table>
**Figure A1.** Qualitative sketch of the reduced pressure of saturated vapour as a function of reduced temperature.
Figure A2. van der Waals isotherms and Maxwell-lines: \( \nu \) as a function of \( \pi \) for fixed \( \tau \). The critical isotherm (\( \tau = 1 \)) has a horizontal tangent of inflection at the critical pressure.

4.

In the general theory of cubic equations in the form (15) one introduces the mathematical control-parameters

\[
\alpha = B - \left( \frac{A}{3} \right)^3
\]

\[
\beta = -\left( \frac{A}{3} \right)^3 + \frac{AB}{6} - \frac{C}{2}
\]

corresponding to the parameters \( a \) and \( b \) in figure 3, respectively.

The number of solutions to the equation is then determined by the sign of the expression

\[
D = b^2 - a^3
\]
\[ \tau 1 + \delta \tau; \pi = 1 + \delta \pi \]

\(\delta \tau\) and \(\delta \pi\) are then related to \(\alpha\) and \(\beta\) by a linear transformation, and for both sets of control-parameters the critical point (the cusp) is located in \((0,0)\). In catastrophe theory it is assumed (based on results by Morse and Thom) that such a transformation leaves the topology of the singularity unchanged, but exactly how the system passes through the critical point depends on the physical mechanism of self-organized-criticality (SOC) that acts by means of the critical density fluctuations and the slaving surface modes, and the mean field theories, like Thom's cusp does not take these fluctuations into account. It has been shown that the mean field description is strictly valid, only in a four-dimensional euclidean space. In all other cases the theory has to be renormalized, and the exact behavior of the renormalized cusp has not been fully determined, not even for such qualitative features as the critical exponents of the power laws that govern the SOC. The "slaving" surface modes that govern the SOC may lead the physical control-parameters to the critical point by some power law, so that even the cusp-shape \((a = -b^{2/3})\) of the mathematical catastrophe set may need renormalization.

Still, the cusp catastrophe and the mean field theory of phase transitions has its paradigmatic merits as the simplest description of a second order phase transition with a critical point.
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On sære matematiske fikser betydning for den matematiske udvikling
af: Claus Dreby, Jørn Skov Hansen, Rune Ulsee Johansen, Peter Melbom, Johannes Kristoffer Nielsen
Vejleder: Mogens Niess

241/93 FOTOVOLTASISK STATUSNOTAT 1
af: Bent Sørensen

242/93 Brovedligeholdelse - bevær mig vel
Analyse af Vejdistrektorats model for optimering af borerparationer
af: Linda Kyndlev, Kære Fundal, Kamma Tulinus, Ivar Zeck
Vejleder: Jesper Larsen

243/93 TANKEEKSPERIMENTER I FYSIKKEN
Et 1.modul fysikprojekt
af: Karen Birkelund, Stine Sofia Korremann
Vejleder: Dorte Posselt

244/93 RADONTRANSFORMATIONEN og dens anvendelse i CT-scanning
Projektrapport
af: Trine Andreasen, Tine Guldager Christiansen, Nina Skov Hansen og Christine Iversen
Vejledere: Gestur Olafsson og Jesper Larsen

245a+b/93 Time-Of-Flight målinger på krystallinske halvledere
Specialrapport
af: Linda Skotak Jensen og Lise Odgaard Gade
Vejledere: Petr Viscor og Niele Boye Olsen

246/93 HVERDAGSVIDEN OG MATEMATIK - LÆREPROCESSER I SKOLEN
af: Lena Lindenskov, Statens Humanistiske Forskningsråd, RUC, IMFUFA

247/93 UNIVERSAL LOW TEMPERATURE AC CONDUCTIVITY OF MACROSCOPICALLY DISORDERED NON-METALS
by: Jeppe C. Dyre

248/93 DIRAC OPERATORS AND MANIFOLDS WITH BOUNDARY
by: B. Booss-Bavnbek, K.P. Wojciechowski

249/93 Perspectives on Teichmüller and the Jahresbericht Addendum to Schappacher, Scholz, et al.
by: B. Booss-Bavnbek

250/93 EULER OG BOILAND - MATEMATISK ANALYSE SET I ET VEDELSKABSTEORETISK PERSPEKTIV
Projektrapport af: Anja Juul, Lone Michelsen, Tomas Nørgård Jensen
Vejleder: Stig Andur Pedersen

