the Semiotic Flora of Elementary particles



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TEKSTER fra



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Abstract

This paper refers (but adds nothing) to the standard model of elementary particles, but present many of these particles in a "botanical" way, like the flowers in a Flora. The vacuum-background for the particles is treated with special emphasis on the zero-point-energy and its measurable effect — the Casimir effect. The special importance of the number 3 in the standard model leads to the idea that classification may be based on C.S. Peirce's *triadic* philosophy of signs — his *Semiotic*. A slightly abbreviated danish version of this article will appear in the collection: Thellefsen and Dinesen (ed.) *Semiotiske Undersøgelser*, Gyldendal, 2003.

Thanks are due to Bent C. Jørgensen for suggesting the botanical metaphor and to Edwina Taborsky for inspiring applications of Peirce's semiotic to physics.

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by Peder Voetmann Christiansen.

Introduction

Most natural sciences start out with a deictic ontology¹, a view that builds on the distinguishability of objects through nomenclature and placing in a system of classification. Thus, a natural science like biology builds on a natural history, like botany that through the classification of Linné allows the naming of plants using a well defined system of indexing — a Flora. The physics of elementary particles is long past the state of natural history by the use of a strong, but heavy mathematical apparatus in Quantum Field Theory and group-representations. As the particles by and by have become as numerous as flowers we can still use a "Flora" for naming and schematically surveying them. A suitable system for this can be found in Peirce's semiotic. This makes it possible to find a shorter way through the mathematical jungle, and certain regularities that still appear enigmatic in the mathematical theory, seem more understandable in the semiotic perspective.

1. the Wild Vacuum.

the physical concept of a particle — a point with mass — is, semiotically speaking, an *icon* — a sign whose object is potential or virtual. The particle as the physical object the icon refers to has definite properties, but not necessarily existence. A virtual particle is just a possibility for excitation of the physical vacuum — the empty space. That space is empty does not mean that it is without properties. It has three types of properties, viz. optical, topological, and metrical properties. The optical properties *)

entail that space has three dimensions and is seen as delimited by a heavenly sphere which has no physical existence. Two parallel lines (light rays) are seen as in the painter's perspective (CP 6.26) intersecting each other in two diametrically opposite points and all possible points of infinity make up "a line in the infinite" i.e. a great circle on the heavenly sphere, called the horizon.***). The topological properties are described by Peirce with four integers, the so called Listing numbers¹ chorisis, cyclosis, periphraxis, and immensity that characterize every three-dimensional object: Chorisis is the number of separate pieces that make up the object. Cyclosis is the number of through-going holes or singularities with axial symmetry (like vortices). Periphraxis is the number of internal, three-dimensional holes, and Immensity is a number that

Peirce uses the name optic for the discipline that is now called projective Geometry. Topology he calls topic. Peirce claims that optic and topic should precede metric.

Every plane bundle of parallel directions of view has a horizon, and all horizons together make up the heavenly sphere.

is only different from zero for an unlimited body. Looking at the whole universe it will have chorisis and immensity equal to one, while its cyclosis and periphraxis are unknown quantities reflecting singularities in the *metric* of space. The field equations of General Relativity that combine the metrical properties with the field of gravitation show that there are possible singularities corresponding to both types: *Cosmic Strings* add to the *Cyclosis* of space and *Black Holes* add to its *Periphraxis*. How many there are of such objects in the visible universe is not known, but observations indicate the both types exist.

Within the normally accessible scales of length and energy the physical vacuum appears completely without structure. It is, though, not without properties, but hides itself under three fundamental constants of nature, viz:

1: $c=3\cdot10^8$ m/s; the velocity of light in vacuum.

- 2: $\hbar = h/2\pi = 10^{-34}$ J·s Dirac's quantum of action. (h is Planck's constant).
- 3: $G=6.67\cdot10^{-13} \text{ N}\cdot\text{m}^2/\text{kg}^2 \text{ Newton's constant of gravitation.}$

Exåressed as here in normal (SI) units the numerical values of these constants are either very big or very small, but that just means that the SI-units (length in meters (m), time in seconds (s), and mass in kilograms (k)) are "human measures", far away from the world of elementary particles. However, it is possible to choose units of length, time, and mass, such that the three constants of nature, mentioned above, all get the value of unity in these new units, the so called Planck-units.

the Planck-length is then:
$$L_p = \sqrt{(-C/C^3)} = 4 \cdot 10^{-36} \text{ m}$$
 the Planck-time is: $t_p = L_p/c = 10^{-44} \text{ s}$, and the Planck-mass is $M_p = \sqrt{(-C/C)} = 5 \cdot 10^{-7} \text{ kg}$.

A natural starting point for pictures of elementary particles is then a sphere with radius one Planck-length and mass one Planck-mass. Compared to ordinary elementary particles (like electrons) the Planck-particle is of very small extension, but very heavy (ca 0.5 mg).

