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HVAD ER MENINGEN MED MATEMATIKUNDERVISNINGEN ?

FIRE ARTIKLER

TEKSTER fra

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Mogens Niss: "Hvad er meningen med matematikundervisningen?" - Fire artikler

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Hæftet består af fire artikler: "Goals as a reflection of the needs of society", "Matematikkens rolle i ungdomsuddannelserne - almendannelse og/eller studieforberedelse?", "Nogle perspektiver for matematikundervisningen i de gymnasiale uddannelser i 199o" og "Considerations and experiences concerning integrated courses in mathematics and other subjects". Artiklerne behandler forskellige sider af spørgsmålene 'af hvilke årsager ønsker samfundet at der skal undervises i matematik i den offentlige (især den gymnasiale) skole?' og 'hvordan skal/bør/kan matematikundervisningens rolle og indretning for de 16-19-årige være?'

I N D H O L D

1. GOALS AS A REFLECTION OF THE NEEDS OF SOCIETY

Marts 198o side 1 - 42

2. MATEMATIKKENS ROLLE I UNGDOMSUDDANNELSERNE almendannelse og/eller studieforberedelse

Oktober 198o side 1 - 14

3. NOGLE PERSPEKTIVER FOR MATEMATIKUNDERVISNINGEN I DE GYMNASIALE UDDANNELSER I 1990

Marts 1979 side 1 - 12

4. CONSIDERATIONS AND EXPERIENCES CONCERNING INTEGRATED COURSES IN MATHEMATICS AND OTHER SUBJECTS

August 198o side 1 - 6

F O R O R D

De fire artikler, der i dette hæfte er samlet under titlen "Hvad er meningen med matematikundervisningen?", danner ikke nogen sammenarbejdet helhed, selv om de alle kredser om den problemstilling, titlen antyder. Der vil derfor kunne findes beslægtede betragtninger på tværs af artiklerne, hvoraf ingen er omarbejdet til lejligheden. Den ene af artiklerne (nr. 3) har været tilgængelig før, dels for Matematiklærerforeningens medlemmer i et særligt hæfte, dels i en lidt forkortet form i Normat (Nordisk matematisk Tidsskrift). To af artiklerne (nr.1 og nr. 4) bliver tilgængelig om kortere, henholdsvis længere tid; den første i Studies in Mathematics Education vol. II (Unesco), der forventes at foreligge omkring nytår 198o/81, den anden i proceedings fra ICME IV i Berkeley, der forventes at foreligge om nogle år. Artikel nr. 2 offentliggøres her for første gang og er ikke planlagt udgivet på anden måde.

Når jeg på trods af denne baggrund har valgt at samle artiklerne i en IMFUFA-tekst er det med det enkle formål at gøre dem samlet tilgængelige for interesserede, der ellers var henvist til at opsøge dem på andre og ofte besværlige måder, f.eks. ved at bede mig om "håndkopier".

Der kan siges at foreligge en vis arbejdsdeling mellem artiklerne. Den første,"Goals as a reflection of the needs of society", forsøger at levere en analyse af samfundets objektive (explicitte og implicitte) interesser i og overordnede hensigter med at give undervisning i matematik i de offentlige skoler. Analysen består af to dele, et generelt afsnit der tjener til at tilvejebringe nogle analysekategorier, og et historisk afsnit, hvor der tegnes et rids af udviklingen i samfundets hensigter og mål med matematikundervisningen. Også den anden artikel "Matematikkens rolle i ungdomsuddannelserne"er delt i to afsnit. I det første benyttes den foregående artikels begrebsapparat til at skitsere den historiske udvikling i hensigterne med matematikundervisningen i det danske gymnasium. Det andet afsnit indeholder træk af mit bud på hvad matematikundervisningens rolle i ungdomsuddannelsen skal være. Selv om den tredje artikel, "Nogle perspektiver for matematikundervisningen i de gymnasiale uddannelser i 199o", er mere end et år ældre end de andre, behandler den en videreførelse af tankegangen fra den foregående, derved at den undersøger nogle mere detaljere spørgsmål og aspekter ved matematikundervisningen under perspektiver af samme art som dem der omtales i artikel nr. 2. Endelig leverer den fjerde artikel nogle overvejelser over et enkelt af de spørgsmål, der berøres i artikel nr. 3: skal matematikundervisningen gives integreret med undervisningen i andre fag? Dette sker under titlen "Considerations and experiences concerning integrated courses in mathematics and other subjects".

Mogens Niss, november 198o

MOGENS NISS:

GOALS AS A REFLECTION OF THE NEEDS OF SOCIETY

An essay

Denne artikel er udarbejdet til offentliggørelse i en let revideret form i publikationen "Studies in Mathematics Education II", der udgives af Unesco, og som ventes at foreligge omkring årsskiftet 198o/81 i en engelsk og en fransk version.

Den her optrykte udgave er originaludgaven, før revision til Unseco-publikationen.

INTRODUCTION

Everything said in the present paper lies in the delicate and controversial twilight zone between mathematics as a subject and education as a social phenomenon. It is difficult to manoeuvre in this zone. Mathematicians, mathematics educators and mathematics educationalists often feel a certain uneasiness about the political and ideological facets inevitably associated with social matters, or they feel inadequately qualified in social science. On the other hand, generalists in education only very seldom possess a knowledge of mathematics and mathematics teaching sufficient to deal satisfactorily with all aspects of the complex of mathematics education. The present author - a mathematician, a mathematics educator and a mathematics educationalist risking his skin by transgressing the preserves of mathematics and entering into considerations involving society - is not superior to this difficulty. In fact, the state of affairs seems even worse. Having a rather general scope the present paper ought to be of relevance to the situations in a multitude of countries in the first, the second and the third world, with different social, economic and political backgrounds, including highly centralized as well as highly decentralized organisational systems. Such a scope is highly ambitious in itself and difficult to do justice to. The author wants to apologize for not having been able to emancipate considerations completely from the limitations of horizon induced be his specific national prerequisites.

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Issues of mathematics education can be considered under two different perspectives, an analytic perspective and a normative perspective. Considerations made under an analytic perspective concentrate on detecting the state of affairs, identifying its fundamental components and determinants, the causal connections within it, and the sources of influence upon it etc. In considerations made under a normative perspective, ideas, principles and suggestions, or whatever, are put forward and advocated (perhaps on a background of analytic considerations).

The present paper is primarily an analytic one. Normative considerations are based on values, including socio-political values. So, it would probably be futile to put forward suggestions or imperatives for the goals and arrangement of mathematics education in a publication of world-wide distribution. This does not mean that this paper is claimed to be neutral of values, values lie in the very core of these matters and influence analytic considerations also, only that a paper concerned with understanding general external factors of importance to the role and position of mathematics education is thought to be more relevant in an international context than concrete proposals.

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The main concern of this paper is with mathematics education of pupils and students (from now on termed "students", all of them), between 6-7 and 18-19 years of age, who run through the official school system of

society, whereas we shall not invest much effort in dealing with very specialized courses of mathematics education for a narrow and selected clientele in specific institutions outside or in the outskirts of the general school system.

METHODOLOGICAL DIFFICULTIES

When analysing the goals of mathematics education as a reflection of the needs of a given society, one is faced with the principal and practical problem of detecting and identifying (and documenting) the actual impact on mathematics education of social needs. Only seldom has "society" articulated its general interests and motives directly and publicly. Most often official and semi-official statements are confined to dealing with specific objectives, at a rather low level of generality, concerning mathematical content, knowledge and skills and perhaps oriented primarily towards examination requirements, the ultimate ends of mathematics education being presupposed, or considered irrelevant. And even if this were not the case, if "society" did articulate its interests, by indicating ultimate ends of mathematics education, we could not be sure that such publicly articulated interests and "the real, objective interests" of society (and what they are is subject to discussion) coincided. In this respect the situation is different from society to society. In some societies the fixing of general educational goals and curricula is

strongly centralized and bears the stamp of political control. In other societies the planning is centralized but the decisions are left to non-political professionals. In still other societies decisions on goal and curricula are taken at a decentralized level, locally or, in the extreme, by the individual school or even the individual teacher. Combinations of these features (for instance, centrally fixed frames, certain decisions concerning content left to the teachers) are also frequently encountered.

The situation described implies, in any case, that analyses of the reflection at issue are bound to follow chiefly paths partly hidden in fog. Furthermore, it implies that conclusions referring to these matters must necessarily be either rather vague or become postulates to some extent. To avoid obscuring the main lines of this paper too much by reservations and modifications we have preferred postulates to vagueness.

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In any society where mathematics education is being given, there are intimate relations between mathematics education on the one hand, and political, ideological and economic interests of that society on the other hand. It might very well be that these relations have not resulted from a well-defined set of conscious considerations, followed by decisions and initiatives (usually the situation is far more complex), but this does not imply that the relations do not exist. In the context of this paper, it probably would not

be a reasonable investment of effort to attempt to prove the existence of such relations - a proof would have to consider, in a very concrete way, the different societies covered by the assertion. We shall confine ourselves to emphasizing that in any society mathematics education in school is part of the total school system of that society. Hence mathematics education is bound to be subject to the interests of society as reflected in the school system, whether the interests imply that school is left to relative autonomy, with only funds provided, or the interests imply that school is kept to some degree or another under central control. Instead, it seems more relevant to our theme and purpose to investigate a little further the possible shapes and characteristics, which the relations in question can take and have actually taken.

Since, in this connection, we are primarily interested in that subset of the total set of relations between mathematics education and society which contains goals, objectives and purposes and their possible social roots, it seems necessary to make some initial distinctions between terms.

TERMINOLOGICAL DISTINCTIONS. GOALS: PURPOSES AND OBJECTIVES

Throughout this paper we distinguish between <u>two kinds of goals:</u>

- "The purposes" of mathematics education will be devoted to indicating the motives and reasons for which mathematics education at all exists (in a given school system in a given country or region), i.e. the ultimate gains the attainment of which justifies the existence of mathematics education (in that system). In other words, the indication of purposes answers the question "why should mathematics education exist?".

The term

- "the objectives" of mathematics education refers to the specific qualifications (in terms of insight, knowledge, accomplishments, behaviour, attitudes etc.) at which mathematics teaching should be aiming.

This distinction represents an instrument for our analysis. Only very seldom in official documents, in commentaries and debates on the goals of mathematics education is such a distinction between "ultimate ends" ("purposes") and other kinds of "ends" made. This state of affairs implies that it becomes difficult to separate in the discussion strata which, in our opinion, ought to be treated as different from each other. This sometimes gives rise to confusion, not only of terminology (the terms "goal", "aim", "purpose", "objective" etc. are used indiscriminately), but also in matters of substance. It should, of course, be admitted that the distinction made is neither theoretically founded

nor sharp (as can be seen also from the examples below), but this does not seem to affect its substance. The fact that official documents on actual systems of mathematics education do not very often operate with the distinction at question, does not imply that the distinction cannot be made in relation to that system, or that purposes cannot be identified, only that complex analysis and interpretation is needed. Indirect indications of purposes are to be presupposed in, and dug out from, indication of goals of all kinds, of which, practically always, some are present.

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To illustrate what is meant by these terms, some examples might be appropriate. Two "vectors" of examples will be given, both constructed, in the sense that they are not taken from any actual directions for mathematics education in any country, even if they are not too far from some of them. Serving an illustrative purpose only, neither of these two vectors pretends to constitute an exhaustive set of goals of a system of mathematics education.

Example 1

"It is a purpose of mathematics education to make it possible to utilize mathematical tools to contribute to technological and economic growth in society."

One corresponding objective might be:

"It is an objective of mathematics education to skill students in the process of structuring non-mathematical

situations, destilling off the mathematical aspects of problems under consideration, formulating and solving the mathematical problems involved, and interpreting solutions with regard to the original problems."

Example 2

goes along a different line:

"It is a purpose of mathematics education to contribute to developing in the individual citizen general mental capacities of use to his own life, such as: capacity of carrying out and assessing logical reasoning, imagination, inventiveness, and consciousness of aesthetical values."

A corresponding objective:

"It is an objective of mathematics education that students acquire insight into the role, character and building up of at least one axiomatized mathematical topic."

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Of course, it is not possible from a given purpose of mathematics education to deduce objectives in a deterministic way. The fixing of the objectives is dependent of several factors, the most important being: the level (including age level) of the students thought of, the character and needs of the society in question, the organisational framework for the governing of mathematics education, the concepts of mathematics and of mathematics education held by the curriculum planners. For instance, to the "purpose" of example 1 the following objectives, different from the one firstly stated, might correspond equally well at some levels or in some

societies: "It is an objective of mathematics education to make students understand and accept that some practical problems might be successfully treated by means of mathematics", or "It is an objective of mathematics education to train students to perform arithmetical operations quickly and with proficiency."

It is a characteristic feature of "purposes", and sometimes also of "objectives", that the achievement of them cannot be measured by any operational means, the reason being that they represent points at infinity which can be approached but never reached.

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The direction from "purpose" to "objective" represents a decrease in generality and social involvement (a decrease which would have continued, had we proceeded into regarding also more specific objectives, content, means and methods of mathematics teaching).

The ultimate interests of society in mathematics education are to be found in the purposes. The objectives are more detailed, precise and concrete specifications of, or components in, the purposes to which they refer. They are vehicles for for the pursuit of purposes.

What matters, when attempting to classify a goal as a purpose, is whether or not the goal is really a purpose in the sense

here defined, i.e. bears the stamp of being an ultimate end in its own right, in the context in which it is put forward.

The ambiguity arising from the possibility of classifying certain goals as either purposes or objectives, or both, constitutes one of the several points of obscurity in considerations and debates on mathematics education.

TYPES OF PURPOSES

In most countries mathematics has always been taught in some schools to some students, the categories of which have varied from time to time and from country to country. Very different (and sometimes even mutually incompatible) purposes to justify the existence of mathematics education can be identified. This state of affairs might seem to indicate that the existence of mathematics education is in fact not a function of purposes, or, in other words, "purposes" are either trivial or have no more importance to mathematics education than mottos of kings have had to their actual rules. Hence, it might be argued, attention should be directed towards specific objectives, contents and methods of mathematics education rather than to its purposes.

It might be that the mere existence of mathematics education as being given to someone in the school system is relatively independent of purposes, but the role and character of mathematics education, its specific objectives, contents and methods, the set of students to whom mathematics is or should be taught, are strongly influenced by purposes, whether they be direct or indirect. In particular the question of detecting or establishing purposes is of crucial importance as far as mathematics education of masses of students is concerned. Until recently, questions of purpose have not been posed in relation to arithmetic teaching, the usefulness of which has been considered evident in almost every society. The difficult questions in relation to purposes regard "mathematics proper", post-elementary mathematics, rather than arithmetic, although also arithmetic teaching may now be facing similar questions in view of the introduction and dissemination of pocket calculators. All in all, there seem to be good reasons for investigating further the purposes of mathematics education.

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There are essentially two main types of purposes of mathematics education, each of them containing a multitude of single purposes.