251/93 Genotypic Proportions in Hybrid Zones
by: Freddy Bugge Christiansen, Viggo Andreasen and Ebbe Thue Poulsen

252/93 MODELLERING AF TILFELDIGE FØDNUKER
Projektrapport af: Birthe Friis, Lisbeth Helmgard Kristina Charlotte Jakobsen, Marina Modsb Johansson, Lotte Ludvigsen, Mette Basse Nielsen

253/93 Kuglepakning
Teori og model
af: Lise Arleth, Kåre Fundal, Nils Kruse
Vejleder: Mogens Niess

254/93 Regressionssømls
Materiale til et statistikkursew
af: Jørgen Larsen

255/93 TID & BETINGET UAFHÆNGIGHED
af: Peter Barremoës

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y: T. Christensen og K.B. Olsen

257/93 Modellerling of dispersion in piezoelektriske kvarantser
af: Pernille Postgaard, Jannik Rasmussen, Christina Specht, Nikko Óstergaard
Vejleder: Tage Christensen

258/93 Supplementende kursemateriale til "Lineære strukturer fra algebra og analyse"
y: Mogens Brun Reesfeldt

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by: Jeppe C. Dyre

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Eksamensopgaver fra 1976-93

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by: Lars Kadison

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af: Bent Sørensen

265/94 SPHERICAL FUNCTIONS ON ORDERED SYMMETRIC SPACES
To Sigurdur Helgason on his sixtieth birthday
by: Jacques Faraut, Joachim Hilgert and Gestur Olafsson

266/94 Kommensurabilitets-oscillationer i laterale supergitter
Fysikspeciale af: Anja Boisen, Peter Bøggild, Karen Birkelund
Vejledere: Rafael Taborsky, Poul Erik Lindelof, Peder Voetmann Christiansen

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af: Charlotte Gjerrild, Jane Hansen
Vejleder: Bernhelm Booss-Bavnbek

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Et projekt om modellering af aorta via en model for strømning i kloaker
af: Anders Marcussen, Anne C. Nilsson, Lone Michelsen, Per M. Hansen
Vejleder: Jesper Larsen

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af: Tine Guldager Christiansen, Ken Andersen, Nikolaj Hermann, Jannik Rasmussen
Vejleder: Jens Højgaard Jensen

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by: Jacob Jacobsen

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Opgaget eller opfundet
NAT-BAS-projekt
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af: Kristian Hoppe og Jesper Guldager
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Verifikation af model
af: Lise Fabricius Christensen, Helle Pilemann, Bettina Sørensen
Vejleder: Mette Olufsen

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3. modul matematik, førår 1994
af: Trine Andreasen, Bjørn Christensen, Christine Green, Anja Skjoldborg Hansen, Lisbeth Helgaard
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Projektrapport 1. modul
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Projektrapport udarbejdet af:
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af: Erwin Dan Nielsen, Jan Danielsen, Niels Bo Johansen
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   (en kaotisk talgenerator)
   af: Erwin Dan Nielsen og Niels Bo Johansen

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   lige ve' det!
   Gymnasieøvelsenens begrundelsesproblem
   En specialerapport af Peter Hauge Jensen
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   Et modul matematik projekt
   af: Anders Mørkussen, Anne Charlotte Nilsson,
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   LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH
   ENERGY SYSTEM
   an example of using methods developed for the
   OECD/IEA and the US/EU fuel cycle externality study
   by: Bent Sørensen

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   af: Lotte Ludvigsen & Jens Frandsen
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   et metaproyekt
   Af: Jesper Duelund og Birthe Friis
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   to fortolkninger af kvantemekanikken
   af: Maria Hermansson, Sebastian Horst,
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   Vejledere: Jeppe Dyre og Peder Voetmann Christiansen

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   denskabelig matematisk model
   Et matematisk modelprojekt
   af: Claus Dreyer, Michael Hansen, Tomas Højgaard Jensen
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   by: Bent Sørensen

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   af ozon
   af: Glenn Møller-Holst, Marina Johannessen, Birthe
   Nielsen og Bettina Sørensen
   Vejleder: Jesper Larsen

302/95 KOMPRESSERER - Analyse af en matematisk model for
   aksialkompressorer
   Projektrapport af: Stine Bøggild, Jakob Hilmer,
   Pernille Poggaard
   Vejleder: Viggo Andreasen