The force of gravity on the surface of such a particle will be so strong, that the particle "swallows itself" and becomes a mini-black hole. This has never been observed, and will probably never be, since the Planck-energy $M_P \cdot c^2 = 10^{18}$ GeV is far beyond the range of even the largest accelerators. Perhaps there has been many of them when the universe was only one Planck-time old, but as "mini-black-holes" quickly evaporate by a process called Hawking-radiation, they have all disappeared long ago. If we could view the physical vacuum through a microscope with a resolution of one Planck-length we would likely see that space on these scales is not without structure, but has both *cyclosis* (from superstrings) and *periphraxis* (from mini-black-holes). Topology (and hence also metric) is *chaotic* on the Planck-scale, both in space and time.

In the holistic "New Age Philosophy's" critique of physical reductionism (as expressed, e.g. by David Bohm) one often sees the assertion that the physical vacuum contains infinite amounts of energy².

Even the smallest volume, like a cubic millimeter should, according to this conception, contain enough of energy to sustain the whole world for many years*)

We shall see how such an idea can arise from a — basically correct — application of physical principles and why it is, despite of this, altogether wrong.

Let us consider a small part of space delimited by two parallel metal plates separated by a distance L. Between such plates there can be a series of electromagnetic oscillation-modes that are standing waves whose half wavelength is a whole fraction of the distance L. Like for an oscillating string or a closed organ-pipe we can distinguish between a ground-tone with the wavelength 2L and an infinite series of overtones, where the nth overtone has the wavelength 2L/(n+1). The ground-tone has n=0 and the overtones have n from 1 to ∞ . The frequency of oscillation of each such mode is found by dividing the wavelength up into the velocity of light c. Thus, the ground-tone has the frequency v=c/2L. Every mode can be considered as a harmonic oscillator, and according to Quantum Mechanics it can only have the discrete energy-values

$$E_m = (m + \frac{1}{2})h\nu$$

where m is a positive integer or zero. We see that the energy is *quantized* with the quantum hv.

Such field-quanta can be regarded as particles, and when it, like here, are quanta of an electromagnetic "light-field" we call the particles photons. Likewise, we speak of phonons when it is a sound-field like the oscillations on a string that are quantized. (c should then be the velocity of sound). If the nth mode is excited to the mth level we say that there are m photons (phonons) in the state n. Thus, the ground state of vacuum is the one where m=0 for all the states. From the above formula for the energy-values we see that the energy of each mode in its ground-state is not zero, but carries the zero-point-energy hv/2. As there are infinitely many modes in the cavity, the total zero-point-energy is infinite. This, however, is a purely formal consideration that does not consider the semantic purport in the concept of energy, namely ability to perform work. If an oscillator is excited to level m it can perform work by delivering a quantum hv to the surroundings whereby the oscillator itself makes a transition to level m-1. This, however, is impossible, if the oscillator is in the ground-state m=0, because there are no lower levels. So, the infinite vacuum-energy turns out to be a fiction, and a "perpetuum mobile of the third kind" is an impossibility like all other kinds of perpetuum mobile.

One should not, however, entirely disregard the zero-point-energy as being unreal, because it shows itself in other ways than the ability to perform work, namely by the *pressure* it exerts on the surroundings. The so called *Casimir-effect* is an experimental demonstration of this pressure.³.

The zero-point-energy has physical actions and is therefore, according to Peirce's pragmatic criterion of meaning, real. This assertion leads naturally to the question "From where did it come?" This is a mischievous question that leads to the mischievous answer: "We

^{*)} A hypothetical engine that can extract the vacuum energy is called a "Perpetuum Mobile of the third kind".

made it ourselves!" There is, namely, a concept-logical connection between *localizing* a particle (to ensure that it is situated in a certain, limited region of space) and to *transfer* energy to it. This connection is expressed in *Heisenberg's uncertainty relation*

$\Delta x \cdot \Delta p > h$

where Δx is the uncertainty of spatial location and Δp the uncertainty of momentum (mass times velocity). If we try to localize the particle strongly, i.e. make Δx very small, then Δp will be, correspondingly, greater. The particle will not rest quietly when we keep it in a narrow cage, and therefore we have to perform work by narrowing its limits — a work that adds to the kinetic energy of the particle. This argument is also valid when there is no particle. For example there are *no photons* when all the oscillatory modes are in their ground-state. The zero-point-energy of the photon-field's ground state is, according to the previous derivation hc/4L, i.e. it increases when we diminish L and the increase comes from the work we do by the compression.

3. The Vacuum Press

Let us perform a thought-experiment wherein we compress the vacuum by means of the apparatus shown in figure 1. The cavity-length L is here the distance between the piston and the bottom of the box.