The first type of purposes refers to needs of society as a totality, such as (a) needs of social and economic development, (b) needs concerning political and administrative government of society, (c) needs of general cultural activity

and achievements and (d) needs concerning values and ideology. These needs do not regard the individual citizen or groups of citizens but society as a system. Therefore, those purposes of mathematics education arisen from these needs are only aiming at individuals because individuals are, in their capacity of being the necessary subjects of mathematics education, also the indispensable carriers of the mathematical answers to these needs. In emphasizing the global orientation, in contradistinction to an orientation towards individuals, of the social needs mentioned and of the related purposes of mathematics education, it is not said that the needs and priorities in this category are generally agreed upon in a given society in question. What matters is that the decision-makers outlining the framework of mathematics education have these needs and priorities in mind, whatever their positions and backgrounds might be, when explicitly and implicitly determining the purposes of mathematics education. The categories (a), (b), (c) and (d) serve primarily as examples and should not be understood in too narrow a sense. Thus, in a given society it might happen that the purpose of mathematics education were to provide an instrument for selecting from the population those persons of the highest capacities to occupy leading positions in that society, irrespective of the presence or absence of mathematics associated with such positions. This purpose may be contained in (a), (b) or (c), depending on the society.

The second type of purposes regards the needs of students as individuals, either in their situation as students or as future adults. It consists of purposes directed towards serving certain needs of individuals, which society (i.e. the decision-makers on mathematics education) looks upon as being of use to the individual citizen in mastering his own social and private life. It may include also potentials which are left to free development by the individual student exploiting the possibilities of mathematics education.

Purposes of the second type regard needs like the following examples: needs of comprehending and judging phenomenae, as well as statements about them, occurring in the natural and social environment of man; needs of active and critical participation in democratic processes; needs of action and creativity; needs of obtaining emotional and aesthetical experiences; needs of obtaining attractive conditions (including material conditions) of private life.

It might well be that, in having a purpose of the second type as a purpose of mathematics education, the interests of the decision-makers, of the curriculum planners, of teachers, of students, and of parents, do not coincide. It might also be that they do. Whether one or the other possibility, or a mixture of them, is true, depends on a multitude of factors, primarily of a social nature. In any case, because of the status of the school system as an institution of society, society determines how mathematics education should come into play in relation to these purposes, also if society determined that the development of one, more or all of the

potentials in question, or similar ones, should be left to free development in school and outside social control.

The following diagram summarizes the classification here presented of purposes of mathematics education:

		Examples of regards
I	Purposes oriented towards NEEDS OF SOCIETY AS A TOTALITY	(a) needs concerning social and economic development (b) needs concerning political and administrative government of society (c) needs concerning general cultural activity and achievements (d) needs concerning values and ideology
II	Purposes oriented towards NEEDS OF INDIVIDUALS IN MASTERING THEIR OWN LIVES	(a) needs of comprehending and judging phenomenae, as well as statements about them, occurring in the natural and social environment of man (b) needs of active and critical participation in democratic processes (c) needs of action and creativity (d) needs of obtaining emotional and aesthetical experiences (e) needs of obtaining attractive conditions (including material conditions) of private life

We do not claim that the above classification of purposes of mathematics education into essentially two classes can serve as a partition of the total class of purposes which have been brought about (explicitely or implicitely, for examples through objectives) in the course of times, or

might be brought about. However, we have found that a large majority of the goals usually discussed are based on one, or both, of the two types operated with here. In a later section of this paper, a selection of representative goals and purposes put forward in literature on mathematics education (debate - and research literature as well as official pronouncements) will be examined more closely.

To specify, for a system of mathematics education (actually existing or fictitious) in a given society, those purposes of each of the two types which are established as purposes of that specific system and which are not (it may even happen that only one of the types is represented), together with possible priorities and interrelations between them, is the first step in characterizing within the present kind of framework of analysis the goals of mathematics education as a reflection of needs of that society.

The two types, I and II, of purposes need not be mutually independent. It will often be the case, in a given system of mathematics education in a given society, that the purpose of fulfilling certain needs of individuals (like, for instance, active and critical participation in democratic processes (IIb)) is perfectly compatible with a purpose of fulfilling needs of society as a totality (like, for instance, needs concerning values and ideology (Id)).

In the same way, the purpose (part of Ia) of providing society with qualified and specialized labour in certain professions at a high level corresponds, in many societies, very well to the purpose of providing certain groups of persons with attractive live conditions (IIe). The ways in and the extent to which purposes of the first type are related to purposes of the second type, still in a given system of mathematics education in a given society, reflect profound political, social, cultural and ideological characteristics of that society.

It could be added that, depending on the characteristics of the society under consideration, it might happen, primarily in societies with a complicated multi-level educational decision system, that differences in interests within the system give rise to adoption of purposes which are mutually incompatible or even contradictory. It might also happen that different levels in the system (for instance, central curriculum planners on the one hand, and teachers on the other hand) emphasize different purposes.

PURSUIT OF PURPOSES

In the course of time several aspects of mathematics education and mathematics teaching have been subject to intense debates among people inside as well as outside mathematics and mathematics education. Why have such debates not come to an end long ago, in consideration of the large amount of experience gained

from mathematics teaching during more than a hundred years? Two main reasons seem to be of relevance to this question. Firstly, as has been previously touched upon, the purposes of mathematics education have always been either rather obscure and implicite or subject to disagreement. Secondly, even when purposes are established, there is no firm foundation, in terms of safe knowledge, on which it can be judged if this or that way of organizing mathematics education will ensure the fulfilment of the purposes. This lack of foundation pertains to purposes of the first as well as purposes of the second type, thus leaving us with fundamental questions unanswered, such as: "On what grounds can we claim that mathematics can provide students with a general sharpening of intellectual capacities, to be actualized in situations outside mathematics?" and "Where do we find evidence for the assertion that teaching mathematics (as distinct from arithmetic and very elementary mathematics) to masses of students is a necessary, or only powerful, instrument to economical development of society? Wouldn't it be more reasonable to concentrate the resources at disposal for students really needing mathematics in their future professions?" The situation described implies that irrespective of which view one holds concerning the capacity of mathematics education to fulfill purposes of the nature here discussed, the view must be founded to some extent on very general, "transcendental", considerations, assumptions and beliefs. As particularly regards purposes

concerning needs of social and economical development (Ia) the problem is of a twofold nature: First of all, the role and function of mathematics as such in social and economic development are far from well illuminated, its involvement being complicated and its effect most often resulting from "action at a distance" (in space and time). On that background it is not surprising that it has caused difficulties to determine the mathematical qualifications required for citizens to participate in processes of social and economic development. Nor is it, furthermore, surprising that curriculum planning is faced with substantial problems in determining mathematics curricula that can meet such qualification demands. These uncertainties have given rise to the adoption of two opposite strategies, as regards the needs at issue varying from time to time and from country to country. Strategy I can be phrased as follows: There are strong reasons to believe that in any society mathematics as a science in addition to the presence of a high level of mathematical competence with various categories of citizens, are factors of vital importance to the development of society. However, only very little of substance is known about the channels and paths of influence of mathematics on society. So, instead of founding our strategy of mathematics education on misleading, or even erroneous, ideas, risking to cause serious damages with far-reaching consequences, we should let mathematics itself, mathematicians and mathematics teachers decide from purely mathematical and didactical consideration not subject to external pressure which mathematics should be taught and how this should be done. The external restric-

tions put on mathematics education should be as few as possible in order to set free a maximal amount of mathematical resources in society.

Strategy II can be phrased as follows: Even if it may very well be true that not very much is known about the ways along which mathematics influences society and its development, we cannot leave mathematics education to mathematics, mathematicians and mathematics teachers alone. Many examples can be given showing that such a strategy tends to produce persons whose mathematical knowledge remains encapsulated from problems of the real world, persons who have serious difficulties in bringing their mathematical knowledge into contact with the kind of problems that occur in other sciences, technology, economy etc. Therefore, and because manpower and economical resources are scarce we have to formulate priorities - and ensuing curriculum plans to fulfill them - which can ensure a sufficient amount of properly qualified students to further professional education. Within this strategy, the role of mathematicians and mathematics teachers is to fill the centrally fixed frames with specific content and teaching. Through central examinations it is controlled that students meet the required standards.

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When discussing general strategies to be adopted in the pursuit of purposes, there is a factor, neither element in the category of purposes nor in the category of objectives but yet reflecting interests of society, which is of importance. The question is about general concepts and values concerning

human nature and individuals in relations to other individuals and to groups of individuals, general ideas and views of what education is and should be, as well as related ideas about which general educational instruments are acceptable or desirable and which are not. In any society such ideological prerequisites are present. There might be several of them, some of them might be mutually conflicting, they might be difficult to identify, but they cannot be dispensed with. The factor described, which could be called the human-ideological factor, is self-contained and independent not only of purposes and objectives of mathematics education but also of mathematics education as a whole. It is, of course, interacting with mathematics education where this is being given, putting restrictions on it, exploiting its potentials etc. As regards this factor, a given purpose (or objective) can be pursued in very different ways.

The factor needs not necessarily be purely ideological (i.e. purely connected with values). It may contain also considerations from psychology and pedagogy on what is phychologically possible, which pedagogical instruments are efficient and which are not, and the like. The point is, in any case, that the concrete character and shape of mathematics education is strongly influenced by this factor, which thus represents another path of influence on mathematics education of social interests.

To illustrate possible contents of the human-ideological factor a few examples would probably be useful:

(1) As one extreme, one could imagine an educational system, where the individual students are considered primarily as necessary carriers of the contributions by mathematics education to the answering of social demands. In a certain sense students are educated to become part of the technology of that society. The educational means adopted by the system are chosen according to their efficiency in pursuing the goals of mathematics education in that system, for instance to obtain a maximal amount of mathematical insight, knowledge and skills with students, with only secondary attention paid to their personal needs in general. Perhaps students are wanted also to subordinate their personal needs to the needs of society as a whole, or to a group, a region, or whatever might be in question. Programmes are established at central quarters, students are not invited to, nor supposed to, discuss the programme, they are supposed to comply with it as well as they can.

(2) As another extreme, one could imagine an educational system, where students are also educated to take part in answering social demands, but where education is considered first of all a process of personal development of individuals, a process where personal potentials are brought to maturity. (Probably this conception is based on the idea that the capacity of students to participate in answering social needs becomes

optimal when their development of personality is brought into focus.) So, the goal of fulfilling social needs is restricted by the priority given to regards to personal integrity and development of the individual students. The educational instruments adopted by the system are chosen according to their capacity of stimulating maximal development of individuals. Students are invited to participate in decisions about the programme, which may even be left completely to local determination, with respect to their personal interests. They are invited to work inductively, to look for themselves, to play and discover, to follow their curiousity, and attention is paid to their feeling of comfort.

There exists no society in which these extreme versions of the human-ideological factor are in function exactly as sketched, but probably in any society the factor is present as a mixture of them.

OBJECTIVES

Although a distinction within the category of goals between "purposes" and "objectives" (or other terms covering the same substance) is normally not encountered in writings on mathematics education, there seems to be a tendency to concentrate on indicating and commenting upon goals which in our classification would become objectives, rather than purposes (in the sense used here), even if this requires interpretation in each case.

Another distinction is, however, frequently made: the division of the goals of mathematics education into two classes, the "utilitarian" goals and the "formative" goals. It seems that this distinction is not a very precise or happy one to make. This is so because the attributes "utilitarian" and "formative" as used in the debate are far from always mutually exclusive. Goals stated to be utilitarian are sometimes of a formative character, and goals referred to as formative are often accompanied by utilitarian reasons, like "this or that formative goal should be a goal of mathematics education because the corresponding capacity is important to modern society".

Recalling that objectives are more detailed, precise and concrete specification of, or components in, the purposes for which they are vehicles, we shall, instead of distinguishing between formative and utilitarian goals, introduce the following distinction between "formative objectives" and "substance objectives":

By a formative objective we mean any objective oriented towards establishing in students general personal characteristics and capacities, such as attitudes, general mental capacities, capacities of performing certain general activities, etc. To such objectives mathematics is solely a vehicle, the subject matter of which being unimportant in itself, the criterion for including a certain content matter being its capability of serving these formative aims. In principle formative objectives might be pursued by any subject other than mathematics, provided it possessed this capability.

The concept "formative" should be taken in its broadest sense, including anything concerned with developing general personal characteristics in students.

By substance objectives we mean objectives of providing students with competences, for which mathematics as such is indispensable. The term "mathematics as such" is not limited to include only specific content or operations, it includes anything involving mathematics substance in one form or another.

Now the relation between purposes and objectives may be schematized as follows:

purposes / objectives	purposes of type I	purposes of type II
formative objectives	"Having the fulfilment of the general social need X as a purpose of mathematics education, the formative objective to develop in students the personal characteristic x is put as an objective of mathematics education."	"Having the fulfilment of the need Y of individuals as a purpose of mathematics education, the formative objective to develop in students the personal characteristic y is put as an objective of mathematics education."
substance objectives	"Having the fulfilment of the general social need Z as a purpose of mathematics education, the substance objective of developing in students the competence z involving mathematics substance is put as an objective of mathematics education."	"Having the fulfilment of the need W of individuals as a purpose of mathematics education, the substance objective of developing in students the competence w involving mathematics substance is put as an objective of mathematics education."

A given objective may refer to type-I purposes as well as to type-II purposes at the same time. For instance, the formative objective "to provide students with the capacity of performing abstraction processes" may refer to a general social demand of a qualified labour force, which in a rapidly changing society is capable of adapting with flexibility to new situations (covered by Ia, b), or to the need of students as individuals to obtain attractive conditions in private life (IIe). It may refer also to a general ideological desire that citizens with consciousness and criticism can identify the core of important and complicated social issues and take positions on them (covered by Id (and b)), or to the needs of students as individuals to participate actively and with criticism in democratic processes (IIb). So, one and the same objective may be chosen to serve several purposes, even of different categories, at the same time. Of course, this need not be the case, we could think of hosts of objectives which refer to only one purpose.

One implication of this, as regards analyses of the impact of social needs on the goals of mathematics education, is that it is difficult to detect from a mere indication of objectives, in official documents or in the didactics debate, the ultimate social interests related to these objectives. In this respect, one could feel tempted to put forward the hypothesis that the more compound, heterogenous and complex the interests and the political organization of a given society are, the farther away from the surface and from

the daylight are the purposes of mathematics education to be found.

ELEMENTS IN THE HISTORICAL DEVELOPMENT OF GOALS

It seems to an established fact that in most countries post-elementary mathematics education in school up to the beginning of the 20th century had development of mental discipline as its foremost goal. Geometry, in particular, was chosen to fulfill this task. Society needed people to occupy leading positions in politics, public administration, industry and science, for whom general mental capacities, such as loyalty to given conditions, ability to distinguish essentials from casual details, capability of analysing situations, judgement, care, precision and diligence in work, appreciation of cultural and moral values of society, were desired. So, in this respect we are dealing with purposes of type I. But, of course, mental discipline was to some extent considered also useful to the individual in mastering his own life (type II-purposes). The distribution of emphasis on type I- and type II-purposes undoubtedly differed very much from country to country, perhaps with Germany and England as extremes in the type I-end and the type II-end, respectively.

Only a small minority of the population got a mathematics education going beyond the level of elementary arithmetic.

Although the instruction of elementary arithmetic aimed primarily at providing pupils with certain computational skills, useful for them in their adolescent lives (purposes of type II), as well as demanded by society in general (purposes of type I), with an increasing emphasis on the latter as industrialization grew, this instruction was aimed also at developing mental discipline. It may be supposed that the purpose of this was to serve the increasing demand of a careful, precise, diligent and loyal labour force, particularly for the growing industry.

The very idea of mental discipline stemmed from faculty psychology, according to which man possesses certain general, context-free faculties, which, once developed in one context, can be freely transferred to any context. In concordance with these ideas, the purposes of post-elementary mathematics education, whether dominantly of type I or of type II, were pursued via objectives of a formative character.