303/95 Masterlignings-modeller af Glasovergangen
   Termisk-Mekanisk Relaksation
   Specialrapport udarbejdet af:
   Johannes K. Nielsen, Klaus Dahl Jensen
   Vejledere: Jeppe C. Dyre, Jørgen Larsen

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   indeholdende bl.a. ordforklaringer, resuméer og
   tabeller
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By: B. Booss-Bavnbek, K. Furutani

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Preprint of a chapter for the forthcoming International Handbook of Mathematics Education (Alan J. Bishop, ed)
By: Mogens Niss

307/95 Habit Formation and the Thirdness of Signs
Presented at the semiotic symposium
The Emergence of Codes and Intensions as a Basis of Sign Processes
By: Peder Voetmann Christiansen

308/95 Metaforer i Fysikken
af: Marianne Wilken Bjergregard, Frederik Voetmann Christiansen, Jørn Skov Hansen, Klaus Dahl Jensen, Ole Schmidt
Vejledere: Peder Voetmann Christiansen og Petr Viscor

309/95 Tiden og Tanken
En undersøgelse af begrebsverdenen Matematik udført ved hjælp af en analogi med tid
af: Anita Stark og Randi Petersen
Vejleder: Bernhilm Booss-Bavnbek

310/96 Kursusmateriale til "Lineære strukturer fra algebra og analyse" (El)
af: Mogens Brun Heefelt

311/96 2nd Annual Report from the project
LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM
by: Håleine Connor-Lajanbe, Bernd Kuemmel, Stefan Krüger Nielsen, Bent Sørensen

312/96 Grassmannian and Chiral Anomaly
by: B. Booss-Bavnbek, K.P. Wojciechowski

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The Logical Function of Idealism in Peirce’s Philosophy of Nature
By: Helmut Pape, University of Hannover

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By: Johnny T. Ottesen

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af: Gunhild Hune og Karina Goyle
Vejledere: Peder Voetmann Christiansen og Bruno Ingemann

316/96 Plasmascillation i natriumklynger
Specialrapport af: Peter Neibom, Mikko Østergård
Vejledere: Jeppe Dyre & Jørn Borggreen

317/96 Punkter og symplektiske algoritmer
af: Ulla Rasmussen
Vejleder: Anders Nøddsen

318/96 Modelling the Respiratory System
by: Tine Guldager Christiansen, Claus Draby
Supervisors: Viggo Andreasen, Michael Danielsen

319/96 Externality Estimation of Greenhouse Warming Impacts
by: Bent Sørensen

320/96 Grassmannian and Boundary Contribution to the -Determinant
by: K.P. Wojciechowski et al.

321/96 Modelkompetencer – udvikling og afprøvning af et begrebsapparat
Specialrapport af: Nina Skov Hansen, Christine Iversen, Kristin Troels-Smith
Vejleder: Morten Blomhøj

322/96 OPGAVESAMLING
Bredde-Kursus i Fysik 1976 – 1996

323/96 Structure and Dynamics of Symmetric Diblock Copolymers
PhD Thesis
by: Christine Maria Papadakis

324/96 Non-linearity of Baroreceptor Nerves
by: Johnny T. Ottesen

325/96 Retorik eller realitet?
Anvendelser af matematik i det danske Gymnasiums matematikundervisning i perioden 1903 – 88
Specialrapport af Helle Pilemann
Vejleder: Mogens Niss

326/96 Bevisteori
Eksemplificeret ved Gentzens bevis for konsistensen af teorien om de naturlige tal
af: Gitte Andersen, Lise Mariane Jeppesen, Klaus Frovin Jørgensen, Ivar Peter Zeck
Vejledere: Bernhilm Booss-Bavnbek og Stig Andur Pedersen

327/96 NON-LINEAR MODELLING OF INTEGRATED ENERGY SUPPLY AND DEMAND MATCHING SYSTEMS
by: Bent Sørensen

328/96 Calculating Fuel Transport Emissions
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by: Viggo Andreason, Juan Lin and Simon Levin

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by: Bent Sørensen

331/96 Viskese fingre
Specialerapport af:
Vibeke Orlin og Christina Specht
Vejledere: Jacob M. Jacobsen og Jesper Larsen