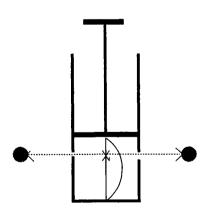


Figure 1 the vacuum press

When we press down the piston we change the wavelength of the ground-mode and thereby increase the zero-point-energy. For sufficiently small values of L the zero-point-energy will be greater than the relativistic rest-energy mc^2 of a particle of mass m. This, however, is not sufficient to create the particle, because, if it emerges within the box it wil have a "localization-kinetic-energy" according to Heisenberg's uncertainty relation, and this energy increases faster (inversely proportional to L^2) when L decreases and therefore there will never be enough of zero-point-energy in the photon-field to create a particle with mass. If there are holes in the box potentially existing particles may escape and then have no localization-energy. There will then be enough of energy in the photon-field to create an electron when L becomes smaller than the *Compton wavelength* of the electron $\lambda_C = \frac{1}{L^2} / mc \approx 3 \cdot 10^{-13} \text{ m.}$, where $m \approx 9 \cdot 10^{-31} \text{ kg}$ is the mass of the electron.

When we try to press the piston to the bottom various particles will sprout from the holes like seeds of an orange when L passes below their respective Compton wavelengths.

The Compton wavelength puts a natural limit to how narrowly a particle may be localized. If we think of the particle as a small hard sphere, we can think of the Compton wavelength as the radius of the sphere. The radius of the electron is then ca 1000 times as small as the radius of a hydrogen atom and ca 2000 times as big as the radius of the atomic nucleus (the proton). In the Planck system of units (where $\hbar=1$ and c=1) the radius of the particle is simply the reciprocal of its mass. A particle of one Planck-mass (a mini-black-hole) will have radius one Planck-length — the smallest distance that can be connected with classical conceptions of space-time.

It may seem contradictory when we claim that the zero-point-energy cannot perform work but is yet able to produce particles. The explanation is, again, that the holes in the box, that allow the particles to escape also makes it possible for the zero-point-oscillation to yield, i.e. decrese its frequency and thereby its energy. Still, we maintain that the work comes from the compression of the piston and the zero-point mode is only an intermediate storage-medium for the energy.

The most efficient method of compressing space consists in providing two massive particles with a high velocity in an *accelerator* and then arranging a *collision* between these particles. In CERN's (newly abolished) LEP (Large- Electron-Positron-Collider) the collision-energies reached about 100 GeV, and that is not quite sufficient to produce the currently most

interesting particles (as the Higgs-boson)*). A new accelerator LHC (Large Hadron Collider) will, within a few years yield significantly higher collision energy by using hadrons (like protons) that are about 200 times more massive than electrons (and thereby also more compressed beforehand).

4. Renormalisation — just smart, or a bit too smart?

The previous discussion of the vacuum press and the Casimir effect (the pressure on the piston) is incomplete, because it only takes into account the ground-mode of the photon-field. Naturally we must also regard the infinity of overtones, but that leads to the problem that the total zero-point-energy (and thereby also the pressure) becomes infinite. The zero-point energy of the *n*th mode is:

$$E_n = \frac{1}{2}hv_n = hc \cdot (n+1)/4L$$

It is therefore clear that the complete zero-point-energy includes a factor that is the sum of all positive integers from 1 to ∞ , and this factor must, for a normal consideration, be infinitely great This we could, perhaps, learn to accept, for, as we have seen, the zero-point-energy cannot perform work, so we could disregard it as being non-energy. But it's not so easy. Every single mode gives rise to an upward-directed force on the piston that is $K_n = -\frac{dE_n}{dL}$ and the sum of all thes forces will contain the same infinite factor, such that the pressure (measurable) becomes infinite, which it clearly isn't in reality.

Casimir's calculations, as well as Spaarnay's experiment even show that the pressure is negative, i.e. the force on the piston is directed downwards. We are, therefore, forced to "explain away" or renormalize this infinity. A way to do this is by using a mathematical technique called analytic continuation. A very important function in Mathematics is Riemann's xetafunction $\zeta(z)$ that is defined for complex numbers z=x+iy in the following way:

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^{z}}$$

This definition is entirely clear for all z whose real part, x, is greater than 1, because then the series converges to a finite value. However, the function has a unique analytical continuation to the whole complex plane, including negative real values of z, where the series is divergent. Formally, we can put z=-1, whereby the infinite sum becomes the previously mentioned sum of all positive integers, and we can assign it a value given by the analytical continuation of the zetafunction to z=-1. In this way we get at the renormalized value $\zeta(-1)$

^{*)} The most interesting particles are those predicted theoretically but not yet found with certainty experimentally.

=-1/12⁴, i.e. not only have we transmutated the infinite factor to something finite we have even given it the correct sign! In a similar way we can "prove" other absurdities, e.g. that ∞=-1/2, for if we put z=0 in the above formula we get a sum of infinitely many 1s, i,e. ∞ , and the analytical continuation $\zeta(0)$ has the value -1/2.

SUch a mathematical renormalization-technique appears "a bit too smart" because it may lead to screaming absurdities, but the method should not be entirely rejected, as it is, in fact, applied and often leads to results that are completely correct. An example is the so called factorial function $n!=1\cdot 2\cdot 2\cdots n$, i.e. the number of permutations of n objects, that is defined for positive integers n. An analytical continuation employing the so called Gammafunction allows us to define $(-1/2! = \sqrt{\pi}, a \text{ result that no mathematician or physicist will cast in doubt})$. We shall not "throw out the baby with the bathing water" by prohibiting renormalization by analytical continuation, but still, I want to go through a physical argument of argumentation reflecting Casimir's calculation and, hopefylly, making it a little less suspect. I shall give a short outline of the argument here, while the details can be found in the appendix.