*

Around the turn of the century the changes in the socio-economic and technological conditions of many countries, first of all in Western and Northern Europe and the USA, effected major changes in the educational needs of these countries. As particularly regards mathematics education, this development led to an increasing emphasis in the debate on "the use" of mathematics in society. In [1] De Vault and Weaver characterize the period 1894-1923 in the USA (for grades K-6) as follows: "... we teach mathematics because adults use mathematics in their everyday work requirements." (page 110). Osborne and

Crosswhite refer in [1] (page 183) (for grades 7-12) to observations made around 1912-17 by an American committee under the International Congress of Mathematicians, that utilitarian aims were becoming increasingly more important and that the formal discipline concept of education was being questioned.

In Germany the trends were similar. In 1915 The Association for the Promotion of Mathematics and Science Instruction ("Der Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts") arranged a prize subject competition on the question "Which demands should be made to the education of the German Youth after the War, and what can mathematics and science instruction contribute to the realization of these demands?" (translation) ([2]). The first prize was awarded to Dr. W. Schmiedeberg, master at "der Oberrealschule zu Bielefeld". His contribution possesses explicitness, thoroughness and clarity to a degree which is indeed rare. Schmiedeberg points out four general purposes of the education in school: education for national defense; education for serious, diligent and conscientious labour; education for working community; education for patriotism. He summarizes these purposes as follows: "We need a strong people which can defend itself, and industrious people which can create economic greatness, a loyal and patriotic people that commits its power and labour consciously to national aims." ([2], page 5, translation), and says further: "If we recall that the future of Germany relies on the maintainance and further development of work of quality, we realize that the educa-

tion to absolute devotion to duty should remain our foremost educational aim for all kinds of schools." ([2], page 14, translation). Various components in these purposes are put forward: "education for regularity and stability", "subordination of the individual to the organism of work", "preparation for adaptation and obedience". The purpose of mathematics education should be to contribute to the fulfilment of these general purposes of education (the foremost means being applications of mathematics).

Even if Schmiedeberg's points of view may appear today as rather extreme, they were apparently approved by the association. We think it not unjustified to suppose that Schmiedeberg expresses views generally held at that time, also in official quarters, at least in Germany. Probably, only his articulate way of expressing these views makes him a singularity in the debate.

So far, it seems to be well founded to claim that purposes of type I obtained an explicit and dominant position during the first decades of the 2oth century, whereas purposes of type II were not in focus. In England, however, the situation was somewhat different. In 19ol John Perry, professor of engineering science, maintained, in opposition to the mainstream in England at that time, that the study of mathematics began because it was useful, continues because it is useful, and is valuable to the world because of the usefulness of its results (quotation from [3]). He indicated eight "obvious forms of usefulness" in the study of mathematics, often quoted in writings on mathematics education:

"(1) In producing the higher emotions and giving mental pleasure. [...]
(2) (a) In brain development. (b) In producing logical ways of thinking. [...]
(3) In the aid given by mathematical weapons in the study of physical science. [...]
(4) In passing examinations.[...]
(5) In giving men mental tools as easy to use as their legs and arms; enabling them to go on with their education (development of their souls and brains) throughout their lives, utilizing for this purpose all their experience. This is exactly analogous with the power to educate one's self through the fondness of reading.
(6) Perhaps included in (5): In teaching a man the importance of thinking things out for himself and so delivering him from the present dreadful yoke of authority, and convincing him, that, whether he obeys or commends orders, he is one of the highest beings. This is usually left to other than mathematical studies.
(7) In making men in any profession of applied science feel that they know the principles on which it is founded and according to which it is being developed.
(8) In giving to acute philosophical minds a logical counsel of perfection altogether charming and satisfying, and so preventing their attempting to develop any philosophical subject from the purely abstract point of view, because the absurdity of such an attempt has become obvious."

These statements have often been interpreted as being in keeping with the general stream of social-utilitarianism of that period. However, this interpretation becomes doubtful at a closer analysis (cfr. [4]). Most of these "forms of usefulness" refer directly or indirectly to the needs and interests of the individual (type II-purposes), with the possible exception of (2) and (7) which might be considered as referring also to some form of social utility (type I-purposes). As to (3) we are not able to tell whether it refers to type I- or to type II-purposes.

Even if Perry's statements do not include specific indications of objectives, we venture the conjecture that

the pursuit of the purposes underlying these statements should take place by chiefly formative objectives.

Although we thus are essentially not, in the case of Perry's suggestions, dealing with purposes of type I, it might well be justified to consider these suggestions, when comparing with the background on which they were put forward, as contributing to the wave requiring utility of mathematics education. Already the requirement of external justification of mathematics education would work in that direction.

The promotion of type I-purposes continued, in England, in the course of the first decades of the century. In 1919 a report ([5]) made under the auspicies of the Mathematical Association stated the following:

"1. That a boys educational course at school should fit him for citizenship in the broadest sense of that word: that, to this end, the moral, literary, scientific (including mathematical), physical and aesthetic sides of his nature must be developed. That in so far as Mathematics is concerned, his education should enable him not only to apply his mathematics to practical affairs, but also to have some appreciation of those greater problems of the world, the solution of which depends on Mathematics and Science.

2. That the utilitarian aspect and application of mathematics should receive a due share of attention in the mathematical course. [...]"

Thus, purposes of type I gained momentum but type II-purposes were still important:

" We assume that the majority of boys will not make any extensive or profound use of Mathematics in after-life, that they will not even be able to follow in detail the mathematical methods of engineering and Applied Science. But they can be so taught that a vista is opened up to them through which they may see the tremendous potentialities of the study whose elements they are mastering. [...] A public must be created able to realise what Science and Mathematics are doing for the world, and to form some general conception of the means imployed. The average man will not be more than a spectator of the world's material progress; we have suggested that he should be an intelligent spectator. But the world needs an increasing number of workers trained in mathematics; there must be specialists. [...]

We do not hesitate to put the pleasure motive among the foremost."

*

During the twenties and thirties purposes of type I maintained and strengthened their central and somewhere dominant position in the industrialized world. Due to the stagnation and recession of industrial and general socio-economic development connected with the world crisis around 193o there was, however, no large-scale qualitative technological expansion which required fundamentally new competences of the labour force. So, there was no external pressure for re-examining the purposes of mathematics education. This effected somewhat of a fossilization of arithmetic teaching, with mechanical routine in computation as a main object of it. Also post-elementary mathematics kept going on along the traditional paths, with a tendency to replace the traditional compartmentalization of the curriculum into independent topics with attempts at a unification as the only innovation. As regards objectives, a trend of switching weight from formative to substance objectives (without dispensing with the former) in the pursuit of purposes of

type I, and type II as well, could be detected in several countries.

*

In the late thirties certain new signals entered the scene. Several observers and commentators, many of which belonged to mathematics quarters, found that too much emphasis was laid on drilling mechanical and somnambulistic routine in operations and in obtaining correct results, and that too little attention was paid to students' understanding of mathematical backgrounds (which did not mean that the purpose of social utility was being seriously questioned). De Vault and Weaver quote in [1](page 121) Brueckner [6] for having written (in 1941): "Arithmetic should be both mathematically meaningful and socially significant". In England the Spens Report (1938) of the Consultative Committee on Secondary Education stated:

> "No school subject, except perhaps Classics, has suffered more than Mathematics from the tendency to stress secondary rather than primary aims, and to emphasise extraneous rather than intrinsic values. As taught in the past, it has been informed too little by general ideas, and instead of giving broad views has concentrated too much upon the kind of methods and problems that have been sometimes stigmatised as "low cunning". It is sometimes utilitarian, even crudely so, but it ignores considerable truths in which actual Mathematics subserves important activities and adventures of civilized man. It is sometimes logical, but the type and "rigour" of the logic have not been properly adjusted to the natural growth of young minds." (Quoted from [3], page 17)

Because of World War II the growing demand, raised by circles of mathematicians and mathematics educators, of students' knowing and understanding the proper mathematical background

of what they were doing, did not come to an immediate unfolding. After the War socio-economic and technological conditions were totally different from those of the preceding period. All over the world the rearrangement of war production to peace production, the needs associated with restauration and economic growth, etc. influenced new requirements and new challenges to the educational sector, and hence to mathematics education as well. Technology was no longer static; it expanded qualitatively as well as quantitatively, with the USA in the front of the wave. One result of this expansion was the demand of a well educated labour force at all levels, capable not only of performing routine operations remaining the same over a longer period of time, but also capable of adapting to more rapid changes in technology and working conditions, and many of whom were to utilize and understand mathematics at a far more sophisticated level than before. As De Vault and Weaver put it when commenting in [1] on the "Sputnik shock": "For both the average citizen and the worker employed in the expanding economy mathematics was essential. Increasingly it was recognized that the man on the street needed to be knowledgeable in mathematics if he was to understand the world in which he lived" (page 135).

The situation described gave rise to an alliance between mathematicians, mathematics educators and educationalists, conducted by university mathematicians, on the one hand, and society, represented by government institutions and

international organisations, on the other hand. Society demanded a system of mathematics education which could serve first of all type I(a)-purposes, secondarily type II-purposes, with particular regard to flexible technical competences of rather high complexity. The communities of mathematicians and mathematics educators offered to serve these purposes by means of a thorough modernization and scientification of mathematics curricula at all levels, chiefly with substance objectives focussing on general mathematical structures as vehicles. It may be conjectured that the post-war economic expansion was in itself less essential to many of these quarters that the opportunity it provided for innovating mathematics education and raising the amount and quality of recruits to university studies in mathematics and natural sciences.

This development constituted the socalled "first wave" of the "new math"-reform. It was carried finally through in most countries at the beginning of the seventies. Since this development is well illuminated by several observers and commentators, we shall not go into further details. Instead, we shall comment upon certain recent trends.

*

During the period of consolidation succeeding the culmination of the "first wave", various new traits have emerged. They originated from the observations that

(1) a strikingly large proportion of students of all grades had serious difficulties in grasping the rather abstract

and structure-oriented set-up which had become common as a result of the "first wave";

(2) the new curricula did not come up to the expectations and promises that these curricula could enable students to utilize mathematics in non-mathematical surroundings with intelligence and flexibility towards varied situations. Institutions and employers receiving students after school began complaining on this;

(3) students, in particular at the expanding secondary level, were increasingly critical against being forced to learn complicated subject matter the relevance of which was obscure to them and being questioned from outside mathematics education quarters.

In reaction to these symptoms of a crisis-like situation mathematics educators and educationalists started to reconsider the goals and curricula of mathematics education.

Bent Christiansen identifies in [7] (1975) the following directions of change in goals:

"2.6.1 An increased emphasis on affective goals.

2.6.2 An increased acceptance of responsibility for the majority of students, not only for an elite.

2.6.3 An increased emphasis on usefulness (in an ordinary daily life sense) and on applicability of the outcomes of mathematics teaching.

2.6.4 An increased concern for the individual learner; for learning more than for teaching; for the learner's experiences, needs and interests, more than for the requests of mathematics as a science.

2.6.5 An increased acceptance of the role of the mathematical activities (involved in processes such as description, systematization, problem-solving, proving;

- - not only knowledge of the results of mathematical activities, but also experiences from a familiarity with the processes that lead to the results are seen as main objectives." (page 1o)

Z.P. Dienes suggests (1978) in [8] (page 82f) "that the main aim of mathematics education, instead of being the knowledge of mathematics, should be the development of some patterns of thinking, certain types of strategies, that people might develop in face of new situations which they have never encountered before." He finds that mathematics education may provide students with four competences: to abstract ("The purpose of getting rid of irrelevancies and cutting through noise (in information theory terms), and getting down to the real message is certainly a particular competence which would be a great asset to people in the modern world."), to generalize ("This kind of enquiry obviously has a great interest in a very fast expanding and modern world" and "..'what would happen if.. ?', 'supposing that..?'. This kind of enquiry is very important in any kind of modern development, let it be government, industry or scientific research. The modern man should always consider the possibility of change. In fact he has no choice. He lives in a world of change and he has to adapt to it."), to decode and encode messages ("So, learning to cope with precise messages or learning to formulate precise messages would be an important side effect of having learned to cope with mathematical messages in general."). In his conclusion (page 94) Dienes says:

"To sum up, the suggestion is being put forward that the learning of mathematics could and should have other pay-offs besides the knowledge of mathematics. [...] Last, but certainly not least, is the social pay-off. Children learn to work together in collaboration instead of in competition with one another while discussing problems such as have been described. They learn to respect the opinions and values of others and thus develop their own opinions and values within a practical and realistic social context."

Letting these few references indicate the latest trends in goals of mathematics education (several other references could be given), how can these trends be interpreted within our framework analysis?

As to purposes, the direction encountered after the first wave is towards attributing increasing importance to needs of individuals, as students and later adults, i.e. type II-purposes. Particular attention is being paid to such needs associated with living in, understanding and participating in a society of a rapidly growing complexity, characterized by alienation and narrow ranges of possibilities of action, originating from, among other things, optimization and control of the exploitation and allocation of scarce recourses. Perhaps one could say that the <u>increasing stressing of type II-purposes</u> attempts at providing individuals with tools for protecting themselves against consequences of certain lines of development in society.

Needs of society as a totality, i.e. <u>type I-purposes, still have a central position</u> in mathematics education. Up to the seventies society demanded manpower which could participate in the technological and economic expansion which took place by extension of the production apparatus and the public sector

and by submitting ever new areas of activity to technological treatment or innovation. This extensive expansion also effected a general increase in the demand of labour. In the seventies, and concurrently with the emergence of the economic world recession, the economy switched from extensive to intensive exploitation of capacity, with automatization, optimization and rationalization as important ingredients. This development involved a decreasing demand of labour.

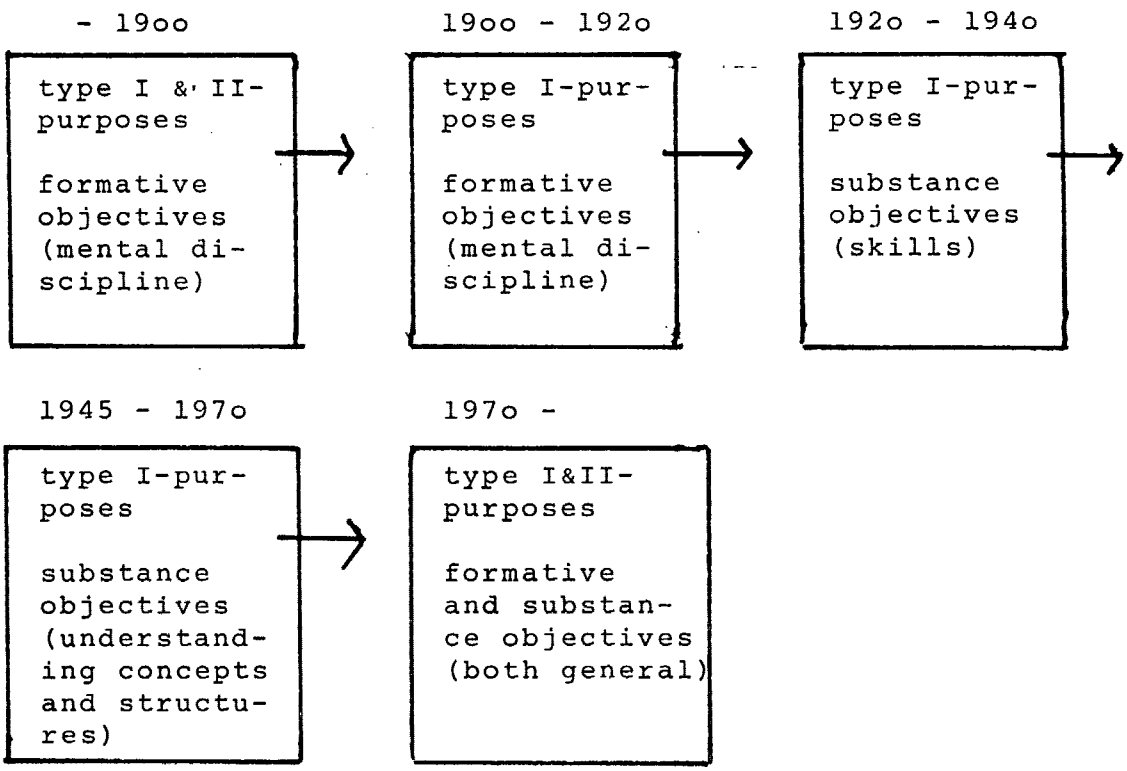
One result of the development described, the consequences of which for the individuals was touched upon above, is increasing requirements to the labour force of adaptation to changing conditions, of general rather than specific competences, also in mathematics, of capability of utilizing general mathematical tools under various circumstances, etc.