332/97 ANOMAL SWELLING OF LIPOIDE DOBBELTLAG
Specialerapport af:
Stine Sofia Korremann
Vejleder: Dorthe Posselt

333/97 Biodiversity Matters
an extension of methods found in the literature on monetisation of biodiversity
by: Bernd Kuenelm

334/97 LIFE-CYCLE ANALYSIS OF THE TOTAL DANISH ENERGY SYSTEM
by: Bernd Kuenelm and Bent Sørensen

335/97 Dynamics of Amorphous Solids and Viscous Liquids
by: Jeppe C. Dyre

336/97 PROBLEM-ORIENTATED GROUP PROJECT WORK AT ROSKILDE UNIVERSITY
by: Kathrine Legge

337/97 Verdensbankens globale befolkningsprognose - et projekt om matematisk modellering
af: Jørn Chr. Bendtsen, Kurt Jensen, Per Pauli Petersen
Vejleder: Jørgen Larsen

338/97 Kvantiiserings af nanolederes elektriske ledningsævne
Første modul fysikprojekt
af: Søren Dam, Esben Danielsen, Martin Niss, Esben Friis Pedersen, Frederik Resen Steenstrup
Vejleder: Tage Christensen

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by: Wolfgang Coy

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by: Carsten Lunde Petersen

341/97 Two chapters on the teaching, learning and assessment of geometry
by: Mogens Niss

342/97 LONG-TERM SCENARIOS FOR GLOBAL ENERGY DEMAND AND SUPPLY
A global clean fossil scenario discussion paper prepared by Bernd Kuenelm
Project leader: Bent Sørensen

343/97 IMPORT/EXPORT-POLITIK SOM REDSKAB TIL OPTIMERET UNNYTELSE AF EL PRODUCERET PÅ VE-ANLEG
af: Peter Melbon, Torben Svendsen, Bent Sørensen

344/97 Puzzles and Siegel disks
by Carsten Lunde Petersen

345/98 Modeling the Arterial System with Reference to an Anesthesia Simulator
Ph.D. Thesis
by: Mette Sofie Olofsen

346/98 Klyngedannelse i en huketode-forstevningsproces
af: Sebastian Horst
Vejledere: Jørn Borggren, NBI, Niels Boye Olsen

347/98 Verificering af Matematiske Modelle - en analyse af Den Danske Eulerske Model
af: Jonas Blomqvist, Tom Pedersen, Karen Timmermann, Lisbet Ehlerschuler
Vejleder: Bernhelm Booss-Bavnbek

348/98 Case study of the environmental permission procedure and the environmental impact assessment for power plants in Denmark
by: Stefan Krüger Nielsen
Project leader: Bent Sørensen

349/98 Tre rapporter fra PAGMAT - et projekt om tal og faglig matematik i arbejdsmarkedssuddannelserne
af: Lena Lindenskov og Tine Wedege

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351/98 Aspects of the Nature and State of Research in Mathematics Education
by: Mogens Niss
The Herman–Swiatec Theorem with applications
by: Carsten Lunde Petersen

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Specialerapport af: Per Gregersen og Tomas Højgaard Jensen
Vejleder: Morten Blomhøj

A GLOBAL RENEWABLE ENERGY SCENARIO
by: Bent Sørensen and Peter Meibom

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by: Carsten Lunde Petersen and Gustav Ryd

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Analyse af en matematisk model til konstruktion af terrenmodeller
Modelpjekt af: Thomas Frommelt, Hans Ravnkjær Larsen og Arnold Skimminge
Vejleder: Johnny Ottesen

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En historisk analyse af arbejdet med Cayley problem fra 1870 til 1918
Vejleder: Jesper Larsen

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Ph.D. Thesis by: Michael Danielsen

Lono-Term Scenarios For Global Energy Demand and Supply Four Global Greenhouse Mitigation Scenarios
by: Bent Sørensen

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En Meta-projekterapport af: Martin Kiss.
Bo-Jakobsen & Tune Bjarke Bonne
Vejleder: Peder Voetsmann Christiansen

Symplectic Functional Analysis and Spectral Invariants
by: Bernhelm Booss-Bavnbek. Kenro Furutani

Er matematik en naturvidenskab? – en udsætning af diskussionen
En videnskabefagsprojekt-rapport af Martin Kiss
Vejleder: Mogens Nørgaard Olesen