Hitherto, we have only considered the electromagnetic modes in the cavity below the piston in figure 1. This infinity of modes, all have wavelengths less than 2L. However, they exist also above the piston, where each of them gives rise to a downwardly directed force that precisely cancels the *upwardly* directed force from the corresponding mode below the piston. In this way we remove the infinity, so what is left?

There are all the modes whose wavelength is greater than 2L, and these modes are only found above the piston. By adding the forces from these modes one finds that the resulting force on the piston is downwardly directed with the finite value

 K_{total} =- $hc/8L^2$ Curiously enough the previous "bit too smart" renormalization argument gives almost the same, viz. $K_{total} = -hc/48L^2$, i.e. the correct sign and only a factor 6 smaller than the right numerical value.

The Casimir "pressure" is thus a "suction" (because K_{total} is negative) but it can only be felt when L is very small (of atomic size). If we accept that the vacuum is empty, so that the energy density is zero in the external vacuum above the piston, we can interpret the suction of the Casimir effect saying that the vacuum below the piston has negative energy density. It thus corresponds to so called exotic matter that is required to make worm holes in space-time to be used by time-travellers⁵.

By combination with the previous "result" ∞ =-1/2 we have thus "proved" that ∞ != $\sqrt{\pi}$, that the product of all positive integers from 1 to∞ has the finite value $\sqrt{\pi}(1.77)$, but then we seem to have renormalized ourselves entirely out of reality - all too smart!.

If the vacuum press in figure 1 shall be able to squeeze particles out of vacuum, the pressure must be positive. The calculation above, giving a negative pressure can therefore only be valid for distances L larger than the Compton wavelengths of the virtual particles.

The reason why I've given a relatively lengthy discussion of vacuum is to avoid that the following enumeration of particles and their properties should be regarded as reductionistic: We need a holistic conception: The whole is more than the sum of its parts — the elementary particles don't have properties that are independent of their context. A particle is a *field-quantum* and interacts with virtual fields in vacuum. We cannot calculate the properties of one single free particle, e.g. its mass, for its properties reflect the wild vacuum, although it is not quite as wild as certain "quantum holists" claim (infinite energy density, etc.)

5. The mysterious number 3.

In a popular "nature-historic" account of the elementary particles⁶ the author seems puzzled about "two mysterious 3-numbers" that have emerged in later years in particle-physics:

- 1) There are three generations of elementary particles, and
- 2) The heavy nuclear particles baryons (as the proton and the neutron) consist of three quarks.

We can easily expand the list over the fundamental roles of the number 3: Space has three dimensions and three types of properties (chapter 1). There are three types of units (length, mass, and time) and three fundamental constants og nature (\uparrow_1 , c, and G). The quarks have three "colours" (red, green, and blue) and strange electric charges that are not built of the electron's charge e as the quantum of charge, but of 1/3e.

Previously one got used to dichotomies, or two-partitions: There are Fermions (as electrons and quarks with half-integer spin, usually 1/2) and bosons with integer-spin (0 for π mesons, 1 for photons, and 2 for gravitons). There are particles and antiparticles, and there are positively and negatively charged particles with even or odd parity. These dichotomies can be understood from the concept of mirroring. The mirror-image of a particle with negative charge and even parity will be an antiparticle with positive charge and odd parity, but the newly discovered trichotomies are not connected to mirroring and therefore appear strange. I shall not attempt to seek a mathematical justification of the trichotomies but instead take departure from a philosophy that is built on trichotomies and thereby make the "mysterious" number 3s appear as something natural and inevitable, namely Peirce's Semiotic.

6. Semiotics of the particle-concept

A previous article by the author⁷ explained how Peirce's triadic doctrine of categories implies that signs can be divided in classes that can be put up in triangular schemes.

On the most elementary level of description a sign is something that mediates between an *object* and an *interpretant*, schematically I---O where the line stands for the sign-vehicle— a physical signal or link between O and I. By using Peirce's categories we can distinguish between three types of links or mediating processes:

- 1: A potential link that only exists as a possibility. The sign is then called an icon.
- 2: An actual link from an existing object is an index.
- 3: A general link referring to a general class of objects defines a symbol.

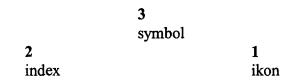


Figure 2 Sign classes for the 1-link relation

On the next level of description the sign-vehicle, or the Representamen R is objectified and gets two links to O and I after the scheme I---R---O. These two links represent the elementary quantum processes Preparation (R---O) and detection (I---R) and each of them can be classified by the three categories, though only such that the category of the detection-link cannot exceed that of the preparation-link. In this way we arrive at the six quantum-semiotic sign classes shown in figure 3:

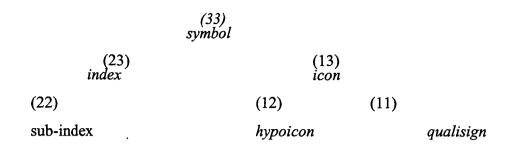


Figure 3 The 6 quantum-semiotic sign classes for the two-link relationI---R7--O.