Therefore, in terms of objectives, there has been a change in stress after the "first wave" of reform from substance objectives to formative objectives, but with "formative" meaning something else than classical mental discipline; now focus is on capacities such as: abstraction, generalization, creativity, general problem-solving behaviour, cooperative behaviour, participation in collective tasks, decoding and encoding of messages, transfer and the like. Also the character of substance objectives has changed. While in the beginning of the reform period substance objectives considered chiefly internal mathematical concepts, they now consider mathematics in relation to its surroundings, in particular model-building and problem solving.

The objectives used to pursue purposes of type II are partly the same formative and substance ones as were mentioned for type I-purposes. Some additional formative objectives, frequently suggested, seem, however, to refer particularly, even if not solely, to type II-purposes: the fostering of curiousity, attitudes of research, critical attitudes, capacity to argue, emotional and aesthetical joys and the like.

The trend of emphasizing the needs of individuals is supported by a parallel trend, in these years, of growing concern about: respect of the integrity of children, discovery, intuition, play, experiment (attributed to the human-ideological factor).

We conclude this section by schematizing (being fully aware of the short cuts implied) the major trends of change in the goals of mathematics education during the periods treated:



Even if this historical sketch identifies only main trends and does this by referring to singular cases, we think it justified to claim that society has always had the serving of needs of society as a totality, in particular type Ia-purposes, as main purposes of mathematics education. Varying with fluctuations in economic, political and ideological conditions, the influence of humanistic ideas etc., type II-purposes have occupied positions of varying strength. But it seems that society has always considered these purposes as in principle subordinate to purposes of type I.

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Mogens Niss
March 198o

E N D

MATEMATIKKENS ROLLE I UNGDOMSUDDANNELSERNE

almendannelse og/eller studieforberedelse?

Indlæg holdt på matematik- og fysik-/kemilærerforeningernes efterårsmøde 11.1o.198o i Århus

Mogens Niss

Arbejdsmanuskript

Denne artikel er manuskriptet til et foredrag med samme titel holdt på Matematiklærerforeningens og Fysik- og kemilærerforeninge(r)n(e)s efterårsmøde i Århus 11.1o.198o.

HISTORISK OVERBLIK

Gymnasiet har altid forstået sig selv som både almendannende og studieforberedende/professionsforberedende:

Gymnasiet er "en fortsat almendannende undervisning, som tillige giver det nødvendige grundlag for videregående studier..." (enslydende formuleringer i 19o3-loven og i 1958-loven).

Selv om dette dobbeltkrav sættes for gymnasiet som helhed og ikke nødvendigvis for det enkelte fag, er der mange vidnesbyrd om at også matematik tænkes at forfølge begge hensigter, i hvert fald for visse elevgrupper. Før jeg går videre med at klarlægge dette, vil jeg forsøge at få hold på hvad "almendannende" nærmere skal betyde og har betydet til forskellige tider; begrebet har jo ikke på forhånd nogen klar mening. Det et f.eks. uklart, om "almendannende" er disjunkt fra "studieforberedende" eller om det måske endda står i modsætning hertil, eller om det tværtimod er en delmængde heraf. Jeg vil diskutere afgrænsningsspørgsmålet ved hjælp af en simpel matrix([1]), hvis rubrikker er udfyldt med eksempler til illustration af hvad der tænkes på:

overordnet HENSIGT / FORMÅL vedrørende:	Forberedelse til livet som PRIVATPERSON OG SAMFUNDSMEDLEM	Forberedelse til UDDANNELSE/ PROFESSION
FAGSPECIFIKT (matematik uomgængelig)	Eks. Mat. modeller i brug til samfundsmæssige beslutninger	Eks. Infinitesimalregning (som forberedelse til ingeniørstudiet)
Matematik som et (i princippet tilfældigt) MIDDEL TIL PERSONLIGHEDSDANNELSE	Eks. Opøvelse af kritisk sans	Eks. Opøvelse af evne til stringent tænkning og prægnant udtryksform

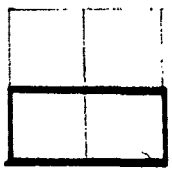
Lad os se på hvordan formålsangivelserne for matematikundervisningen til forskellige tider har lagt vægt på de forskellige aspekter af denne opdeling.

Først de sproglige. På den sproglige linje har der altid siden dens oprettelse været undervist i matematik, med undtagelse af årene 1953-63. I enhedsgymnasiets dage, dvs. indtil 1871, havde alle en betragtelig mængde matematik. Med indførelsen af det ny gymnasium efter 1958-loven genindførtes matematik for sproglige elever.

I perioden frem til de seneste årtier kan denne matematikundervisning ikke have tjent noget fagspecifikt studieforberedende formål, fordi de aftagende studier og professioner ikke havde noget matematikindhold af betydning. Det er først igennem 6o'erne og 7o'erne at matematiske og statistiske betragtninger har vundet bredt indpas i discipliner, der rekrutterer sproglige studenter (økonomi, lingvistik, psykologi, sociologi m.m.). Medicinstudiet har i et vist omfang indeholdt sådanne ingredienser i en længere periode. F.eks. forudsattes sproglige studenter i den matematikløse periode at gennemføre et suppleringskursus i matematik, hvis de ønskede at studere medicin, og det var i øvrigt nærmest af hensyn til dette studium, at matematik blev genindført for sproglige fra 1963.

19o6-bekendtgørelsen (sproglige):
"Formaalet med Undervisningen på de to sproglige Linier skal ikke saa meget være at bibringe Eleverne omfattende Kundskaber i Matematik,...,som at skole Elevernes Tænkeevne ved at indøve den gennem Matematikkens stringente Betragtningsmåder. Ved undervisningen bør man derfor særlig lægge Vægt på en udtømmende og omhyggelig teoretisk Behandling af de optagne Afsnit."

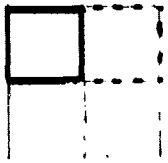
Denne formålsangivelse kan symbolsk repræsenteres således i forhold til den ovenstående matrix:



19o3

1935-anordningen (sproglige):
"Formaalet for Undervisningen er at give Eleverne Kendskab til visse vigtige Anvendelser af Matematikken, Af de teoretiske Afsnit medtages saa meget, at dette Formaal kan opfyldes."

Symbolsk:

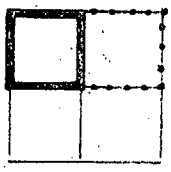


1935

1953. Med ordningen af dette år bortfaldt sprogliges matematik.

1961-bekendtgørelsen (sproglige):
"Formålet med undervisningen er dels at give eleverne et indtryk af matematisk tankegang og metode, dels at give dem nogle matematiske hjælpemidler i hænde, som kan være dem til nytte inden for andre fag i skolen og under deres senere virke."

Symbolsk:



1961

Til yderligere belysning af vægtfordelingen mellem de forskellige aspekter kan følgende citater fra betænkningen "Det ny Gymnasium" fra 1960 (på hvilket grundlag bekendtgørelsen blev udformet) tjene:

"Den almendannende undervisning...hviler frem for alt på 4 store fagområder...Det tredje er de matematiske og naturvidenskabelige fag. Kundskaber på disse områder er i vore dage af afgørende betydning for forståelsen af den verden, hvori vi lever."

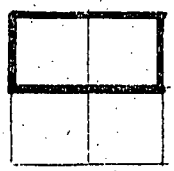
Og et andet sted i samme betænkning:

"I vor tid kan humanismen ikke nøjes med at bygge på vidnesbyrdene fra tidligere tiders kultur og de egentlige humanistiske fag, men må også tillige tage hensyn til naturvidenskabens betydning for menneskets situation."

1971-bekendtgørelsen (sproglige):
"Undervisningen har til formål at opøve eleverne i anvendelsen af matematisk tankegang, metode og viden til formulering, analyse og løsning af problemer på forskellige områder.

Undervisningen skal endvidere give eleverne en elementær forståelse af og evne til kritisk at analysere den måde, hvorpå matematikken anvendes inden for forskellige fagområder."

Symbolsk:

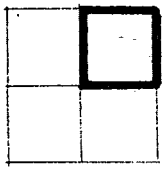


1971

Denne formulering afspejler den begyndende studie-/professionsforberedende hensyntagen, der blev aktuel med det voksende islæt af matematik i en mangfoldighed af videregående uddannelser, der tidligere var uden kontakt med matematiske betragtningsmåder.

Hvad angår matematikerne ser forløbet således ud:

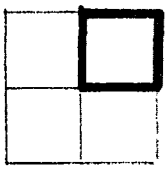
1906-bekendtgørelsen (matematikere):
Denne bekendtgørelse anfører intet formål for undervisningen i matematik på den matematiske linje. Et indtryk af formålet kan man imidlertid få indirekte ved at iagttage, hvad undervisningsinspektør Tuxen skrev i sin indberetning af 1914: "den matematiske (Retning), hvis Hovedformaal som saadan er at forberede til Polyteknisk Læreanstalt." I alt forekommer det berettiget at repræsentere situationen symbolsk i følgende form:



1906

1935-anordningen (matematikere):
"Formaalet for Undervisningen er at bibringe Eleverne Kendskab til de reelle Tal og disses Anvendelse til Beskrivelse af Funktioner, samt Kendskab til simple Figurer saavel i Planen som i Rummet. Eleverne skal lære at arbejde med det matematiske Formelapparat og opnaa Sikkerhed og Færdighed i numeriske Beregninger."

Symbolsk:



1935

1953-anordningen (matematikere):
"Formålet for undervisningen er at bibringe eleverne kendskab til et fundamentalt område af matematikken,

(symbolsk): 

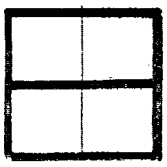
og gennem arbejdet hermed at udvikle og skole deres evne til stringent tænkning og prægnant udtryksform."

(symbolsk): 

Til yderligere støtte for at den øverste venstre rubrik er tillagt vægt kan fra bekendtgørelsen af samme år citeres:

Det vil for forståelsen af kultursammenhængen være af betydning, om der af matematikkens historie medtages træk, der har almenmenneskelig interesse."

I alt



1953

1961-bekendtgørelsen (matematikere):
"Formålet med undervisningen er

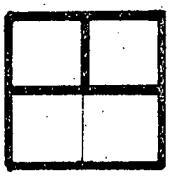
- at give eleverne kendskab til en række fundamentale matematiske begreber og tankegange,

- at søge deres fantasi og opfindsomhed udviklet,

- at øve dem i behandlingen af konkrete problemer, herunder udførelse af numeriske regninger,

samt
- at gøre dem fortrolige med anvendelser af matematikken inden for andre fagområder."

I alt



1961

1971-bekendtgørelsen (matematikere):
"Undervisningen har til formål

- at give eleverne kendskab til en række fundamentale begreber, tankegange og metoder,

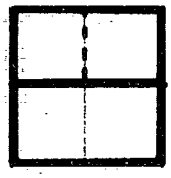
- at opøve eleverne i anvendelse af matematiske begreber, tankegange og metoder til formulering, analyse og løsning af problemer inden for forskellige fagområder,

- at opøve klarhed og logisk sammenhæng i bevisførelse og udtryksform,

- at udvikle fantasi og opfindsomhed,

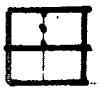
- at give en forståelse af og evne til kritisk at analysere den måde, hvorpå matematikken anvendes inden for forskellige fagområder."

I alt



1971

Udviklingsgangen kan sammenstilles således:

	sproglige	matematikere
1906		
1935		
1953		
1961		
1971		

For sproglige var undervisningen i 1906 udelukkende orienteret mod personlighedsdannende (formaldannende) formål (nederste række i matricen). Det er ikke uden videre muligt at afgøre, om dette skulle tjene individcentrerede eller studie-/professionsforberedende hensigter. Antagelig begge dele, som det også er skematiseret ovenfor. I 1935 skiftede perspektivet brat til forfølgelsen af fagspecifikke formål med hovedsagelig individcentrerede hensigter (aftagerstudierne indeholdt efter alt at dømme praktisk talt ikke matematisk stof). Den samme orientering blev fastholdt i 1961, mens studie-/professionsforberedende hensigter fik fodfæste i 1971. For matematikere var undervisningen indtil 1953 udelukkende rettet mod fagspecifik studieforberedelse. I 1953 udvidedes perspektivet ved at formaldannende formål blev anført (og er forblevet der siden) og ved at undervisningen nu også skulle rette sig mod individernes private og samfundsmæssige liv.

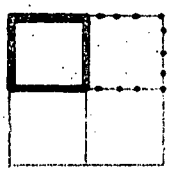
For at komplettere billedet: ser vi til sidst på HF.

HF (1967-kendtgørelsen):

Fællesfag:
"Formålet med undervisningen er at give eleverne et indtryk af matematisk tankegang og metode samt nogle matematiske kundskaber, som kan være dem til nytte inden for andre fag

og under deres øvrige virke."

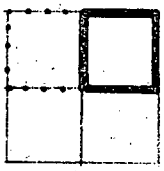
Symbolsk:



1967

Tilvalg:
"Formålet med undervisningen er at give eleverne kendskab til en række fundamentale matematiske begreber og tankegange og at gøre dem fortrolige med anvendelsen af matematikken inden for andre fagområder, således at de får det faglige grundlag for at gennemføre uddannelser, der forudsætter matematiske kundskaber."

Symbolsk:

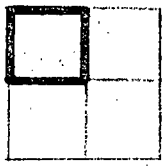


1967

HF (1974-bekendtgørelsen):

Fællesfag:
"Formålet med undervisningen er, at de studerende opnår nogle matematiske kundskaber, som kan være dem til nytte i andre fællesfag og i deres øvrige dagligdag, samt at de får et indtryk af matematisk metode- og tankegang."

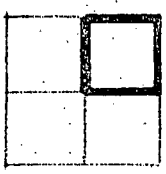
Symbolsk:



1974

Tilvalgsfag:
"Formålet med undervisningen er, at de studerende opnår en sådan indsigt i matematik og et sådant kendskab til fundamentale matematiske begreber, metoder og tankegange, at de får det faglige grundlag for videregående uddannelser, der anvender matematik."

Symbolsk:



1974

Det fremgår heraf, at der på intet tidspunkt i matematikundervisningen på HF er fastlagt formaldannende formål. Der har fra starten været en arbejdsdeling mellem fællesfaget og tilvalgsfaget, således at fællesfaget er orienteret mod at tilgodese individcentrerede hensigter, mens

tilvalgsfaget er eksplicit orienteret mod at forfølge studieforberedende hensigter. Denne arbejdsdeling var svagt sløret i 1967, helt klar i 1974.

DEN AKTUELLE SITUATION

Den aktuelle situation er præget af forskellige former for opbrud, dels inden for matematikundervisningen i særdeleshed, dels på de 16-19-åriges uddannelsesområde som helhed. Jeg vil ikke ved denne lejlighed gå i detaljer med en klarlæggelse af disse opbrudstendenser. Blot et par træk skal fremhæves. Inden for matematikundervisningen, som den for tiden er placeret i de gymnasiale uddannelser, har der igennem de seneste år været lagt øget vægt på anvendelser, øget vægt på heuristisk-induktive argumentationsformer forbundet med reducerede krav til stringens og deduktion, øget vægt på samarbejde med andre fag, eksperimenteren med arbejdsformer forskellige fra den traditionelle klasseundervisning. Der kan desuden spores en tendens til at forlade formaldannende formålsangivelser og begrundelser for faget. I forbindelse med det mere omfattende opbrud, der finder sted inden for de 16-19-åriges uddannelser i disse år, først og fremmest forårsaget af massetilgangen til de gymnasiale uddannelser, og med de strukturovervejelser og -eksperimenter, der er sat i værk forskellige steder, har der også rejst sig spørgsmål om matematikkens placering i det fremtidige mønster. Dels hvad angår tilstedeværelsen og placeringen af faget (og begrundelserne herfor) i dette mønster, dels hvad angår balancen mellem almendannende hensigter og studieforberedende hensigter og de respektive midler dertil.

Der er på denne baggrund at behovet for at gennemføre nye grundlæggende overvejelser over matematikundervisningens rolle og indretning for de 16-19-årige har meldt sig, i øvrigt med tiltagende styrke. Det følgende skal være konturerne af mit bidrag til sådanne overvejelser.

FREMTIDEN

Vi må se i øjnene at fastlæggelsen af matematikundervisningens hensigter og rammer fundamentalt er af politisk art, dvs. beror på en analyse af og stillingtagen til samfundsudviklingen. Matematikundervisningen er altså ikke politisk neutral; hvoraf ikke følger, at man fra modsatte politiske standpunkter aldrig kan nå til fælles konklusioner . Gymnasielærerne i de eksakte naturvidenskabelige fag har ingen traditioner for at anlægge samfundsmæssige endsige politiske betragtninger på fagene. Men det bliver nødvendigt at begynde på noget sådant nu. Med de omvæltninger, eller i det mindste ændringer, i de 16-19-åriges uddannelser, vi kan vente os i de kommende årtier, er det nødvendigt at være rustet til et offensivt bidrag til debatten og arbejdet med forandringerne. Afværgende forsvarsforanstaltninger for traditionen rækker ikke. En oprustning omfatter to dele:

(a) at komme til klarhed over en egen stillingtagen til hvil-

ke overordnede hensigter (i et samfundsmæssigt lys) matematikundervisningen skal tjene, og hvilke konsekvenser der skal drages af det.

(b) komme til klarhed over matematikundervisningens randbetingelser, dvs. samfundets objektive interesser i matematikundervisningen og de mekanismer, der er virksomme i fastlæggelsen af rammerne for den.

*

Jeg skal nu give et kort rids af mit bud på en sådan oprustning. Træk af dette bud har været antydet i en kronik "For matematik og fysik i fremtidens gymnasium" i dagbladet "Information" i foråret (med Jens Højgaard Jensen).

Udgangspunktet for betragtningerne er, at samfundsudviklingen i disse år er præget af

styring og kontrol (med stadig mere brutale midler) af fordelingen og udnyttelsen af knappe ressourcer. Det sker på en måde, der

- øger uligheden mellem forskellige befolkningsgrupper
- skaber politisk disciplinering og apati
- fremmedgørelse under et politiserende ekspertvælde.

Jo større styringskravene er, jo mere teknokratiseret bliver styringen, og jo mere uigennemsigtig bliver den. Som et samlet resultat af dette udviskes grænserne mellem teknik, økonomi og politik.

I dette felt spiller matematik en vigtig, men kompliceret og uigennemskuelig rolle. Fordi rollen udspiller sig i samfundets understrøm og kun sjældent ses på dets overflade, har mange svært ved overhovedet at erkende den. Det er f.eks. én af grundene til at så mange gymnasiereformatorer uden baggrund i matematik eller fysik er tilbøjelige til at undervurdere disse fags rolle i udviklingen og dermed underbetone deres opgaver i ungdomsuddannelserne til fordel for samfundsfag.

Mange elever kommer til at mærke matematikkens rolle i de omtalte processer på deres krop som fremtidige økonomer, teknikere, planlæggere, videnskabsmænd og undervisere. Alle kommer til som individer i samfundet at mærke efffekten af denne rolle som en del af rammen for deres liv.

På denne baggrund er matematikundervisningens vigtigste opgave i mine øjne at forsyne alle med forudsætninger for at opdage, forstå, tage stilling til, og handle over for denne matematikkens rolle i samfundet.

Denne rolle omfatter ikke blot misbrug af matematik, selv om det også indgår. Var der kun tale om misbrug, var parolen jo simpel: "Ned med matematikken i samfundet". Men matematikkens rolle er bestemt af, at matematikken i mangfoldige forbindelser kan noget. Den kan under visse omstændigheder bruges til at repræsentere, forklare eller behandle anliggender uden for

matematikken selv. Jeg vil ikke ved denne lejlighed gå i detaljer med omtalen af de midler, der skal til for at opfylde den beskrevne hensigt. Lad mig nøjes med at nævne, at aktiv modelbygning, arbejde med eksisterende modeller, arbejde med matematikken som fag i interne, historiske og samfundsmæssige belysninger er nødvendige elementer bag den forståelse, jeg efterlyser her. For at behandle disse elementer rimeligt er der mindst brug for den tid, der i øjeblikket er sat af til matematik; for visse elevgrupper er der brug for mere end nu. Når jeg ved forskellige lejligheder har givet udtryk for skepsis over for mat-fys-grenens matematikundervisning i denne forbindelse, er det altså ikke fordi der på denne gren er for megen matematik. Min kritik retter sig mod det forhold, at faget her har en tendens til at blive klaustrofobisk indadvendt. Der er alt for lidt ydre perspektiv på faget, nemlig praktisk talt intet. Den afsatte tid benyttes først og fremmest til intern faglig fordybelse, en fordybelse der for mange elever bringer matematikken til at fremstå som et kompliceret frit svævende spil, hvor bevæggrundene for at spille det står hen i det uvisse, og hvor alt forekommer lige vigtigt, fordi det er del af det samme spil. Man kan næsten sige, at eleverne står i fare for at blive sneblinde, eller, anderledes sagt, for at få en umoden matematikopfattelse, der ikke udgør en tilstrækkelig vaccination mod teknokratiske tilbøjeligheder. Sådanne tilstande er ikke efterstræbelsesværdige, hverken for de mat-fys-elever, der siden skal videre til naturvidenskabelige eller tekniske studier, eller for dem der skal videre til andre uddannelser.

*

Den matematikundervisning jeg advokerer for, skal altså lægge hovedvægten på at være almendannende, i betydningen at være orienteret mod at sætte eleverne i stand til at håndtere deres private og samfundsmæssige liv (første søjle i den tidligere omtalte matrix:

)

Genstandene for en sådan undervisning skal efter min opfattelse ikke fastlægges i pensumkategorier, men i de kategorier (modelbygning m.v.), som er antydet ovenfor. Matematikundervisningen må være eksemplarisk, dvs. de nærmere objekter, der indgår i den må være udvalgt efter deres evne til tilsammen at sige noget dækkende, og hver for sig noget karakteristisk (generaliserbart til et eller andet område) om matematikkens karakter og samspil med omverdenen. Hvilke matematiske emner, der nøjere kommer til at indgå i en sådan undervisning kommer i anden række. Det vigtige er, at de er af et passende omfang og niveau, og at det der faktisk indgår behandles ordentligt, også matematisk set. Det indebærer f.eks. at emnernes resultater ikke kun nås med plausibilitetsargumenter (men selvfølgelig også med sådanne).

Nu kunne man selvfølgelig spørge, om det ikke trods alt er

muligt og ønskeligt på forhånd at udpege visse basale emner, som under alle omstændigheder med stor sandsynlighed vil komme til, eller bør, indgå i en matematikundervisning, der skal sætte eleverne i stand til at forstå træk ved matematikkens involvering i samfundet og de egenskaber ved matematikken, der ligger bag denne involvering. Hvis det er rigtigt at man må forstå hvad matematik egentlig er som fag betragtet (og det er efter min mening rigtigt) for at kunne forstå denne involvering, må man så ikke have indsigt i visse emner, som derfor lige så godt kunne være udpeget på forhånd? I et vist omfang er det faktisk muligt at angive emner, det er svært at komme uden om i en undervisning af den art, jeg plæderer for her. F.eks. er uligheder, linearitet, eksponentiel vækst, grænseprocesser, statistik og sandsynlighedsregning sikkert af denne art. Til en lang række formål tillige infinitesimalregning og differentialligninger, til andre formål lineær optimeringsteori, grafteori og lignende. Men selv om det i et eller andet omfang forholder sig på denne måde, tror jeg at det er vigtigt at forsøge at undgå at beskrive undervisningen i sådanne termer. De har nemlig en ulyksalig tendens til at blive selvstændiggjort som emner for deres egen skyld, ikke mindst i hænderne på matematiklærere. Emnerne bliver let de faste holdepunkter i en kompleks undervisning med højtliggende mål; man kan da i hvert fald føle sig garderet som lærer, hvis man har gennemført en forsvarlig undervisning i de udpegede emner. De øvrige ingredienser er jo alligevel så diffuse og luftige. Men hvis undervisningen hænges op på et sådant skelet af emner, skal der meget til for at forhindre at undervisningen i faget bliver lig med undervisningen i emnerne. Det er klart at skadens omfang er en funktion af emnernes mængde. Men den skal ikke være stor, før det der er sket er en simpel pensumomlægning.

Hvad matematikkens karakter angår, er den jo selvfølgelig et mangehovedet uhyre, som ikke lader sig bringe på en simpel form. Men jeg tror, at denne karakter har mindre at gøre med stofvalget end med behandlingen af det. Hvis man anlægger et historisk syn, ser man også at der har været meget relativt hvad der har været anset for uomgængeligt i matematikundervisningen. Det er ikke mange år siden sfærisk geometri var noget man da måtte kende, eller hvad med aksiomatisk euklidisk rumgeometri, komplekse tal osv? Selv om meget stofvalg har mere at gøre med faglig konvention end med uomgængelige fundamentalier, ville det selvfølgelig være nonsens at hævde, at matematikken er det rene modellervoks, der kan formes efter forgodtbefindende. I en given ramme er visse ting selvfølgelig fundamentale og visse ting mere fundamentale end andre. Men hvorfor bekymre sig så meget om at angive dem, hvis de virkelig er fundamentale? Hvorfor ikke have tillid til at fundamentaliteten giver sig til kende, hvis der i øvrigt arbejdes seriøst med stoffet (hvilket selvfølgelig må forudsættes), f.eks. under perspektiver af den art, der her er diskuteret?

Jeg kender selvfølgelig godt svaret: niveaukontrol og stofsammenlignelighed af hensyn til aftagerne. Men disse hensyn er i princippet fremmede hensyn i en almendannende undervisning. For en almendannende undervisning er det nærliggende

at lade genstandsvalget ske ved anvendelsen af det eksemplariske princip. Det bringer os over i det næste punkt til overvejelse:

*

Det er selvfølgelig naivt at tro, at man kan få lov til at undervise alle 16-19-årige i matematik i betydeligt omfang alene med den hensigt at udstyre dem med midler til at deltage ændrende i samfundets udvikling og at værge sig mod træk i den. Det er næppe først og fremmest i sådanne hensigter, at samfundets interesse i en matematikundervisning for de 16-19-årige er at finde. Samfundets interesse i en stærk matematikundervisning er først og fremmest knyttet til dens professionsforberedende bidrag. Det er selvfølgelig nødvendigt at se dette i øjnene og at tage højde for det. Vi er f.eks nødt til at undgå at et snævert studieforberedende elitegymnasium fraspaltes det samlede gymnasiale uddannelsessystem; det ville være en uddannelsespolitisk katastrofe. Dels fordi et sådant gymnasium vil fremme uddannelsen af horisontforsnævrede teknikere og teknokrater, dels fordi det tilbageblevne standardgymnasium vil være sårbart over for krav om reduktion af indholdet af de "hårde" naturvidenskaber.

Der er derfor brug for at undersøge, om der faktisk er modsætninger mellem at forfølge de beskrevne almendannende hensigter og studie-/professionsforberedende hensigter. Afgørende for tilstedeværelsen af sådanne modsætninger er, om de midler der tages i brug for at leve op til hensigterne er uforenelige, af principielle eller praktiske grunde.

På forhånd og pr. tradition kunne man forvente eksistensen af sådanne modsætninger, fordi tilgodeseelsen af studieforberedende hensigter i matematikundervisning sædvanligvis har været knyttet sammen med udpegelsen af et bestemt fagligt stof, som aftagerne så har kunnet regne med som tilegnet af produktet. Hvor det for forfølgelsen af almendannende hensigter er nærliggende at benytte eksemplariske midler, har det for forfølgelsen af studieforberedende hensigter været sædvane at fastlægge undervisningen ved at udpege et velafgrænset kernestof:

almendannende	~ eksemplariske genstande
studieforberedende	~ velafgrænset kernestof

Imidlertid forekommer det mig for en nærmere betragtning, at der ikke er de store problemer i at forfølge studieforberedende hensigter med "eksemplariske midler". En første, og overordnet,grund hertil er, at der er et vist sammenfald mellem det der skal til for at forstå matematikkens rolle i samfundet på baggrund af 'inside insight' i matematikkens karakter, og det der skal til for at forberede sig på at deltage i udøvelsen af denne rolle. Men hermed er problemet selvfølgelig ikke løst.

Ser man nøjere på det velafgrænsede kernestof som aftagerne kan forvente indlært hos studenterne, indgik dette kernestof tidligere i en arbejdsdeling mellem på den ene side det matematisk-naturvidenskabelige gymnasium og på den anden side de naturvidenskabelige studier ved universiteterne og de teknisk-naturvidenskabelige ved Polyteknisk Læreanstalt. I princippet kunne arbejdsdelingen sagtens have ført til et andet stofvalg, men nu var det tilfældigvis funktionsteori, trigonometri og infinitesimalregning der udgjorde kernestoffet. Dette stof er så blevet stående som centralt i en periode, hvor aftagerne har mere differentierede behov end før, alene fordi der nu er så mange forskellige af dem. Selvfølgelig er det bekvemt for enhver aftager at vide nøjagtigt hvad der kan bygges på. Men hvad betyder det nærmere under nutidens forhold? Skal det velafgrænsede kernestof være

$$\bigcap_{p \in P} \text{det for uddannelse } p \text{ ønskelige stof,}$$

der vel nærmest er tom? Eller skal det være

$$\bigcup_{p \in P} \text{det for uddannelse } p \text{ ønskelig stof,}$$

der vel har det realistiske som en ægte delmængde?