As the interpretant I is to be regarded as a sign-vehicle (representamen for a new interpretant J, we are lead on to consider a three-link relation J---I---R---O consisting of the two two-link relations1:I---R---O, and 2: J---I---R, where we see that the I of the first relation appears as representamen in the second relation, and R in the first relation appears as object in the second relation. When we, as before assign the links categories that cannot increase when we move left in the diagram we find Peirce's 10 classes of signs (CP2.256) arranged in a *Pythagorean Tetraktys* (figure 4)

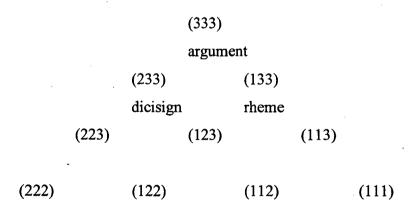


figure 4 the 10 sign classes for the three-link-relation. The 6 classes from figure 3 are the ones, that have 1 in the first place: (111)-(133).

We shall mainly employ the 6 quantum-semiotic sign-classes on figure 3. A particle is a quantum of a field and therefore has two "handles" or links corresponding to the two fundamental quantum processes preparation and detection.

There is a clear line of development in the scheme of figure 3, starting with the qualisign 11 and continuing with successive actualizations of potential links (1 to 2) and generalizations of actual links (2 to 3) without skipping any intermediate stages on the way to the symbol 33. I shall briefly sketch a "nature-historic" interpretation of the sign classes in the right order with hints to particle physics.

11-the qualisign is the empty space, only containing *virtual* fields and particles, like the electromagnetic modes, that can be occupied by photons, but otherwise are in the ground state.

12-the hypoicon can be thought of as a *superstring* — en string i vacuum⁸, having certain field-and particle-properties, reflecting how the string is wound up in other dimensions than the one it is stretching in, The theory of superstrings operates with 8 hidden spatial dimensions and is *supersymmetrical* with regard to Fermion- or Boson-properties of the strings.

22- sub-index: the string closes upon itself to a ring with area.

13-icon: the ring moves in space and thereby establishes a three-dimensional container-like region. Thus appears the three-dimensional continuum of quality that is the premiss for a sign to refer to an object by likeness as an icon does. If the ring is creased thre will be bumps in the container, corresponding to a non-euclidean metric that reflects gravitation.

23-index: The iconic particle may collide with another and produce a lot of unspecified particles through the action of the vacuum press. This is *indexical* semiosis, not yet generalized to lawlike behavior.

33-symbol: As we learn about the properties of particles we become able to predict results of collisions between known particles and interpret their traces. On this stage both preparation and detection are generalized and we have reached the symbolic level of description.

This account of the evolution of signs is inspired bu Edwina Taborsky 9)

7. Contents of the botany-box.

When you go out to botanize, identifying the names of flowers in a *Flora* there are certain concepts you need in your head and certain tools to keep in your box, besides the flora and the lunch-packet, e.g. a magnifying glass for counting petals and stamens. Similarly, the elementary particles are classified by their *internal properties* and their *interactions* with other particles and fields.

The internal properties are spin, mass, and charge.

The interactions are *strong*, *electromagnetic*, *weak*, *and gravitational*, here listed after decreasing strength.

All particles, also the massless, as the photon, interact gravitationally. This universal force is described in Einstein's General Theory of Relativity, but because it is so weak and hitherto has evaded a quantum mechanical treatment it has mostly been ignored by elementary particle- (or high energy-) physics. It is known, though, that the field quantum of gravitation, the *graviton* is massles and has spin 2.

All charged particles (including quarks and "heavy leptons") interact electromagnetically. The strength of this interaction is determined by the electron's charge e which was long believed to be the universal quantum of charge, until it was discovered that the charges of quarks are 2/3 or -1/3 e.

All hadrons (quarks) and baryons (nuclear particles consisting of three quarks) interact strongly and weakly. The strong interaction was earlier described as mediated by medium-heavy bosons, called mesons (π and K), but now we have learned that mesons too are compounds (by a quark and an antiquark). So what is left of the strong interaction is the force between quarks, whose field-quanta are called gluons (8 kinds with spin 1)

Quarks and leptons interact with each other through the *weak interaction* whose mediating field quanta are the *intermediate vector-bosons* (3 kinds with spin 1). The uncharged light leptons, the *neutrinos* only interact weakly (and gravitationally).