Ingen af disse forstillinger kan danne basis for en strategi i dag. I alle de uddannelser, hvor matematik <u>bruges</u> (og ikke bare forudsættes), bruges der mere end man lærer i gymnasiet. Jeg tror at forestillingen om et studieforberedende kernestof har mere at gøre med aftagernes bekvemmelighed, med ønsket om at kunne starte et bestemt sted hvert år, end med noget fagligt dybtliggende. Langt vigtigere end et bestemt pensum er det at studenterne møder med nogle <u>repræsentative erfaringer</u> med forskellige former for matematik og med matematik i forskellige forbindelser, at disse erfaringer er solide og seriøse og indvundet ved selvstændigt arbejde. Evnen til at arbejde med matematik med gåpåmod, overblik og gennemslagskraft, evnen til at skille væsentligt fra uvæsentligt i faglige sammenhænge, er vigtigere end nok så grundige rutineerfaringer med et bestemt bekvemt pensum. Dertil kommer, at jo mere en videregående uddannelse indeholder af matematik, jo mindre afgørende er det, hvilke emner studenterne har været igennem i gymnasiet.

Alt i alt forekommer det mig, at videregående uddannelser i matematik og naturvidenskab ikke har noget væsentlig at frygte af, at studenterne ikke møder med et velafgrænset kernestof i baggagen. Men også på dette punkt forholder det sig som ovenfor sagt: Det forhold at man ikke udnævner et bestemt kernestof som kanonisk og obligatorisk betyder ikke, at der ikke i praksis vil opstå konturerne og elementerne af et sådant stof, derved at visse emner byder sig naturligt til i forbindelse med undervisningens overordnede genstande. Men farerne ved at fixere sådanne emner er de ovenfor beskrevne.

SLUTNING

Jeg vil slutte med nogle pessimistiske betragtninger.

Med den målbevidsthed, hvormed Danmark i disse år søger at bringe sig i spidsen for gruppen af afviklingslande, og med den negligering af uddannelsernes betydning i samfundet der florerer i mange kredse, kan man frygte, at der opstår en uhellig alliance mellem på den ene side underholdningspædagogikkens dominerende fokusering på elevernes umiddelbare lystfølelse og på den anden side det politiske system, om en nedskæring af "de hårde fags" omfang for majoriteten af eleverne i de gymnasiale uddannelser. Ganske vist spiller disse fag en rolle i samfundet, der er tiltagende i betydning, men med de centraliseringstendenser der findes, vil rollen blive varetaget af et snævert lag af stadig mere højtuddannet ekspertise. Hvorfor ikke nøjes med i gymnasiet at udsætte den elite, der siden skal varetage disse funktioner, for de hårde fag, men så til gengæld gøre det virkelig intensivt, og fritage majoriteten for pinslerne, som i øvrigt er vældigt kostbare at holde gående? Sådanne ræsonnementer er næppe helt utænkelige i det politiske system. Nogle kredse har altid ment noget i den retning, andre kan blive fristet til det i nedskæringspanik over den økonomiske krise. Lægger vi hertil, at folkeskolens regne- og matematikundervisning, som tidligere skulle forberede eleverne til med søvngænger-agtig sikkerhed at regne hurtigt og rigtigt i handels- og kontorfag, nu med computer- og lommeregner-revolutionerne har mistet denne objektive funktion, kan man frygte at erhvervslivet og det politiske system som helhed vil nå til det standpunkt, at der vist ikke er brug for en hel befolkning der kan regne mere end det nødtørftigste; det bliver den jo bare utilfreds og kritisk af. Det er min skrækvision, at sådanne for den demokratiske kontrol af samfundet dystre udsigter ikke er så utænkelige, som de trods alt i øjeblikket forekommer at være. Men hvis og når sådanne tilstande indtræder, er det for sent at anskue matematikundervisningen politisk. Det er nødvendigt at begynde nu.

Mogens Niss
oktober 1980

[1] Mogens Niss: "Goals as a reflection of the needs of society" (artikel nr. 1)

Denne artikel er manuskriptet til et foredrag holdt på to efteruddannelseskurser for matematiklærere på Magleås of Ry Kursuscenter i marts 1979. I den foreliggende form har artiklen været distribueret til matematiklærerforeningens medlemmer i rapporten fra de pågældende kurser "Rapport fra de fagdidaktiske kurser om videreudvikling af matematikundervisningen i lyset af U9o". I en let forkortet udgave har artiklen været publiceret under samme titel i Normat, 198o hæfte 2.

MOGENS NISS (Roskilde Universitetscenter, IMFUFA):

NOGLE PERSPEKTIVER FOR MATEMATIKUNDERVISNINGEN I DE GYMNASIALE UDDANNELSER i 199o.

Nu og da rejses der tvivl om berettigelsen af at folk uden egen forankring i gymnasieskolen gør sig kloge på dens problemer og forhold, hvad enten det sker generelt eller i forhold til enkeltfag. En sådan tvivl er ikke ny. I hele sidste århundrede var den lærde skole underlagt universitetets kontrol (undervisningsinspektionen udgjordes af universitetsprofessorer), hvilket af mange af datidens skolefolk med god ret blev opfattet som et tungt åg, et åg som først blev fjernet med gennemførelsen af almenskoleloven af 19o3.

Der kan efter min mening være megen rimelighed, også i dag, i en sådan tvivl om udenforståendes muligheder for at sige noget brugbart om i dette tilfælde matematikundervisningens placering og indretning. Når jeg desuagtet vover at forsøge mig i genren, er det af to sammenknyttede grunde.

Matematiklærerne i gymnasiet er, når de tager deres profession alvorligt, sådan belagt med arbejde med at få undervisningen til at fungere både efter forskrifterne og efter mere ideale fordringer, at de ikke har så megen tid som de gerne ville have, til sammenhængende at overveje og tage stilling til deres fags karakter, rolle og indretning i skolen. Udarbejdelsen af alternative materialer, gennemførelsen af forsøgsforløb og fagsamarbejde er i de senere år svulmet stærkt op. Dertil kommer, at ændringerne i gymnasiets samfundsmæssige opgaver og situation i disse år foregår med en sådan hast, at de overvejelser man kan nå at gøre sig ind imellem det daglige arbejde, hurtigt forældes. I dette perspektiv er der et arbejde at gøre for folk, typisk interesserede universitetslærere , der opfatter det som en naturlig del af deres arbejde at gennemføre og publicere forsøgsvise overvejelser på mere overordnede planer end det daglige arbejde i skolen giver lærerne rum for.

Imidlertid ville der også være behov for sådanne overvejelser fra udenforstående, selv om lærernes forpligtelser i højere grad end tilfældet er efterlod dem tid til at gøre dem. Jeg tror nemlig ikke, at man skal underkende værdien af udenforståendes mere distancerede iagttagelser, bedømmelser og forestillinger, ikke mindst i en tid der i så høj grad er præget af opbrud. Jeg taler ikke om objektivitet i disse iagttagelser, bedømmelser og forestillinger, men om de anderledes proportioner tingene får på lidt afstand, end de har når man står midt i dem.

Hertil skal lægges et mere tilfældigt,men ikke dermed uvigtigt, forhold ved lærernes mulighed for at give sig ind på overvejelser om skolens struktur. Sålænge en gymnasielærers eksistens som fuldtidsansat er endegyldigt afhængig af, om der kan skrabes et ugeskema på 22 timer sammen, er lysten til at realitetsoverveje strukturændringer, der kan få mere gennemgribende timetalskonsekvenser, og dermed måske konsekvenser i form af afskedigelser, for én selv eller kolleger, sikkert af

begrænset omfang.

Med et positivt svar på spørgsmålet om berettigelsen i at udenforstående gør sig kloge på matematikundervisningens problemer, er selvfølgelig ikke sagt, at disse udenforstående er velegnede til at give konkrete anvisninger. Det de kan bidrage med, er perspektiver og måske konkrete forslag på enkelte punkter, men den egentlige planlægning, stillingtagen til og gennemførelse af undervisningen kan kun foretages af den enkelte lærer over for den enkelte gruppe af elever.

Skulle jeg sammenfatte det sagte i et slagord, kunne det tage formen: ingen praksis uden filosoferen, ingen filosoferen uden korrektiver fra praksis.

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Rammerne for hvad der bliver sagt i det følgende, er en antagelse om, at i de kommende år vil 4o-6o%, måske flere, af en ungdomsårgang gennemgå gymnasiale uddannelser i 16-19 års alderen. Med gymnasiale uddannelser forstås her samtlige almene, ikke-erhvervsrettede uddannelser for det pågældende alderstrin, altså både gymnasium og HF og andre parallelle former som måtte opstå, selvstændigt eller integreret med de eksisterende, og der er ikke forudsat noget om grendeling eller niveaudeling af betydning for matematikundervisningen.

Overvejelserne vil være organiseret omkring besvarelsen af en række spørgsmål, der er afgørende for matematikundervisningens placering og udseende i de betragtede uddannelser.

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Skal der for alle elever på det gymnasiale niveau undervises i matematik i 199o?

Ja. Som det efterhånden er blevet fremhævet til kedsommelighed (bl.a. af mig), anvendes matematik til flere og flere formål, i flere og flere forbindelser og i flere og flere professioner, hvor der ikke tidligere benyttedes matematik. Der er dels tale om, at matematikken inden for en række fag inddrages som et hovedmiddel til at præcisere begreber, der er vigtige i faget, dels om at matematikken bidrager til at producere eller understøtte udsagn, resultater, i faget. Desuden er det tale om at matematikken benyttes til at skaffe baggrund for prognoser, planlægning, beslutningstagning og styring i offentlige og private sektorer, og endelig selvfølgelig om at den som altid benyttes i store dele af teknik og produktion, men med en større aktionsradius end før. Denne voksende involvering af matematikken får bl.a. til konsekvens, at langt flere personer end før i tiden kommer til at omgås matematik under forskellige former.

Denne stigende inddragelse af matematik i ikke-matematiske områder, skyldes ikke bare en aktuel modestrømning - selv om dette element også indgår her og der - men har sin basis i, at ma-

tematikken faktisk har noget at tilbyde af relevans for den ikke-matematiske omverden. Til gengæld har dette tilbud også sine grænser, men hvor de går kan ikke afgøres generelt.

Disse forhold begrunder i sig selv, at alle på det gymnasiale niveau skal undervises i matematik, men siger ikke så meget om de synsvinkler, hvorunder det skal ske. For at fastlægge dem betragtes tre farer, der spiller en rolle i forbindelse med den vidtstrakte anvendelse af matematik. Den første er faren for ekspertificering af matematikanvendelsen. Den opstår, hvis anvendelsen af matematik inden for de forskellige sagsområder ikke i hovedsagen varetages af sagsområdets egne folk, i stedet for at være overladt til eksperter i matematikanvendelse. I det omfang dette sidste sker, bliver matematikkens anvendelse mystificeret og uigennemskuelig for alle andre end end snæver kreds. Den anden fare er, at de ikke-matematikere der faktisk selv anvender matematik, kommer til at anvende matematikken receptagtigt og dermed ukritisk og uden blik for mulighederne og begrænsningerne i matematikanvendelsen, på grund af en for snæver og pigtrådsagtig indsigt. Den tredje fare er af en anden art, idet den ikke vedrører folk, der selv kommer til at anvende matematik, men almindelige mennesker, der på forskellige måder bliver "ofre" for matematikanvendelsen, enten som samfundsborgere berørt af almene politiske eller administrative tiltag eller argumenter i den offentlige debat, eller som berørt af foranstaltninger der har betydning for dem som privatpersoner.

Det er vigtigt at alle mennesker gennem deres skoleuddannelse sættes så meget ind i matematikkens indhold, anvendelse og funktion, at disse farer kan modvirkes effektivt, og som antydet er alle berørt af en eller flere af dem. En matematikundervisning, der på den ene side sætter folk i stand til på rimelig måde at udnytte matematiske betragtningsmåder, når situationen er til det, og som på den anden side sætter dem i stand til at tage kritisk stilling til andres anvendelse af dem, må tillægges stor alment demokratisk betydning, hvis man lægger vægt på borgernes selvstændige stillingtagen til dagliglivets og samfundslivets anliggender.

Ud over det anførte er der andre grunde til at undervise alle på det gymnasiale niveau i matematik, grunde der har at gøre med matematikken som orienteringsmiddel, f.eks. over for verdensbilledet (det fysiske rums egenskaber, bl.a.), og ideologi (det frigørende i at "se selv"). Disse grunde vil jeg her lade ligge, fordi de nærmest har karakter af sidegrunde til de først omtalte.

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Det problem der rejser sig med et bekræftende svar på det indledende spørgsmål, bliver så: under hvilke rammer og på hvilken måde kan der tilrettelægges en matematikundervisning, der tilgodeser det beskrevne hensyn? Dette spørgsmål består jo i virkeligheden af en række spørgsmål, hvor jeg først vil behandle formålet med undervisningen.

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Hvad skal være formålet for matematikundervisningen?

I lyset af de ovenfor refererede overvejelser vil jeg foreslå formålet med matematikundervisningen fastlagt således:

Formålet med matematikundervisningen i de gymnasiale uddannelser er

- at eleverne skal erhverve kendskab til og forståelse af matematikkens anvendelse, og baggrunden for den, i ikke-matematiske forbindelser.

  De skal herunder opnå forståelse af hvilke faktorer, såvel ved matematikkens tankegange, begreber og opbygning, som uden for matematikken selv, der spiller en rolle for denne anvendelse, dens muligheder og begrænsninger.

- at eleverne skal kunne foretage kritiske analyser og bedømmelser af gjorte anvendelser af matematik i ikke-matematiske forbindelser.

- at eleverne skal opnå erfaring med selvstændigt og på ikke-receptagtig måde at anvende matematik som middel ved behandlingen af ikke-matematiske problemstillinger.

Ser man på de formål der gennem tiderne har været sat for matematikundervisningen i gymnasiet, viser det sig, at matematikundervisningen meget ofte bl.a. har skullet tjene til at opfylde såkaldt formaldannende formål. Med formaldannende formål vil jeg her forstå sådanne formål, som sigter mod at udvikle generelle intellektuelle eller karaktermæssige evner hos personligheden, der når de først er udviklet kan bringes til udfoldelse i allehånde typer af situationer, uanset deres sagsindhold. Blandt de formale evner, det i tidens løb har været sat som matematikundervisningens opgave at opbygge, er først og fremmest "evnen til at tænke logisk og stringent", "fantasi og opfindsomhed" og "en klar og prægnant udtryksform". Jeg vil her begrunde hvorfor disse ingredienser ikke indgår i det tredelte formål, jeg her har sat op for matematikundervisningen på det gymnasiale niveau.

Nu er det vel temmelig uklart, hvad man skal mene med evnen til at tænke logisk og stringent. Der kan være tale om et spektrum af betydninger, fra evnen til at undgå formallogiske fejl såsom omvending af implikation, ombytning af kvantorer osv., til evnen til at tænke systematisk, struktureret og målrettet, med blik for væsentligt og uvæsentligt, materielle og formale aspekter, hovedspørgsmål og bispørgsmål m.m. I dagligdags sprogbrug er det vel nok især den sidste betydning, der tænkes på. Efter min opfattelse er der ikke belæg for at tro, at selv en succesfuld matematikundervisning udstyrer folk med denne evne; dertil kender jeg for mange modeksempler. At matematisk træning kan være en støtte, på linje med andre erfaringer, er nok tænkeligt, men denne faktor er i så fald næppe den dominerende. Derimod er der for mig ikke tvivl om, at en matematikundervisning, der er bare delvis lykkedes, giver folk opmærksomhed over for formallogiske brist i dagligdags argumentation. Til gengæld har netop disse sider

af dagligdags ræsonneren eller ræsonneren inden for andre områder kun marginal betydning for problemstillingen i langt de fleste tilfælde. I de fleste argumentationsforløb er de materielle aspekter dominerende.

Om matematikkens muligheder for at udvikle fantasi og opfindsomhed, er der vel især at sige, at en sådan opfattelse beror på en antagelse om, at der findes generelle kontekstubundne evner, der kan kaldes henholdsvis fantasi og opfindsomhed, og det tror jeg man bør tvivle på. En udviklet fantasi og opfindsomhed i forbindelse med matematisk virksomhed har ikke nødvendigvis noget at gøre med fantasi og opfindsomged i forbindelse med f.eks. politiske, kunstneriske, musikalske eller dagligdags anliggender. Det faktum, at mange, der haft succes med beskæftigelsen med matematik, også er levende og opfindsomme mennesker på andre områder, ganske som alle andre mennesker kan være det, kan ikke bruges til at slutte, at det er matematikbeskæftigelsen, der er årsagen. Mindst lige så rimeligt ville det være at antage, at der findes en række fælles årsager i tilknytning til almindelig intellektuel udvikling, interesse, åbenhed og gåpåmod, frembragt ved større komplekser af erfaringer.

Hvis man skal pege på en formaldannende effekt af matematikundervisningen, ville jeg være mere tilbøjelig til at hæfte mig ved, at matematikundervisningen, når den lykkes, kan bidrage til at producere en antiautoritær holdning af en bestemt art. Den der har lært at arbejde med matematik,har nogle førstehåndserfaringer om, at f.eks. lærerens påstand om hvad der gælder,kan kontrolleres, hans autoritet er forbundet med hans evne til at bevise det han siger. Matematikundervisningen kan (om den gør det er et andet spørgsmål) på denne måde producere tro på "egne øjne" og selvsikkerhed i god betydning. Men dette kan højst være en tendens, for også her gælder det, at sagsforhold som er forskellige fra dem, hvor træningen er udviklet, rummer masser af aspekter, hvor matematiske tænkeformer ikke kan gøres gældende.

Måske kan der også være tale om, at en vellykket matematikundervisning forsyner én med så meget uimponerethed og selvtillid, at man tør kaste sig ud i andre områder, efter devisen "det kan da ikke være så svært, i hvert fald ikke værre end matematik".

Under alle omstændigheder forudsætter sådanne formaldannende effekter, at matematikundervisningen er lykkedes til bunds. Og det gør den jo som bekendt kun for de meget få. For dem der ikke er på toppen, konverteres de omtalte effekter til de modsatte, ikke mindst hvis der blandt kammeraterne er en eller flere som behersker faget. For disse almindelige elever findes der næppe noget mere autoritært fag end matematik, og selvtillid må hentes et andet sted fra.

På baggrund af disse overvejelser, som i følge sagens natur må være temmelig spekulative, slutter jeg at matematikundervisningens muligheder for at realisere de traditionelle formaldannende formål er nok så usikre. Derfor finder jeg ikke, at der bør sættes sådanne formål for matematikundervisningen. Om man de

facto kan være heldig at tilgodese et eller flere af dem ved den konkrete undervisning ville jo være glimrende, men det må stå hen.

Efter min opfattelse må det ovenfor foreslåede tredelte formål være det centrale og eneste fornødne for en matematikundervisvisning, der rette sig mod alle elever på det gymnasiale niveau, hvad enten man forestiller sig dem niveau- eller grendelt eller ej.

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Hvad skal matematikundervisningens genstande være?

Det er vist temmelig usædvanligt at selve et fags tilstedeværelse og indretning systematisk gøres til objekt for undervisningen. Dette er efter min mening beklageligt. For det første bør det vel være et selvfølgeligt krav nu om dage, at man, når man underkaster mennesker en så stærk påvirkning som undervisning er, drøfter og argumenterer omkring dette forhold med dem. Jeg er ikke i tvivl om, at noget sådant allerede foregår i en vis udstrækning mange steder, men i lyset af det i øvrigt pressede program kommer det nok mest til udfoldelse i mere afslappede stunder og ikke på en systematisk måde.

Jeg forestiller mig ikke, at eleverne møder op med eller har chance for at få udviklet en fuldt moden opfattelse af indretningen af matematikundervisningen og baggrunden for dens eksistens. Men jeg mener, at det at bringe dette anliggende frem i dagens lys til artikuleret behandling,er et særdeles brugbart middel til efterfølgelse af det opstillede formål. Jo flere lag og nuancer der indgår i elevernes matematikopfattelse, jo mere reflekteret, sidevindsstabil og brugbar for dem selv bliver den.

Af disse grunde, og for i øvrigt at vænne lærerne til at argumentere med kortene på bordet og offensivt frem for defensivt om deres fag,mener jeg, at matematikundervisningens tilstedeværelse i skolen, dens indretning, rammer og baggrund, bør gøres til eksplicit genstand for behandling, og det på forskellige trin, efterhånden som elevernes almindelige og faglige udvikling lægger op til det.

Altså:

<u>Genstand nr.1 Matematikundervisningens tilstedeværelse i skolen, dens indretning, rammer og baggrund.</u>

Ud fra det tredelte formål for matematikundervisningen, jeg har foreslået ovenfor, giver det sig selv, at en central undervisningsgenstand må være matematikkens muligheder,og begrænsningerne i disse, for at bidrage til at formulere, beskrive, forklare eller løse problemstillinger uden for matematikken selv. Eftersom kontakten mellem matematik og "virkelighed", hvorved jeg her forstår alt hvad der ligger uden for matematikken selv, kommer i stand via <u>matematiske modeller</u>, vil jeg som den næste undervisningsgenstand anføre:

<u>Genstand nr. 2 Matematiske modeller og modelbygning for konkrete</u>

ikke-matematiske problemstillinger, herunder samspillet mellem disses egenskaber og modellens egenskaber og mulighederne, og begrænsningerne i disse, for ud fra modellen at udsige noget om virkeligheden.

Jeg har i anden sammenhæng udpeget nogle for mig at se fundamentale faktorer bag matematikkens gennemslagskraft over for behandlingen af ikke-matematiske problemstillinger. Jeg har i den forbindelse fundet, at matematikkens deduktive opbygning indgår som et afgørende moment i denne gennemslagskraft. Dette moment må derfor tages i betragtning af en matematikundervisning, der skal tilgodese de ovenfor opstillede formål. Og dette kan ikke ske på rimelig måde, hvis de matematiske begreber og resultater fremstår og formidles som isolerede enkeltbrokker uden et vist minimum af teoretisk sammenhæng. Denne sammenhæng må derfor, selv om den ikke behøver at være "global" men kan nøjes med at være "regional", gøres til en af matematikundervisningens genstande. At gøre matematikkens sammenhæng, dens arkitektur, til undervisningsgenstand er ikke det samme som at gøre den til genstand for indlæringsekser-cits; hvor og i hvilken udstrækning dette sidste skal være tilfældet, må bero på konkrete undervisningsbeslutninger, truffet bl.a. ud fra en bedømmelse af de konkret foreliggende randbetingelser vedrørende elevgruppen og dens erfaringer fra andre sammenhænge. Under alle omstændigheder er det i praksis ikke muligt at studere opbygningen af matematikken som helhed, hvis formålet skal tilgodeses i sin bredde, man må lade sig nøje med eksemplarisk behandling af enkelte områder.

Altså:

Genstand nr. 3 Matematikkens begreber, metoder og opbygning undersøgt med henblik på fagets sammenhæng.

I hovedparten af al matematikundervisning formidles matematikken som et udviklingsløst, færdigt fag, løsrevet fra tid og rum og samfund, bestående af et sæt begreber, resultater og metoder. Dette bestemmer et matematikbillede uden fylde i form af bindinger til kulturelle, samfundsmæssige og historiske omgivelser, matematikken bliver "flad" og statisk. De fleste af dem der beskæftiger sig professionelt med matematik ved, på et højere eller lavere artikulationsniveau, at et sådant billede er meget fortegnet. Matematikken har en historie, ikke bare en indre begrebs- og teorihistorie, men også en ydre historie om dens skiftende institutionelle, økonomiske, praktiske og sociale forankringer. Eller m.a.o., matematikken har gennemløbet og gennemløber stadig forskellige former for udvikling. Gøres disse udviklinger ikke til genstand for undervisning induceres et forvrænget matematikbillede; ikke at jeg mener, at der eksisterer et færdigt, korrekt matematikbillede som skal serveres, men en bortskæring af disse aspekter reducerer matematikken til en én- eller to-dimensional projektion af den flerdimensionale organisme, den faktisk er.

Ud fra disse betragtninger vil jeg som det sidste undervisningsobjekt sætte:

Genstand nr. 4 Matematikkens udvikling og samfundsmæssige placering anskuet under historiske synsvinkler.

Der kan måske være behov for at gøre det klart, at den rækkefølge hvori genstandene optræder, ikke skal angive en rækkefølge for undervisningen i dem. Jeg mener faktisk ikke at undervisningen skal behandle disse genstande som indbyrdes afgrænsede afrundede genstande, betragtet i en eller anden rækkefølge. Der er i stedet tale om genstande, som skal indgå i de sammenhænge der i øvrigt måtte blive programsat. Heller ikke en prioritering mellem genstandene er der tale om med den angivne rækkefølge, ligesom jeg heller ikke har gjort mig forestillinger om det relative omfang af dem. I korthed er der tale om noget parallelt med, og delvis sammenfaldende med, de knudepunkter, som udgør den ene dimension af matematikuddannelsen ved RUC. Dette er i øvrigt ikke nogen tilfældighed.

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Både det formål og de genstande der her er foreslået, er i følge sagens natur uden klare konturer eller grænser: taget bogstaveligt udtrykker de et ubegrænset ambitionsniveau. Meningen er selvfølgelig heller ikke at de skal tages bogstaveligt (men nok alvorligt). De skal snarere opfattes som pejlepunkter, der kan give mening og udgøre centrum for arbejdet på i princippet ethvert aktivitets- og fordybelsesniveau.

Jeg finder altså såvel formål som genstande brugbare for enhver gymnasial matematikundervisning, uanset hvilke strukturelle forhold, med eller uden grene osv., skolen måtte være underlagt.

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I det foregående har jeg forsøgt at undgå at udtrykke mig på en måde, som forudsætter eller tager stilling til om matematikken skal indgå i den gymnasiale undervisning som et selvstændigt fag, dvs. som et eller flere velafgrænsede tidsrum, hvad enten det er som et antal timer pr. uge eller sammenhængende længere perioder, afsat alene til beskæftigelse med matematik, eller om i stedet matematikken alene skal forefindes som bragt i anvendelse ad hoc over for situationer uden for matematikkens selv. De konklusioner jeg ovenfor har draget i forhold til de behandlede spørgsmål er i princippet ikke anfægtet af svaret på dette spørgsmål, hvorimod det omvendte er tilfældet.

Skal matematik i fremtiden eksistere som et selvstændigt fag i den gymnasiale skole?

Lad mig straks udløse spændingen ved at svare ja til dette spørgsmål. Inden jeg begrunder dette svar (det fordrer faktisk en begrundelse, traditionen er ikke svar nok), vil jeg skynde mig at slå fast, at jeg også mener, at matematikken skal bringes i anvendelse ad hoc i integrerede problemsitua-

tioner uden for matematikkens og matematikundervisningens rammer. Mit svar indebærer alene, at der skal eksistere visse afgrænsede tidsrum afsat alene til matematik, ikke at al matematikundervisning skal foregå på denne måde.

Min begrundelse for at der skal undervises i matematik som et selvstændigt fag, ligger allerede i høj grad i de overvejelser, der ovenfor er gjort over de foreslåede genstande for matematikundervisningen. Således kan matematikkens opbygning, anskuet globalt eller regionalt, ikke på meningsfuld måde behandles ad hoc i tilknytning til arbejdet med integrerede, ikke-matematiske problemstillinger. En fokusering på denne genstand forudsætter, at der reserveres tidsrum hvor opbygningen af teorien kan finde sted og studeres, og dermed står vi allerede med et selvstændigt fag. Noget tilsvarende gælder i en vis udstrækning for genstand nr. 1 og i høj grad for genstand nr. 4. Fælles for de tre genstande 1,2 og 4 er jo, at de vedrører forskellige sider af matematikkens som manifestation, i forskellige af dens relationer til omverdenen.

Jeg mener alstå ikke, at matematikundervisningens formål (som her foreslået fastlagt) og de derudfra afstukne undervisningsgenstande,kan tilgodeses, med mindre matematikken eksisterer som et selvstændigt fag. Hvis ikke tyren tages ved hornene, vil matematikken, dens virkemidler, funktioner og samfundsmæssige rolle fortabe sig i uklarhed og mystifikationer, med fortsat usikkerhed, benovelse eller blind skepsis over for faget hos størsteparten af befolkningen til følge.

Et andet argument er,at hvis matematik kun skal eksistere som et anvendelsesfag, bragt i spil ad hoc, vil det antagelig slet ikke komme i spil særlig ofte. Først og fremmest fordi eleverne, hvis de ikke har modtaget undervisning i faget, næppe af egen drift vil overveje, om matematiske betragtningsmåder kunne bidrage til behandlingen af de problemstillinger, de arbejder med. Dernæst fordi erfaringerne viser, at i projektorganiseret arbejde med komplekse problemstillinger er de aspekter, hvor en matematisk behandling giver mening, kun sjældent dem der har mest umiddelbar interesse for eleverne, i hvert fald hvis inddragelsen af matematik ikke af andre grunde er forudsat.

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Andetsteds har jeg behandlet nogle af de midler i form af organisation, metoder og andet, der kan komme på tale ved gennemførelsen af en matematikundervisning med de her anlagte perspektiver. I den forbindelse har jeg advokeret for problemorienteret projektarbejde, i en skala med forskellige grader af radikalitet, som et velegnet instrument. Jeg skal ikke komme yderligere ind på disse anliggender her, men i stedet vende mig mod matematikundervisnigens aktuelle situation og udviklingstendenser for at behandle spørgsmålet

Går det i den rigtige retning (defineret som den jeg har afstukket ovenfor)?

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Den aktuelle situation

Matematikundervisningens aktuelle situation på det gymnasiale niveau er præget af relevansparadokset. Relevansparadokset opstår ved modstillingen mellem på den ene side matematikkens voksende samfundsmæssige betydning og den deraf følgende nødvendighed af at så mange som muligt får adgang til at kikke matematikken i kortene, hvilket altså gør en matematikundervisning objektivt relevant, jvf. også de foregående udviklinger, og på den anden side det forhold, at flere og flere elever i de gymnasiale uddannelser er frustrerede over en matematikundervisning, der forekommer dem irrelevant, uden forbindelse med og betydning for den verden de færdes i og interesserer sig for. Årsagerne til disse frustrationer ligger især i to grupper af forhold. For det første i gymnasiets ændrede karakter. Gymnasiet har ændret sig fra at være et elitegymnasium til at være en masseuddannelsesskole (incl. HF); matematikundervisningen gives ikke længere kun til elever, for hvem gymnasiet er en forgård til videregående studier i naturvidenskab eller teknik. Hertil skal lægges de almene antiautoritære strømninger, elevernes større indflydelse på undervisningen og aktive involvering i planlægningen, deres krav om relevans af og begrundelse for det de lærer osv. Det andet forhold er det, at den gymnasiale matematikundervisning i høj grad befinder sig i ingenmandsland, mellem kvalificeret regning, som i det store hele er overstået med folkeskolen, og seriøse, konsekvensrige anvendelser, som stort set ikke kan gennemføres med det stofindhold, der faktisk forefindes i de gymnasiale uddannelsers matematikundervisning. Ser vi f.eks. på differential- og integralregningen, tjener de først og fremmest til at muliggøre begrebslig repræsentation og præcisering af fænomener og begreber fra fysikken med tilgrænsende områder, dvs. til at give sådanne fænomener og begreber klarhed. Derimod kan differential- og integralregningen kun i begrænset omfang bruges til at forklare noget ved, eller til at handle ved hjælp af. Kun ved ekstremumsproblemer og areal- og rumfangsbestemmelser, eller hvis grænserne overskrides til differentialligninger, kan infinitesimalregningen bidrage til forklaringer eller handlingsanvisninger.