After this short account of *interactions* now follows a survey of the most important *internal properties* of the particles:

The (rest-) mass m and the Energy E are connected through Einstein's relation (hvile) massen m og energien E er forbundne med Einsteins relation

$$E = mc^2$$

In the Planck system of units, where c=1 they are identical. The most commonly used unit for this quantity is the electronvolt eV, that is the energy an electron gets by accelerating through a voltage-drop of 1 volt (1 $eV=1.6\cdot10^{-19}$ J). Besides, we have the multiple units keV (kilo=1000), MeV (Mega = 10^6) and GeV (Giga = 10^9). The electron's mass is ca 500 keV = $\frac{1}{2}$ MeV, corresponding to m= $9.1\cdot10^{-31}$ kg. The photon and the graviton are massless and can therefore only move with the velocity of light. Earlier it was believed that also the neutrinos are massless (they were observed almost simultaneously with the light from the supernova in the big Magellanic Cloud in 1987), but recent experiments in Japan have shown that at least one of the three neutrinos has a mass about 10eV about one hundredthousandth of the electron-mass. As a rule of thumb one may assume that the greater mass, the later is the discovery of the particle, because the accessible accelerator-energies have increased gradually from the keV to the GeV-range.

Spin is an internal angular momentum (length times momentum) and is quantized in units of h. Earlier it was believed that particles are small hard spheres and that the spin expressed the sphere's rotation about its own axis, but it has turned out that only integral spin-values can be interpreted in this way. Now we will say that spin is concerned with how the symbolic representation of the field is changed by a rotation of the coordinate-system used in the description. Spin can be integral or half-integral as the particles are, respectively bosons or fermions. Fermions are exclusive; there can only be one fermion of a given type in a given quantum-state. Bosons, on the contrary, are "social". They are prone to go together in the same

state and form a *condensate* as known from superfluid systems and laser-light (a condensate of photons). The exclusivity of fermions makes it tempting to regard them as the most evident fundamental bricks in an atomic description of matter. Boson-condensates are more apt for describing classical fields. Every fermion has an anti-particle that is different from itself, whereas a boson's anti-particle often (not always) is the same particle. The theory of superstrings (ref. 8) — the newest candidate for a unifying theory of particles and fields is supersymmetrical (hence the name), i.e. it postulates that every boson has a fermion-partner, and vice versa. The photon's supersymmetric partner is called the *photino* it has not yet been seen.

Spin ½ particles (like quarks and leptons can have the spin pointing either forwards or backwards in the direction of movement. This means that they possess parity or helicity, i.e. they are different from their mirror images. Neutrinos are lefthanded (spin against movement), and antineutrinos are righthanded.

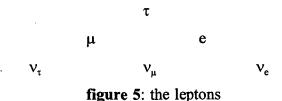
An important theoretical result, the CPT-theorem establishes that the theory must be invariant for the combination of three mirror-operations C, P, and T, i.e. changes of sign for, rewspectively charge (C), parity (P), and the direction of time (T).

8. Semiotic classification of elementary particles.

We must distinguish between proper elementary particles and compound particles that are built of elementary particles. Elementary particles are, for historical reasons, divided in three generations, 1, 2, and 3, whose order corresponds to the order of their discovery, which is connected with the circumstance that the mass (and thereby the necessary accelerator-energy for the production of then) increases from generation 1 to generation 3.

Each generation consists of two leptons and two quarks. So, altogether we have 6 leptons and 6 quarks, which makes it tempting to place them in the semiotic classification with 6 sign-classes in figure 3. All these 12 particles are spin ½ fermions, each having their own anti-particle. So, there are really 24 different particles, but in the following we shall disregard

the anti-particles. I cannot, at present, give a proper semiotic reason for the placing of every single particle in the scheme, so I'll just use a heuristic rule that uses the evolutionary sequence of the 6 sign-classes and assume that this order reflects an increasing mass of the particles. Thus, we get the schematic placement of the 6 leptons:



Generation 1 consists of the electron e and its neutrino v_e .

Generation 2 consists of the muon μ og its neutrino ν_{μ} .

Generation 3 consists of the tauon τ and its neutrino v_{τ}

The three "ons" all have the same (negative) charge as the electron -e, while the neutrinos are uncharged. As mentioned it is now shown that at least one of the neutrinos has a restmass about $10 \ eV$, which doesn't give a clue for ordering after increasing mass, so I've just assumed that their mass increases from generation 1 to generation 3 and ordered them accordingly in the lowest row of the scheme.

considering now the *quarks* that are charged spin $\frac{1}{2}$ fermions, we have the following distribution on the generations: For each of the 6 quarks is noted its mass, measured in MeV. Their charges are given in the left column:

	generation 1	generation 2	generation 3
+2/3 <i>e</i> :	u (up) (5)	c (charm) (350) t (top)	(>80)
-1/3e:	d (down) (9)	s (strange)(160) b (botto	om) (4800)

Figure 6 the three generations of quarks.

We note that for both leptons and quarks the charge-difference between the two particles in a generation is always one electron-charge.

The placement of quarks in the semiotic scheme then looks like this:

b c s d u figure 7: the quarks.

The top-quark t is here placed highest in the scheme, because it is the latest discovered, although its mass is not known very precisely.