Lægger vi hertil at matematikundervisningen sætter eleverne i stand til at løse simple algebraiske ligninger, til at repræsentere forskellige fænomener i et koordinatsystem og til at finde sandsynligheder for begivenheder knyttet til trækning af kugler fra urner eller til kast med terninger, empiriske middelværdier og varianser af talmaterialer, er anvendelseshorisonten vist indkredset,

Hårdt sagt kan man frygte, at kun de elever der kan lære på forventet efterbevilget relevans eller som kan fascineres af de begrebslige eller strukturelle aspekter ved faget, får i realiteten noget ud af undervisningen.

Den gymnasiale matematikundervisnings placering i det nævnte ingenmandsland er ikke af ny dato. Når den er blevet et prob-

lem nu, skyldes det de gymnasiale uddannelsers ændrede karakter.

En ikke ringe del af matematiklærerne i de gymnasiale uddannelser synes at være enige i, at relevansparadokset forefindes og er et stort problem for matematikundervisningens dagligdag og perspektiver. I stigende grad er de begyndt at lade lærebogstilegnelse træde i baggrunden til fordel for arbejde med utraditionelle materialer, indgåelse af fagsamarbejde eller -integration, gennemførelse af projektorganiserede forløb, arbejde med konkrete modeller for ikke-matematiske fænomener. Forsøg der i større eller mindre grad overskrider de generelle rammer, florerer vist som aldrig før. Der sker altså noget, eller sagt på grundtvigiansk: "der er grøde". Jeg røber næppe nogen hemmelighed ved at udtrykke min sympati for og tilslutning til alle disse bestræbelser på at finde brugbare svar på problemerne, og det sidste ord er ikke sagt endnu. Alligevel kan jeg ikke , selv om jeg er en udenforstående iaggtager, lade være med at nære en vis bekymring for nogle mulige farer i udviklingen, som jeg på forskellige steder synes at kunne spore. Jeg tænker her på faren for, at der i de nødvendige bestræbelser for at bringe matematikken i kontakt med omverdenen, sker en negligering af de træk af matematikken, dens opbygning (genstand nr. 3), som overhovedet gør en sådan kontakt meningsfuld og frugtbar. Negligeringen har sine nærliggende grunde i de omtalte frustrationer hos eleverne og deres krav om undervisningens synlige relevans, men den er også udtryk for en defensiv strategi over for matematikundervisningens problemer. Overses matematikkens opbygning og sammenhæng, og forsynes eleverne kun med recepter og plausibilitetsargumenter, bliver deres bevæbning med indsigt, forståelse og kritisk analysekapacitet kun tilsyneladende. I realiteten vil deres udstyr på mange punkter blot blive et tryllekunstnersæt, fortrinsvis til egen illusionering. Og det holder efter min opfattelse ikke i længden. For det første fordi det ikke gør det muligt at tilgodese det formål, jeg her har opstillet. For det andet fordi det kan komme til at gå i spænd med nogle ildevarslende videnskabsfjendske og populistiske strømninger i tiden, med krav om umiddelbar brugbarhed af det lærte til erhvervslivets kortsigtede formål. Og dette er strømninger og krav, som man ikke bare finder i Glistrups og Erhard Jacobsens partier.

Endelig er der fare for, at en negligering af matematikkens opbygningsmæssige aspekter vil gøre matematikundervisningen svag over for et back-lash i konservativ retning, med krav om tilbagevenden til en ren og streng disciplintilegnelse og færdighedsindøvelse hos dem der kan hænge på, mens fanden kan tage resten.

Lad mig understrege, at vi efter min mening <u>ikke</u> er kommet derhen hvor det går galt, langtfra, men vi kan komme det, hvis vi betragter tidens glade budskaber, hvor jeg selv er leveringsdygtig i en velassorteret gaveæske, som mirakelkure. Sådanne findes ikke på disse gebeter.

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Afslutningsvis vil jeg fremhæve et par punkter, hvor der må

ske ændringer, hvis de perspektiver for fremtidens matematikundervisning, som jeg her har skitseret, skal blive til virkelighed.

- Matematikkens gennemslagskraft over for behandlingen af seriøse ikke-matematiske problemstillinger må dokumenteres for eleverne, ikke bare postuleres.

- Mangfoldigheden i de fremtidige skæbner for eleverne i de gymnasiale uddannelser sætter spørgsmålstegn ved betimeligheden i et for alle fælles indhold og et kanonisk pensum. Hvad der der behov for er ikke en vidtdreven grendeling af eleverne, heller ikke i matematikundervisningen. Matematikundervisningen må være eksemplarisk, med genstandene som de faste holdepunkter snarere end matematiske områder i sædvanlig forstand. Vægten må lægges på kvaliteten af arbejdet med genstandene, snarere end på hvor langt der nås i pensumtermer.

En matematikundervisning, der skal leve op til centralt fastsatte standarder i form af krav til indhold og eksamensfærdigheder kan ikke realisere de nævnte hensigter. Det først fornødne er måske derfor at få disse centralt fastsatte krav beskåret, ikke for at svække det ømfindtlige "niveau" i matematikundervisningen, men tværtimod for at give det den styrkelse der følger med et bortfald af åndenøden.

Mogens Niss
27.3.1979

CONSIDERATIONS AND EXPERIENCES CONCERNING INTEGRATED COURSES IN MATEMATICS AND OTHER SUBJECTS

Mogens Niss
associate professor of mathematics
IMFUFA, Roskilde University Centre
Denmark

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Denne artikel er manuskriptet til et indlæg, jeg som panelist gav i sessionen "How effective are integrated courses in mathematics and science for the teaching of mathematics?" på Fourth International Congress of Mathematical Education, Berkeley, California, 1o.-16.8.198o. Manuskriptet ventes at udkomme i proceedings fra kongressen i løbet af et par år.

The increasing interest within the community of people involved in mathematics education in proposing and discussing courses where mathematics is taught integratedly with other subjects, seems to stem primarily from two different types of concern, which may be phrased as follows:

(1) Mathematics teaching does not function too well as it has developed. It is probably necessary to recognize, even if only with uneasiness, that an increasing number of students do not feel sufficiently motivated to work with abstract mathematical topics or constructed, simplified application examples. Is it possible to restore students' motivation for the study of mathematics as a subject by demonstrating its role in tackling genuine, extra-mathematical problems, for instance by letting it be taught in integration with one or more other subjects, probably most adequately science subjects?

To this concern the idea of integrated courses represents a (new) strategy for pursuing the ultimate end: success for the teaching of mathematics as a subject in itself.

(2) Mathematics teaching does not function too well as it has developed. One major reason for this is that it displays mathematics as being a purpose in itself, as a formal game, too far removed from the rest of the world. We should admit that the teaching of mathematics cannot serve as its own end. An important ingredient in the rationale for teaching mathematics is its role as a powerful tool in tackling problems outside mathematics itself. And this aspect has not gained a position which corresponds to its importance, a point which is also reflected in the often reported fact that students have major difficulties in applying the mathematics they have been taught, to "real life situations" at and after school. Is it possible to improve this state of affairs by letting mathematics be taught in direct contact with subjects for which it is a fundamental tool?

To this concern the ultimate interest in mathematics teaching is based on the view that mathematics is valuable in a broader context than just as a subject. This concern aims at improving the understanding and utilization of mathematics in extra-mathematical situations. The idea of integrated courses represents a (new) strategy to this end.

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How are these concerns present as regards our session? Its title is "How effective are Integrated Courses in Mathematics and Science for the Teaching of Mathematics?" This formulation seems to presuppose implicitly that the final goal of our endeavours is success for the teaching of mathematics as a subject in itself. According to this interpretation this session is devoted, then,

to discussing to what extent integrated courses provide a useful means for pursuing this goal, how the integration should be carried out if the case should arise, and what problems are likely to be encountered in the implementation of it. This suggests that we are dealing with the first-mentioned concern.

The subtitle asks the question: "How to tackle the dilemma between the need for preparing children/students to deal with useful mathematics in natural situations, and the need for teaching them mathematics as a systematic structure." It takes the need of teaching students mathematics as a systematic structure for granted but introduces an additional need, with which the first might be in potential conflict and with which it hence should be balanced: the need of preparing students to use mathematical tools in coping with natural situations. This brings into the discussion elements of the second-mentioned concern. So, we are supposed not only to treat the theme of our session under the perspective of the first concern but also under the perspective of the second one.

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In my view we should, however, not continue without having examined a little bit the purpose of giving mathematics education to large groups of children and students, since any answer to the questions of our session is bound to reflect somehow a basic conception of the purpose of mathematics education. If this purpose were taken for granted wihtout further comment, the discussion might end up with talking at cross-purposes.

I agree with those who reject that mathematics education should be given for its own sake. (Moreover, I consider it over-optimistic to believe societies to accept "its own sake" as a sufficient argument for assigning to mathematics education the dominant position it actually possesses in almost any country. (For further comments on this matter, cfr. [1])) The reason for giving mathematics education to a majority of children and students at every level must refer to regards outside mathematics itself.

Having not enough space to go into a detailed analysis of these

issues, I shall restrict myself to pointing out what to me is the main justification for giving extensive mathematics education to everybody.

Mathematics plays a central and increasing role in the function and development of fundamental aspects of our societies. It is of democratic importance, to the individual as well as to society at large, that any citizen is provided with instruments for understanding, judging and utilizing this role of mathematics. Anyone not in possession of such instruments becomes a "victim" of societal processes in which mathematics is a component. So, the purpose of mathematics education should be

to enable students to realize, understand, judge, utilize, and sometimes also perform, the application of mathematics in society, in particular to situations which are of significance to their private, social and professional lives.

Twogoals can be derived from this purpose:

(a) students should acquire understanding of those factors within mathematics (such as ideas, concepts, edifices of theory, methods etc.), as well as those outside mathematics, which are of importance to the applicability of mathematics, its potentials and limitations;

(b) students should themselves acquire experiences with applying, independently and in a non-mechanical manner, mathematics in treating extra-mathematical situations.

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Let us, on the background of these premises, rephrase the question of our session as follows: Are integrated courses in mathematics and science the right vehicle for the pursuit of these aims?

My answer to this question is "yes" and "no": integrated courses are necessary but not sufficient. The arguments for this answer are partly based on concrete experiences in Denmark with integrated courses on different levels, partly on theoretical considerations of a more principal nature.

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So far, we have not tried to define what to mean by "integrated courses in mathematics and other subjects". At this stage a few

comments on terminology might be adequate.

By an integrated course in mathematics and other subjects (one or more) we shall understand an educational course in which the non-mathematical subjects occupy a salient position and are taken seriously, i.e. are not serving as only an excuse for doing mathematics, or as a source of illustrations to it, but their integrity and particular requirements are respected.

Using this definition, we can distinguish between two principally different types of integrated courses:

(1) integrated courses where the main interests lie with the non-mathematical subjects and where mathematics is a service subject introduced only when and only to the extent that it serves these main interests.

(2) integrated courses intending to serve also purposes paying particular regard to aspects of mathematics.

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At the Natural Science Basic Education Programme at Roskilde University Centre (my own university) in Denmark students are engaged in half of the time in two years with performing problem-oriented projects on complex situations, which are not from the outset defined by subject categories but in which mathematics and/or different natural sciences become involved under various perspectives, according to the situation. These projects may be considered as (extreme) instances of integrated courses of the first kind.

One marked experience which we have gained from these projects, is that students do not, in general, feel much inclined to introduce mathematical considerations, unless this was explicitly planned from the very beginning. In projects where mathematics or matters closely related to it do not form a determined part of the problem in focus of attention, students tend to avoid mathematics, and even if they don't the involvement tends to be narrowly directed towards exploiting methods or results as recipes, without much regard being paid to their justification, let alone their embedding in a systematic theoretical context.

If, on the other hand, we are talking about projects, where the applicational role of mathematics is part of the initial problem, for instance through explicit intentions to construct or examine a mathematical model, things are markedly different. Students usually treat the issue of the project,and the mathematics to be learnt inconnection with it,with a maturity, competence and sense of proportions, which are not often seen with students who learn mathematics and applications seperately.

Experiences similar to those gained from the Natural Science Basic Education Programme are reported from various experimental courses of the same character but of a far smaller scale, at upper secondary level (16-19 years) in Denmark.

In the programme leading to a final degree in mathematics at Roskilde University Centre (a programme presupposing and continuing the basic educational programme mentioned above) all activities have, of course, mathematics as a subject (considered in a broad context) in focus of attention.Within a complicated organisational framework, which I shall abstain from describing here (for a reference, cfr. [2]), courses where mathematics is integrated with other subjects from part of the programme. Also in this programme we have obtained encouraging experiences with projects where mathematical models in biology, economics and physics are constructed or investigated, encouraging also as regards the learning of the associated mathematics, which proceeds faster and with stronger motivation than is usually seen with a similar mathematical content. However, there seems to be a tendency to a weakened long-term fastening of the mathematics learnt in this way, unless it is supported by additional"fastening activities", like traditional exercise sessions.

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The experiences reported show that if mathematics is left alone to integrated courses of the first kind it tends to reach an only marginal position, and a position which is insufficient for a successful pursuit of the purposes for mathematics education for which I am advocating. In contradistinction, inte-

grated courses of the second kind seem to be most valuable in this respect. Moreover, it is difficult to imagine how students could acquire the sort of understanding and personal experiences with mathematics in extra-mathematical situations, required by the goals stated, if not from integrated courses (in the sense here defined) forming part of their mathematical education.

This poses a new question: Couldn't we content ourselves with giving mathematics education alone in the form of integrated courses (of the second kind)? My reasons for answering "no" to this question are the following:

The capacity and power of mathematics in treating problems of "the real world" is not an illusion. Mathematics indeed offers remarkably powerful instruments for understanding and treating a large variety of complex situations outside mathematics itself. Any mathematics education pursuing the goals put forward here must aim at understanding the origin and character of this capacity. It is probably well justified to claim that a determining factor in this capacity are the abstract, deductive structures forming the edifice of mathematics. Therefore this edifice as such should be an object of study in mathematics teaching. This is why the dilemma mentioned in the subtitle of our session is a genuine and not an artificial one. It is necessary for preparing children/students properly to deal with useful mathematics in natural situations that they obtain insight in and knowledge about mathematics as a systematic structure. Such insight and knowledge do not automatically result from integrated courses. Actually our experiences suggest that this is not the case. But balanced with courses on mathematics as a systematic structure they constitute a most valuable, and in my view even indispensable, component of any mathematics teaching aiming at giving not only mathematics instruction but mathematics education.

[1] Mogens Niss: Goals as a reflection of the needs of society
To appear in "Studies in Mathematics Education II", Unesco

[2] Mogens Niss: The 'crisis' in mathematics instruction and a new teacher education at grammar school level
Int.J.Math.Educ.Sci.Technol., 1977, vol 8, No.3(303-321)

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