As earlier mentioned the quarks participate, both in the strong, the electromagnetic, and the weak interactions. Weak interactions are mediated by a *vector-field* analogous to the electromagnetic, whose quanta are spin-1-bosons, as the photon. However, they are not massless as the photon, but, on the contrary, very heavy (90-100 GeV), which has been difficult to understand, but is now explained by assuming that they interact with an uncharged, spin 0 field (a condensate) of so called *Higgs-bosons*, that are not yet seen with certainty in experiments.

The weak interactions make it possible that the heavy quarks may decay to the lighter, e.g. the process

d--->u+W will be possible, when W has a single negative charge -e like the electron. Correspondingly, we have the process

c--->d+W⁺ where the positive W⁺ is the anti-particle to W⁻. Finally we have the possibility s--->d+ Z_0 where Z_0 is uncharged, as s and d both have the charge -1/3 e. All these three vector-bosons are discovered at CERN. Because Z_0 is the heaviest of the three, the s-decay runs slower than the d- and c-decays.

9. Compound particles.

The greatest merit of the quark-theory is that it can explain the properties of the heavy-fermions — the baryons and the intermediately heavy bosons — the mesons. Earlier it was believed that mesons (especially the pions π^+ , π^- , and π^0) were elemintary quanta of the field that mediates the strong interactions between baryons, but now we know that all these particles are compounds:

A baryon (spin ½ or 3/2) consists of three quarks

A meson (spin 0 or 1) consists of a quark and an anti-quark.

Both quarks and anti-quarks have spin ½, so a meson may have spin 0 or spin 1.

Traditionally known strong interactions, mediated by π -mesons involve only generation 1 quarks u and d. For spin 0 mesons we then have the three possibilities (antiquarks are denoted with a stroke above the quark-symbol): $(u, \overline{d}) = \pi^+$, $(d, \overline{u}) = \pi^-$, and π^0 or $\eta = (d, \overline{d})^*$, i.e. two charged and two uncharged particles The diagrams show immediately that π^+ and π^- are each other's anti-particles, while π^0 (og η) are its own anti-particle.

Spin 1 mesons are very unstable and decay rapidly to spin 0 mesons. This family contains, besides π -mesons, also combinations that contain s quarks, namely the K-mesons (kaons)

 $K^+ = (u, \overline{s}), K^- = (s, \overline{u}), \text{ and } K^0 = (s, \overline{d}).$ It is seen that K^+ and K^- are each other's antiparticles, while K^0 is different from anti- K^0 .

For baryons it is valid, as mentioned, that each of them consists of three quarks and, thus, may have spin ½ or 3/2. For spin 3/2 baryons all combinations of the three lightast quarks (u,d, and s) are possible. We may then assign the three quarks the Peircean categories: u=1, d=2, and s=3 and apply the scheme of 3-link sign-relations (figure 4) to the following figure 8:

The two uncharged particles π_0 and η are both quantum mechanical mixtures (superpositions)

of (u, \overline{u}) and the heavier $(d\overline{d})$.

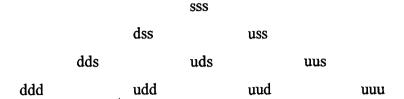


figure 8 the spin-3/2 baryon-decuplet.

This figure is called the spin 3/2 decuplet (because it contains 10 particles) is one of the quark-theory's great successes, because it predicts the existence of a hitherto unknown particle, the topmost in the scheme, (sss), called the Ω -particle. It carries a negative unit of charge, (as s has the charge -1/3) and is rather stable (long-living) because it only decays via the weak interaction with the heavy Z^0 particle.

Spin ½ baryons cannot contain three identical quarks. This rule is coming from Pauli's exclusion principle, that forbids two fermions to occupy the same quantum state. Using again figure 4 (or 8) as our starting point we have to omit the three corners of the triangle, so there are only 7 spin ½ baryons, of which the most important is the *proton* (uud) (charge +e) and the *neutron* (udd) (charge 0). Spin 3/2 baryons (except Ω) decay rapidly to spin ½ baryons via the strong interactions.

It was a great obstacle for the early quark-theory, that one never sees a single (free) quark. It seems that they only exist three or two at a time, confined in the prison of baryons or mesons. In order to explain why the 3 always were together it was invented to assign them a *colour* or colour-charge, r,g, or b (red,green, or blue). The ruke then is, that only "white" particles can appear as free, namely the baryons (rgb) or the mesons (colour+complementary colour, e.g. blue+yellow). In the modern theory of strong interactions, called quantum-chromodynamics (due to the colours) it is assumed that thew quarks attract each other with forces that are weak at short distances, but strong at large distances. These forces, that "glue" the quarks together in "white" bundles are mediated by field-quanta that are called *gluons*, yhat like photons are massless spin-1-particles. As a force between two quarks act between 3·3 colour-combinations, one should think there would be 9 different gluons, but it turns out that the

photon is hiding among these combinations, so there are only 8 gluons. With the high accelerator-energies that are available today, it is possible to tear quarks out from baryons and mesons. They are then seen as "jets", i.e. long stripes, consisting of quarks, anti-quarks, gluons and a lot of other particles, that are created when one with brutal force tear the quarks loose from their attraction.

10. Conclusion.

The particles that are here classified by means of semiotic schemes are all described in the so called *standard model* of elementary particles. we have looked at 63 particles(24 leptons and quarks with anti-particles, 10 spin 3/2 baryons, 7 spin ½ baryons, 4 spin 0 mesons, 6 spin 1 mesons, 3 vector bosons, 8 gluons, and 1 Higgs boson). The Higgs-boson is, as mentioned, not yet found with certainty, and that is regarded as a problem for the standard model, because it plays an important role for the understanding why certain particles (as vector-bosons) have mass. The theory indicates, that the Higgs-boson is related to the heavy b-quark and therefore only is produced at very high energies (over 100 GeV).

The semiotic approach is a *schematization*, not a physical theory, like the standard model, that has its own difficulties to fight against, notably the lack of supersymmetry. Superstring theory solves this problem and also includes gravitation, which the standard model has avoided. But, probably, there will still be some use for a "nature-historic" account, as the semiotic, that "steal around" the heavy mathematical apparatus, that the theories require.

Appendix

Casimir-renormalization

As we have seen, a cavity between two plates separated by a distance L may contain an infinity of standing waves whose wavelengths are all smaller than 2L. The zero point energy of these modes would give rise to an infinite positive (upwards directed) force on the upper plate if it wasn't for the fact that the same modes exist *above* the plate and each of the modes above provide a negative, downwards directed force that precisely cancels the force from below. There are still modes above the plate, not yet taken into account, namely all the modes with wavelengths greater than 2L. So, let us imagine a second plate with a distance L' from the first plate. The nth mode in the cavity between L and L' will have the wavelength

$$\lambda_n = \frac{2(L'-L)}{n}$$

the frequency of this mode is

$$v_n = \frac{c}{\lambda_n} = \frac{cn}{2(L'-L)}$$

and its zero-point energy is

$$E_n = \frac{1}{2} h v_n = \frac{hcn}{4(L'-L)}$$

which gives rise to the force

$$K_n = -\frac{\mathrm{d}E_n}{\mathrm{d}L} = -\frac{hcn}{4(L'-L)^2}$$

The force of the mode as a function of its wavelength λ is then

$$K(\lambda) = -\frac{hc}{2(L'-L)\lambda}$$

In order to sum all these forces we have to find the density of modes. First, the number of modes dn in the frequency-interval dv is

$$\frac{\mathrm{d}n}{\mathrm{d}v} = \frac{2(L'-L)}{c}$$

So, the number of modes per wavelength interval is

$$\left|\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right| = \frac{\mathrm{d}n}{\mathrm{d}\nu}\left|\frac{\mathrm{d}\nu}{\mathrm{d}\lambda}\right| = \frac{2(L'-L)}{\lambda^2}$$

The total force from all the modes with wavelengths greater than 2L can the be calculated as

$$K_{total} = \int_{L}^{L'} K(\lambda) \frac{\mathrm{d}n}{\mathrm{d}\lambda} \mathrm{d}\lambda$$

with the result

$$K_{total} = -\frac{hc}{8L^2} \left(1 - \frac{4L^2}{L^2}\right)$$

It is seen that the total force is downwards (negative) when L < L'/2, and in the limit, when the external cut-off, L' goes to infinity we get the earlier quoted renormalized result

$$K_{total} = -\frac{hc}{8L^2}$$

Notes and references.

(danish references are marked with a (D)).

- 1. Roberto Poli, Foreword, Axiomathes, vol. 12, p. 1-5, Kluwer, Dordrecht, 2001
 - 1. The New Elements of Mathematics by Charles S. Peirce vol. III, p. 112, ed. Carolyn Eisele, 1976.
- 2. Ken Wilber (ed.) the Holographic Paradigm and other Paradoxes interview eith D. Bohm, ch. 5, Shambala Publications, Boston, 1982
- This effect, predicted by the dutch physicist H.G.B. Casimir in 1948 was verified experimentally by M.J. Spaarnay in 1958.
- 4. Handbook of Mathematical Functions, ed. M. Abramowitz and I,A. Stegun, Dover, N.Y. 1965.
- 5. (D) See, e.g., the article by Michael Agermose Jensen in the journal of the Niels Bohr Institute, Copenhagen, Gamma, nr. 110 from june 1998.
- 6. (D) Jens Lyng Petersen: Elementarpartikler om felter, kvarker og kvanter. Gyldendals Intro-serie, 1991.
- 7. (D) Peder Voetmann Christiansen, Fysik og semiotik semiotikken i fysikken, *Anvendt semiotik*, red. Keld Gall Jørgensen, Gyldendal, 1997.
- 8. (D) Bent Sørensen: Superstrenge—en teori om alt og intet. Munksgaard, Nysyn, 1987.
- 9. Edwina Taborsky: Energy and Evolutionary semiosis. Sign Systems Studies 30.1, University of Tartu, 2002.

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